Problem 1

Use the equation $m(dv/dt + v/\tau) = -eE$ for the electron drift velocity v to show that the conductivity at frequency ω is :

$$\sigma(\omega) = \sigma(0)\left(\frac{1 + i\omega\tau}{1 + (\omega\tau)^2}\right)$$

Show what $\sigma(0)$ is.

Problem 2

Find an expression for the imaginary part of the wavevector in the energy gap boundary of the first Brillouin zone, in the approximation that to the Eq.48 in chapter 7 of Kittel. Give the result for the imaginary part of wave vector at the center of the energy gap. Find the expression for the square of imaginary part of the wavevector.

Problem 3

(a) Simon 4.4
(b) Simon 15.1

Problem 4

We consider an alkali metal to be a set of N fixed ions (per unit volume) and N free electrons (one free electron per atom) of mass m and immersed in a vacuum ϵ_0.

A. This electron gas is submitted to an alternating electric field of frequency ω with form $E = E_0 e^{i\omega t}$. Find the equation of motion for each electron knowing that there exists a damping force that is proportional to their velocity v and of the form$-\dfrac{m}{\tau}\vec{v}$, τ is the relaxation time. Find the steady state solution.

B. Express the conduction current density corresponding to the motion of N electrons per unit volume N? Taking into account the displacement current density (which is that of the vacuum), find the expression for the total current density.

C. Show that alkali metals can be characterized electrically either by the complex relative dielectric constant $\tilde{\epsilon}_r(\omega) = \epsilon_1(\omega) - i\epsilon_2(\omega)$ or by a complex electrical conductivity $\tilde{\sigma}(\omega) = \sigma_1(\omega) + i\sigma_2(\omega)$. Simplify the expressions obtained using $\omega_2^p = Ne^2/m\epsilon_0$.

D. Numerical application for potassium: Specify the value of the plasma frequency ω_p as well as the corresponding energy (in eV). Mass density (K)=870 kg/m^3, atomic mass= 39.1, and τ= 2.64×10^{-14} s.

E. Sketch the evolution of $\epsilon_1(\omega)$and $\epsilon_2(\omega)$. Specify values when $\omega = 0$ and $\omega = \infty$. Give the value of ω at which $\epsilon_1 = 0$.

F. When initially mono-energetic electrons ($E^0 \approx 20 - 50$keV) are transmitted through a thin metal film, they excite the collective plasma oscillations of the free elections in the film via the longitudinal electric field they induce. Thus their energy loss is proportional to $\epsilon_2/(\epsilon_1^2 + \epsilon_2^2)$. Indicate the characteristics of this energy loss by specifying the position of its maximum ΔE and its full width at half maximum $\Delta E(1/2)$. What are the numerical values of ΔE, $\Delta E(1/2)$ and E_F for potassium? (To simplify the calculations take into account that $\omega_p \tau \gg 1$).

G. The electric field is now transverse field of an electromagnetic plane sine wave linearly polarized in x and propagating from origin O in the positive z-direction. Determine the equation of the electromagnetic wave propagation in the plasma. Solve it using $\bar{E}_T$ its complex aptitude in z=0 (where $\mu = \mu_0$). Show graphically the dispersion relation $\omega = f(k)$ for $\omega > \omega_p$ assuming that the relaxation time is infinite. Find the expression for the phase velocity of the wave as a function of ω, and ω_p. What is the nature of the wave when $\omega \leq \omega_p$?

H. Continuing in the hypothesis that $\tau = \infty$, find the expression for the magnetic excitation associated with the previous electric field as function of ϵ_r and next of ω_p and w. What is the wave impedance Z_p in the plasma?

I. In fact the metal only occupies the half-space of positive z and the wave calculated above is only the part of the incident monochromatic linearly polarized electromagnetic wave that propagates in a vacuum ($\epsilon_0, \mu_0, z < 0$) and is partially reflected at the $z = 0$ plane. After recalling the expressions of incident waves($\bar{E}_i$ and $\bar{H}_i$: complex amplitudes) and the reflected waves ($\bar{E}_r$, $\bar{H}_r$), determine, from boundary conditions on the reflection coefficient $r = \bar{E}_r/\bar{E}_i$ as a function of Z_0, Z_p and the next as a function of $\tilde{\epsilon}_r$, and ϵ_0

J. Show the variation of the ration R between the reflected and incident intensity ($R = r \,.\, r^x$) as a function of ω after having first calculated the ration ω/ω_p for which $R = 1/m$. Carry out a numerical calculation using m=16. Indicate the frequency domain where a total reflection is obtained and the frequency domain where the wave can be fully transmitted. Determine for potassium numerical value of the wavelength λ_0 of the incident electromagnetic wave in a vacuum that marks the boundary between these two regions.

K. Show that the results obtained in I, J could be deduced immediately from the expression for the coefficient of optical reflection (in magnitude) at normal incidence: $r = \dfrac{1-\tilde{N}}{1+\tilde{N}}$, where the plasma may be described by a complex optical index $\tilde{N} = n - ik$ that is a function of ϵ_1 and ϵ_2.

Problem 5 (Bonus)

(a) Kittel Chapter 8, problem 1
(b) Kittel Chapter 8, problem 3