This series of homework is based on the fifth chapter of Introduction to solid state physics by Charles Kittel. Before answering the problems, make sure to study the related materials thoroughly from the two primary references. You may think and work on problems with your classmates, but **you must submit the answers you only write**; otherwise, you will receive no points. The last question is for bonus points.

Problem 1

(a) Consider a dielectric crystal made up of layers of atoms, with rigid coupling between layers so that the motion of the atoms is restricted to the plane of the layer. Show that the phonon heat capacity in the Debye approximation in the low temperature limit is proportional to T^2.

(b) Suppose instead, as in many layer structures, that adjacent layers are very weakly bound to each other. What form would you expect the phonon heat capacity to approach at extremely low temperatures?

Problem 2

The crystal basis of graphene and of diamond is composed of two carbon atoms in nonequivalent position. Thus, their dispersion curves are composed of acoustical branches and optical branches, and their acoustical branches are assumed to obey to the Debye approximation: $w = v_S|k|$ and their optical branches are assumed to obey to the Einstein model $w = w_E$ = Cst. Deduce the numerical values of their Debye and Einstein temperatures from their common sound velocity v_s=18,000 m/s with also v_E (Einstein frequency) at about 4×10^{13} Hz.

Problem 3

Kittel: chapter 5, Problem5.

Problem 4

Simon: 2.1, 2.7

Bonus Problem

In a simple model, superconductivity is modeled as made up of two paired electrons obeying Bose-Einstein statistics. The superconductor; therefore, consists of two fluids, one formed by normal free electrons with density n_e , and the other formed by such pairs of density n_p.

1. In the normal state, but at very low temperatures(we neglect the dependence of the Fermi energy E_F and assume $T= 0$ k), find the expression for the density of free electrons n_e as a function of $E_F(N)$
2. In the superconducting state, the Fermi energy is slightly lower $E_F(S) = E_F(N)(1-\delta)$ where $0<\delta<<1$. What becomes the expression of n_0 for n_e? Deduce the expression for the density of pairs as a function of n_0 and δ.
3. In an independent evaluation (based on Bose statistics), we find that the density of pairs is $n_p = 2.6\left(\frac{m k_B T_c}{\pi \hbar^2}\right)^{(3/2)}$ where m is the mass of electrons and T_c the critical temperature. Numerically find n_e and np for the case of lead where T_c = 7.2 K and $E_F(N)$ = 6 eV. Deduce the numerical value of δ.