Quantum Correlations and their measures

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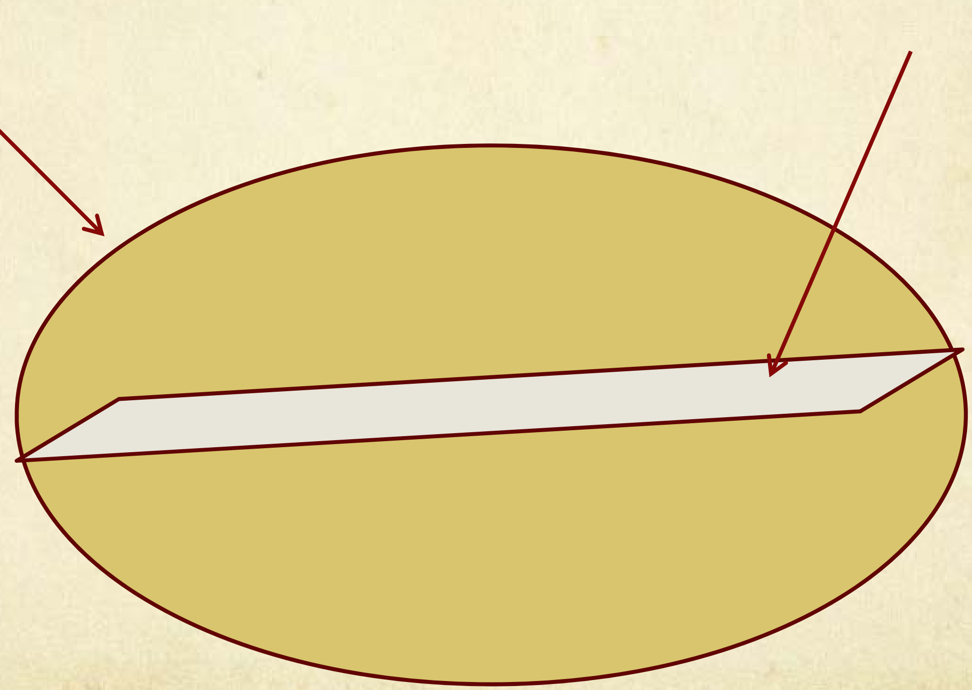
Entanglement of pure states

$$|\psi\rangle = a|00\rangle + b|01\rangle + c|10\rangle + d|11\rangle$$

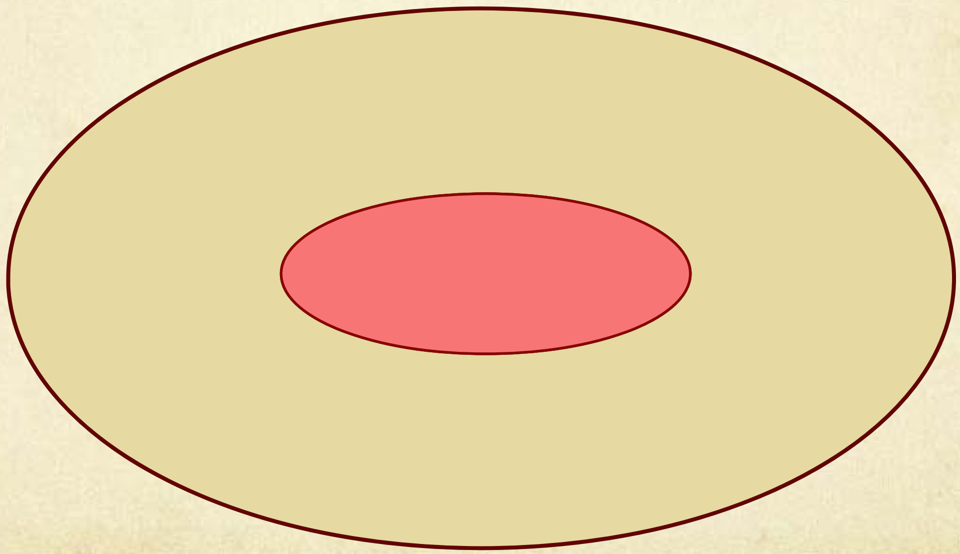
$$ad - bc = 0$$



Measure Zero, but dense somewhere

Separable states

$$\rho = \sum_i p_i \ \sigma_i \otimes \sigma'_i$$



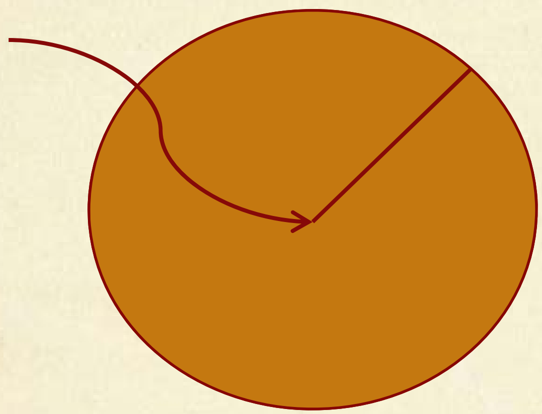
The intuitive reason

$$\rho^{T_A} \geq 0$$

Peres, (1998)
Horodecki's, (1998).

The constructive reason

$$\frac{1}{4}I$$



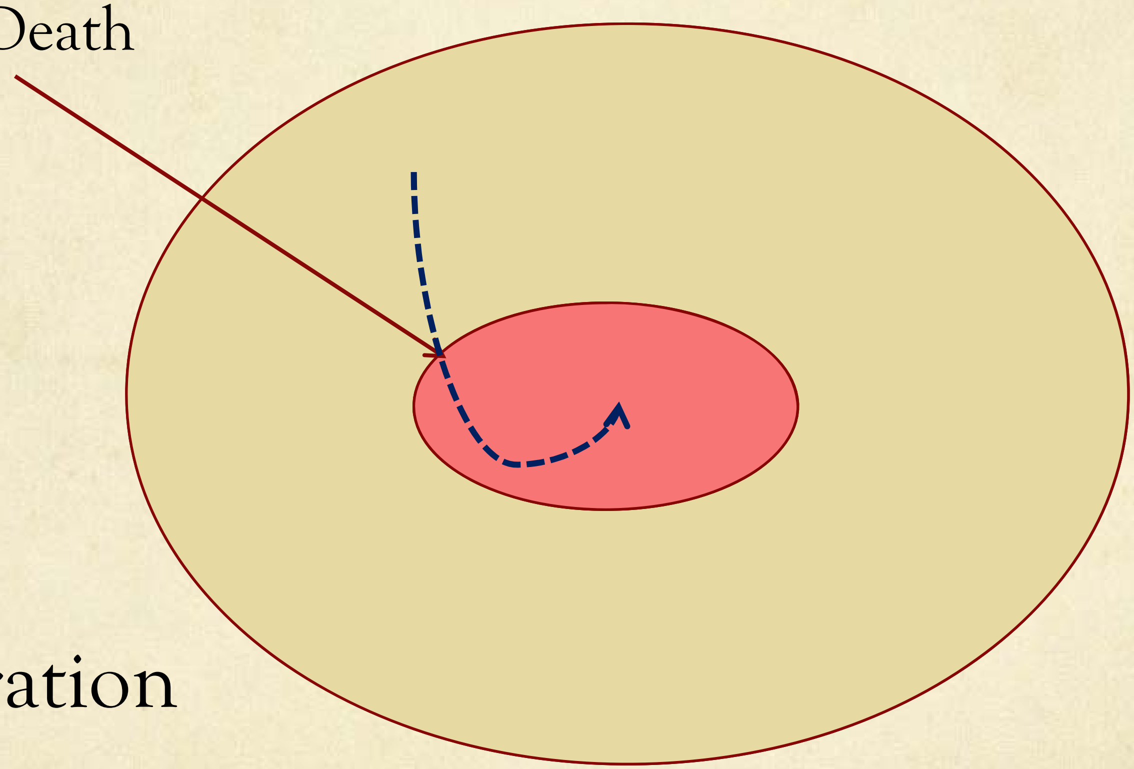
$$\varepsilon \leq \frac{1}{15}$$

$$\rho = (1-\varepsilon)\frac{I}{2^n} + \varepsilon\ \rho_1$$

Braunstein, Caves, Jozsa, Linden, Popescu, and R. Schack (1998)

Implications

Entanglement
Sudden Death



Preparation

Robustness

What is Quantum Discord?

Correlations between classical probability distributions

$P_a(i)$

$P_{a,b}(i,j)$

$P_b(j)$

$P_b(j|0)$

$P_b(j|1)$

$P_b(j|2)$

$P_b(j|n)$

$$H\left(P_b\right) - \sum_i P_a(i) H(P_{b|i})$$

$$CC := H\left(P_b\right) - \sum_i P_a(i) H(P_{b|i})$$

$$= H(A) + H(B) - H(A,B)$$

Total Correlation!?

ρ_a

ρ_{ab}

ρ_b

$$TC := S(\rho_a) + S(\rho_b) - S(\rho_{ab})$$

An Example

$$\left|\varphi_{ab}\right\rangle = \frac{1}{\sqrt{2}}\left(\left|00\right\rangle + \left|11\right\rangle\right)$$

$$TC = 1 + 1 - 0 = 2$$

Another example

$$\rho_{ab} = \frac{1}{2}\left(|00\rangle\langle 00| + |11\rangle\langle 11|\right)$$

$$TC = 1 + 1 - 1 = 1$$

Operational Meaning

I

σ_z

$$\left|\phi^{+}\right\rangle = \frac{1}{\sqrt{2}}\left(\left|00\right\rangle + \left|11\right\rangle\right)$$

$$\left|\phi^{-}\right\rangle = \frac{1}{\sqrt{2}}\left(\left|00\right\rangle - \left|11\right\rangle\right)$$

$$\rho_{ab} = \frac{1}{2}\left(\left|00\right\rangle\left\langle 00\right| + \left|11\right\rangle\left\langle 11\right|\right)$$

$$\rho_{ab} = \frac{1}{2}\left(|00\rangle\langle 00| + |11\rangle\langle 11|\right)$$

$$I$$

$$\sigma_x$$

$$\rho_{ab} = \frac{1}{2}\left(|00\rangle\langle 00| + |11\rangle\langle 11|\right)$$

$$\rho'_{ab} = \frac{1}{2}\left(|10\rangle\langle 10| + |01\rangle\langle 01|\right)$$

$$\rho_{ab} \rightarrow \frac{1}{2} I \otimes \frac{1}{2} I$$

Groisman, Popescu, and Winter (2005)

Reminder

$P_a(i)$

$P_{a,b}(i, j)$

$P_b(j)$

$$H\left(P_b\right) - \sum_i P_a(i) H(P_{b|i})$$

$P_b(j \mid 0)$

$P_b(j \mid 1)$

$P_b(j \mid 2)$

$P_b(j \mid n)$

A new measure of Classical Correlation

$P_a(i)$

$P_{a,b}(i, j)$

$P_b(j)$

$$S(\rho_b) - \sum_i P_a(i) S\left[\rho_b(i)\right]$$

$$CC := \max\left\{ S(\rho_b) - \sum_i P_a(i) S\left[\rho_b(i)\right] \right\}$$

$\rho_b(0)$

$\rho_b(1)$

$\rho_b(2)$

$\rho_b(n)$

Ollivier and Zurek (2001)
Henderson and Vedral (2001)

Properties of CC

$$CC(\rho_a \otimes \rho_b) = 0$$

$$CC\left[(U \otimes V)\rho(U^* \otimes V^*) \right] = 0$$

$$CC\left(|\psi\rangle\right) = E(|\psi\rangle)$$

CC should not increase by local actions

Quantum Correlation?

Or Quantum Discord

$$QC \coloneqq TC - CC$$

Discord

$QC \neq$ Entanglement

For Pure States

$$TC(|\psi\rangle) = 2S(\rho_a)$$

$$CC(|\psi\rangle) = S(\rho_a)$$

$$QC(|\psi\rangle) = S(\rho_a)$$

Examples of zero-discord states

$$\rho_{ab} = \frac{1}{2}\left(|00\rangle\langle 00| + |11\rangle\langle 11|\right)$$

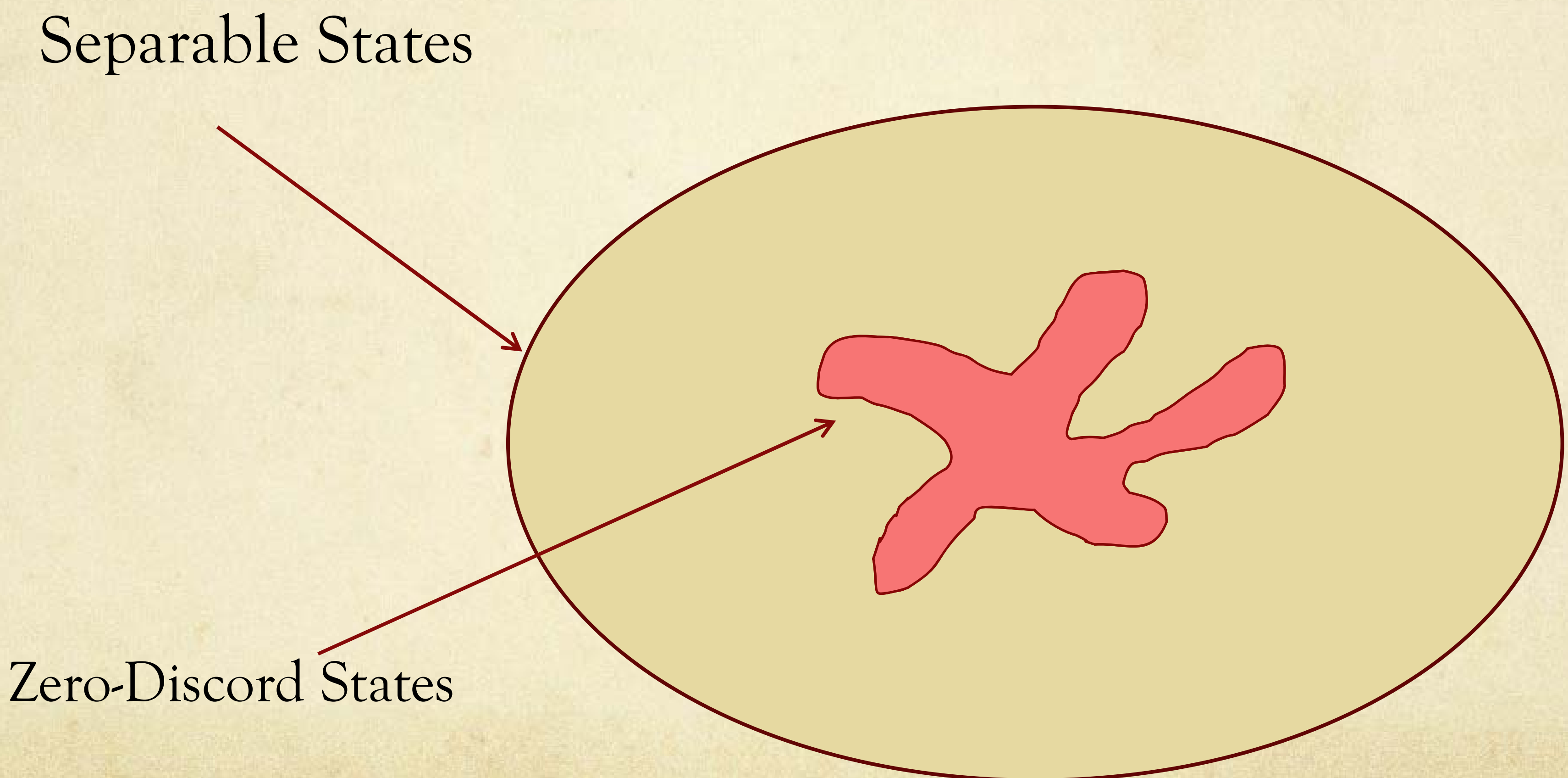
$$D\left(\rho_{ab}\right) = 0$$

$$\rho_{ab} = \sum_i p_i \, |i\rangle\langle i| \otimes \rho_i$$

Examples of non-zero discord states

$$\rho = \frac{1}{2}\left(|00\rangle\langle 00| + |+1\rangle\langle +1| \right)$$

How many zero-discord states?



Ferraro, Aolita, Cavalcanti, Cucchietti, and Acın (2010)

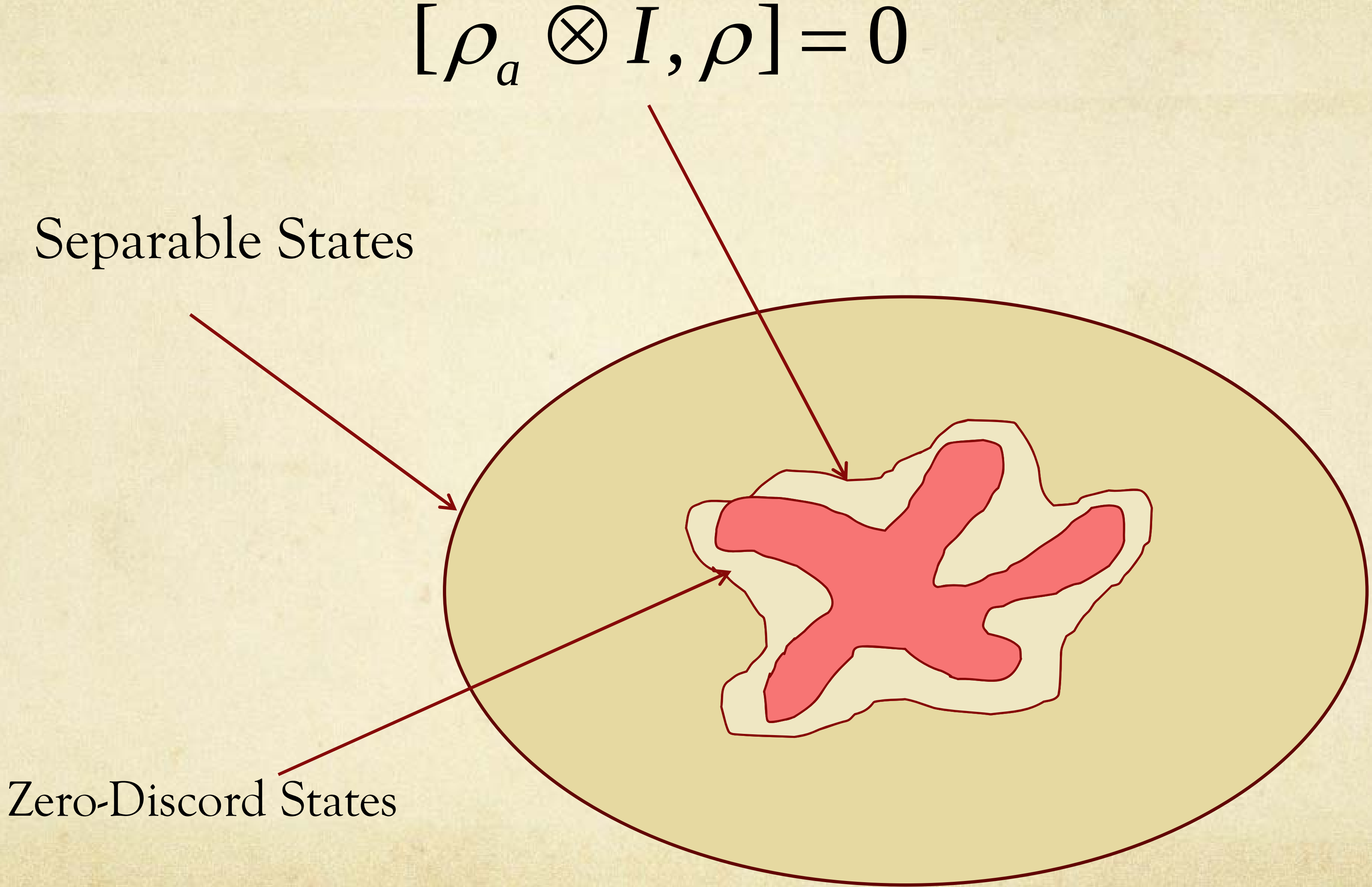
Almost all states have quantum correaltion

$$D_A(\rho) = 0 \quad \Leftrightarrow \quad \rho = \sum_i p_i \pi_i \otimes \sigma_i$$

$$\rho_A = \sum_i p_i \pi_i$$

$$\rho_A \otimes I = \sum_i p_i \pi_i \otimes I$$

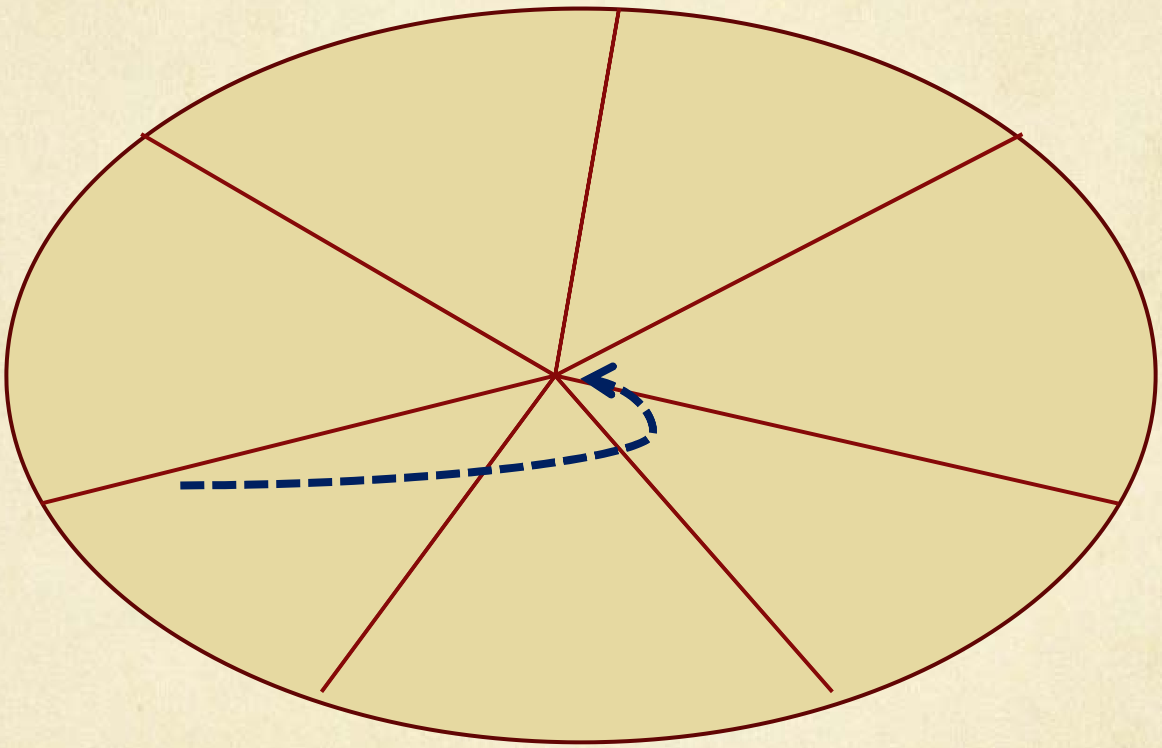
$$[\rho_A \otimes I, \rho] = 0$$


$[\rho_a \otimes I, \rho] = 0$
Separable States
Zero-Discord States

Zero-Discord States

- Measure zero
- No-where Dense

An intuitive picture



Implications

Computable Measures

- Based on Distance
- Based on Fidelity
- Based on Local Uncertainty

Geometric measure

$$D(\rho) = \min D(\rho, \sigma_{cc})$$

$$D\left(\rho, \sigma\right) = \| \rho - \sigma \|_2$$

$$\rho = \frac{1}{4}\left(I + r.\sigma \otimes I + I \otimes s.\sigma + t_{ij}\sigma_i \otimes \sigma_j\right)$$

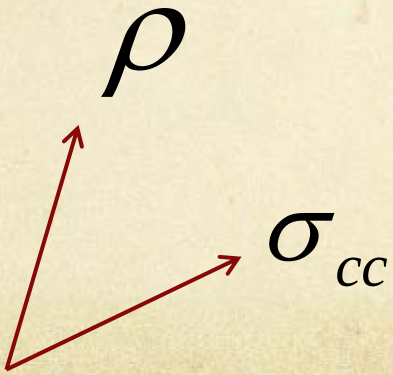
$$D = \frac{1}{4}(\lambda_1(M) + \lambda_2(M))$$

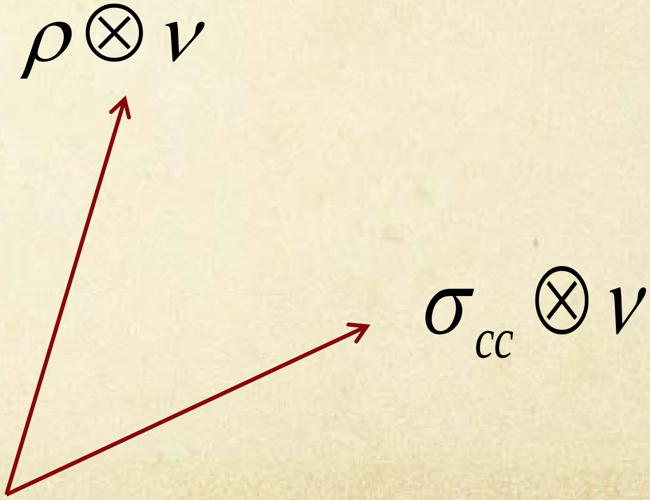
$$M = rr^T + TT^T$$

Dakic, Vedral, and Breuckner (2010)

The problem with this measure

$$\rho \rightarrow \rho \otimes \nu$$





Piani (2012?)

Based on Fidelity

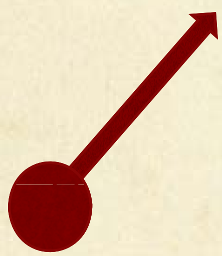
$$D\left(\rho\right) = 1 - \max F(\rho, \sigma_{cc})$$

$$F(\rho, \sigma_{cc}) = Tr(\sqrt{\sqrt{\rho}\sigma_{cc}\sqrt{\rho}})$$

Bruss, Kampermann and Stretslov (2012)

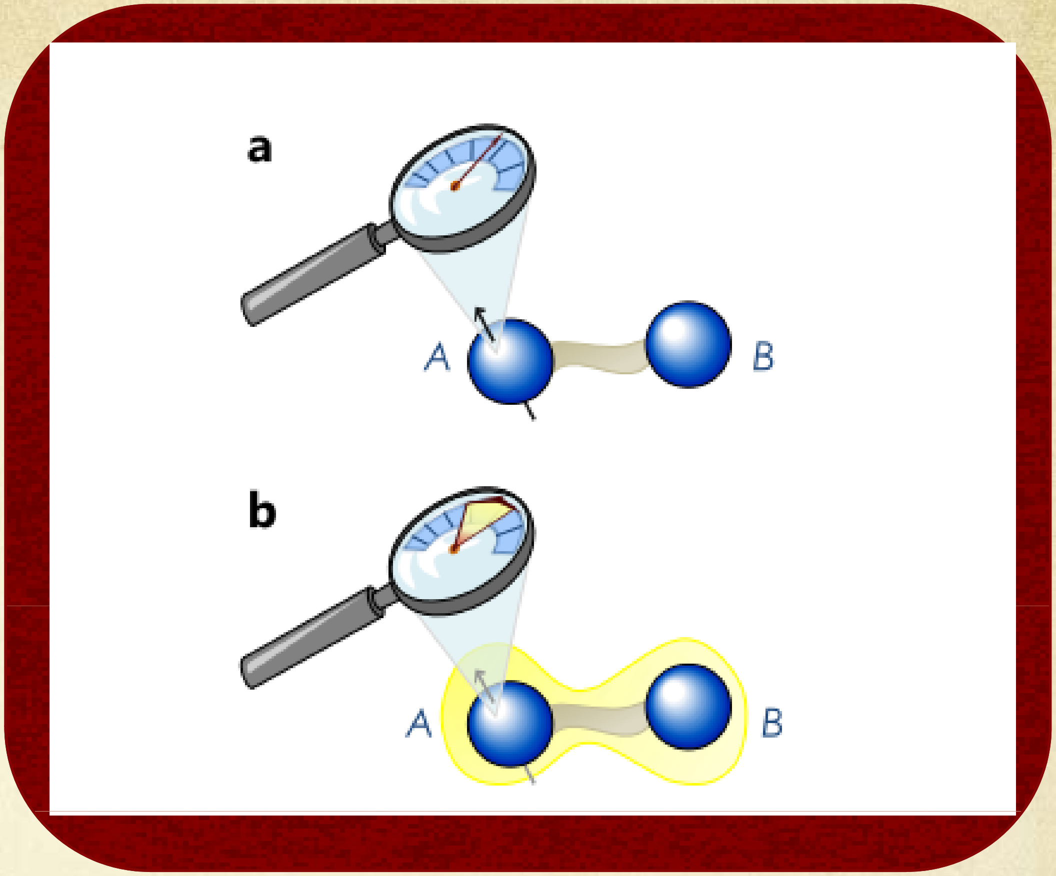
Abad, Karimipour and Memarzadeh (2012)

A radical view
Local Quantum Uncertainty
(LQU)

$|n\rangle$  $\sigma . n$ No uncertainty

Girolami, Tufarelli, and Adesso(2013)

$$|\varphi_{ab}\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$



Girolami, Tufarelli, and Adesso(2013)

$$\rho_{ab} = \sum_i p_i \ |i\rangle\langle i| \otimes \rho_i$$

$$\rho_{ab} = \sum_i p_i \ \sigma_i \otimes \rho_i$$

$$LQU_A(\rho_{AB}) = 1 - \lambda_{\max}\{W_{AB}\}$$

$$(W_{AB})_{ij} = Tr\left[\sqrt{\rho_{AB}}(\sigma_i \otimes I)\sqrt{\rho_{AB}}(\sigma_j \otimes I)\right]$$

Thank you for your attention

Local Uncertainty

$$I(\rho, K) = -\frac{1}{2} Tr\{[\rho^{\frac{1}{2}}, K]\}$$

$$I(\rho, K) \geq 0$$

$$I(\rho, K) \leq \left\langle K^2 \right\rangle_\rho - \left\langle K \right\rangle_\rho^{\ 2}$$