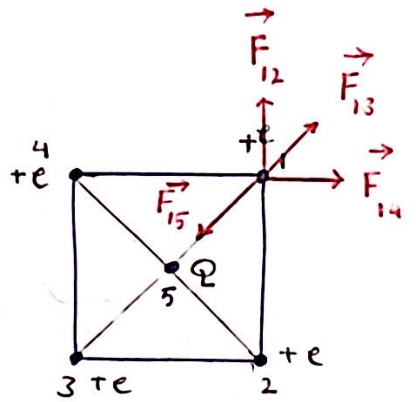
سؤال ۱



$$\left|\vec F_{12} + \vec F_{14} + \vec F_{13}\right| =$$

$$\frac{2e^2}{4\pi\varepsilon_0 a^2}\cos\frac{\pi}{4} + \frac{e^2}{4\pi\varepsilon_0(\sqrt{2}a)^2}$$

$$= \frac{2e^2}{4\pi\varepsilon_0 a^2}\,\frac{\sqrt{2}}{2} + \frac{e^2}{8\pi\varepsilon_0 a^2} = \frac{e^2}{4\pi\varepsilon_0 a^2}\left(\sqrt{2} + \frac{1}{2}\right)$$

جهت این نیرو در راستای $\vec F_{13}$ است .

$$\left|\vec F_{15}\right| = \frac{Qe}{4\pi\varepsilon_0\left(\frac{\sqrt{2}}{2}a\right)^2} = \frac{Qe}{2\pi\varepsilon_0 a^2}$$

جهت این نیرو باید خلاف $\vec F_{13}$ باشد بنابراین Q بار منفی است .

$$\frac{Qe}{2\pi\varepsilon_0 a^2} = \frac{-e^2}{4\pi\varepsilon_0 a^2}\left(\sqrt{2} + \frac{1}{2}\right) \rightarrow \boxed{\begin{aligned} Q &= -\left(\frac{\sqrt{2}}{2} + \frac{1}{4}\right)e \\ &\simeq -0.96e \end{aligned}}$$

②

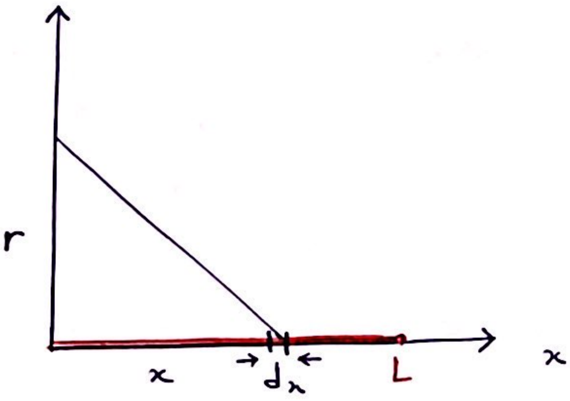
$$U = \sum_{i<j}\frac{q_i q_j}{4\pi\varepsilon_0 r_{ij}} = 4\left(\frac{e^2}{4\pi\varepsilon_0 a}\right) + 2\left(\frac{e^2}{4\pi\varepsilon_0(\sqrt{2}a)}\right)$$

$$+ 4\left(\frac{Qe}{4\pi\varepsilon_0\left(\frac{\sqrt{2}}{2}a\right)}\right)$$

$$= \frac{e^2}{4\pi\varepsilon_0 a}\left[4 + \sqrt{2} - (4 + \sqrt{2})\right] = 0 \rightarrow \boxed{U_{\text{کل}} = 0}$$

②

سؤال 2



$$dq = \lambda \, dx = \alpha x \, dx$$

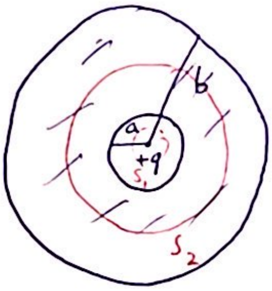
$$dV_p = \frac{dq}{4\pi\varepsilon_0 (x^2+r^2)^{1/2}} = \frac{\alpha x \, dx}{4\pi\varepsilon_0 (x^2+r^2)^{1/2}}$$

$$\rightarrow \quad V_p = \frac{\alpha}{4\pi\varepsilon_0} \int_0^L \frac{x \, dx}{(x^2+r^2)^{1/2}} = \frac{\alpha}{4\pi\varepsilon_0} \sqrt{x^2+r^2} \Big|_0^L$$

$$\rightarrow \quad V_p = \frac{\alpha}{4\pi\varepsilon_0} \left(\sqrt{L^2+r^2} - r \right)$$ ③

$$E_y^p = -\frac{\partial V_p}{\partial r} = \frac{\alpha}{4\pi\varepsilon_0} \left(1 - \frac{r}{\sqrt{L^2+r^2}} \right)$$ ①

سؤال 3



$r<a$

$$\Phi = \oint \vec{E}\cdot\hat{n}\,ds = 4\pi r^2 E = \frac{q_{enc}}{\varepsilon_0} = \frac{q}{\varepsilon_0}$$

$$\rightarrow \quad E = \frac{q}{4\pi\varepsilon_0 r^2} \qquad ①$$

$a<r<b$

$$\Phi = \oint \vec{E}\cdot\hat{n}\,ds = 4\pi r^2 E = \frac{q_{enc}}{\varepsilon_0}$$

$$= \frac{1}{\varepsilon_0}\left[q + \int_a^r \frac{A}{r'}\,(4\pi r'^2 dr) \right]$$

$$= \frac{1}{\varepsilon_0}\left(q + 4\pi A \int_a^r r' dr \right)$$

$$= \frac{1}{\varepsilon_0}\left[q + 2\pi A(r^2 - a^2) \right]$$

$$\rightarrow E = \frac{q}{4\pi\varepsilon_0 r^2} + \frac{A}{2\varepsilon_0}\left(1 - \frac{a^2}{r^2}\right). \qquad ③$$

سوال 4

$$\vec{E} = -\frac{\partial V}{\partial x}\hat{x} \rightarrow \vec{E} = \begin{cases} 0 & ; \quad 0 < x < 1 \\ -3\hat{x} & ; \quad 1 < x < 2 \\ 0 & ; \quad 2 < x < 3 \\ +2\hat{x} & ; \quad 3 < x < 4 \\ 0 & ; \quad 4 < x < 5 \end{cases}$$

2

نیروی وارد بر الکترون به شکل $\vec{F} = -e\vec{E}$ خواهد بود:

$$\vec{F} = \begin{cases} 3e\,\hat{x} & ; \quad 1 < x < 2 \\ -2e\,\hat{x} & ; \quad 3 < x < 4 \\ 0 & ; \quad \text{بقیه نقاط} \end{cases}$$

1

$$v_2^2 - \cancelto{0}{v_1^2} = 2a(x_2 - x_1) \rightarrow v_2^2 = 2\frac{F}{m}(2-1) = 6\frac{e}{m}$$

$$v_3 = v_2$$

$$v_4^2 - v_3^2 = 2a'(x_4 - x_3) \rightarrow v_4^2 = v_3^2 - 4\frac{e}{m} = (6-4)\frac{e}{m} = \frac{2e}{m}$$

$$\boxed{v_5 = v_4 = \left(\frac{2e}{m}\right)^{\frac{1}{2}}}$$

~~$\frac{e}{m} = 1.8\times10^{11}$~~

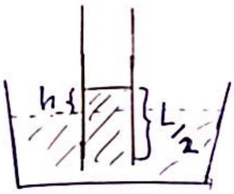
1

~~$v_5 \simeq \sqrt{1.8\times10^{11}} \simeq 4.2\times10^{5}$~~

همچنین از جهت پایستگی انرژی نیز می توان به دست آورد

$$-eV_1 = -eV_5 + \frac{1}{2}mv_5^2 \rightarrow v_5 = \sqrt{\frac{2e(V_5 - V_1)}{m}} = \sqrt{\frac{2e}{m}}$$

سؤال 5

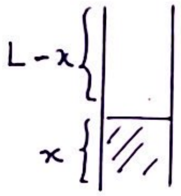


الف)

$$C = \varepsilon_0 \frac{L(\frac{L}{2})}{d} + \varepsilon \frac{L(\frac{L}{2})}{d} = (\varepsilon_0 + \varepsilon)\frac{L^2}{2d} \quad ①$$

$$= (K+1)\varepsilon_0 \frac{L^2}{2d}$$

ب) اگر مایع به اندازه x آرش ε وارد فضای بین صفحات خازن شده باشد



$$C = \varepsilon_0 \frac{L(L-x)}{d} + \varepsilon \frac{Lx}{d}$$

$$= \varepsilon_0 \frac{L^2}{d} + \frac{(\varepsilon - \varepsilon_0)Lx}{d}$$

$$U = \frac{Q^2}{2C} \rightarrow F_x = -\left.\frac{\partial U}{\partial x}\right|_{x=\frac{L}{2}} = -\frac{Q^2}{2}\frac{\partial}{\partial x}\left[\frac{d}{\varepsilon_0 L^2 + (\varepsilon - \varepsilon_0)Lx}\right]\Bigg|_{x=\frac{L}{2}}$$

$$= \frac{2dQ^2}{L^3}\frac{\varepsilon - \varepsilon_0}{(\varepsilon + \varepsilon_0)^2} = \frac{2dQ^2}{L^3\varepsilon_0}\frac{(K-1)}{(K+1)^2} \quad ②$$

$$F_x = mg = \rho g h L d$$

در حالت تعادل

$$\rightarrow \begin{cases} h = \dfrac{2Q^2}{\rho g L^4 \varepsilon_0}\dfrac{K-1}{(K+1)^2} \\ Q = C_0 V = \varepsilon_0 \dfrac{L^2}{d} V \end{cases} \rightarrow \boxed{h = \frac{2V^2}{\rho g d^2}\frac{\varepsilon_0 (K-1)}{(K+1)^2}} \quad ①$$

ظرفیت قبل از فرو بردن
خازن در مایع دی الکتریک

[illegible]

$$\boxed{\cancel{h = \frac{2V^2}{\rho g d^2}\frac{\varepsilon_0 (K-1)}{(K+1)^2}}}$$