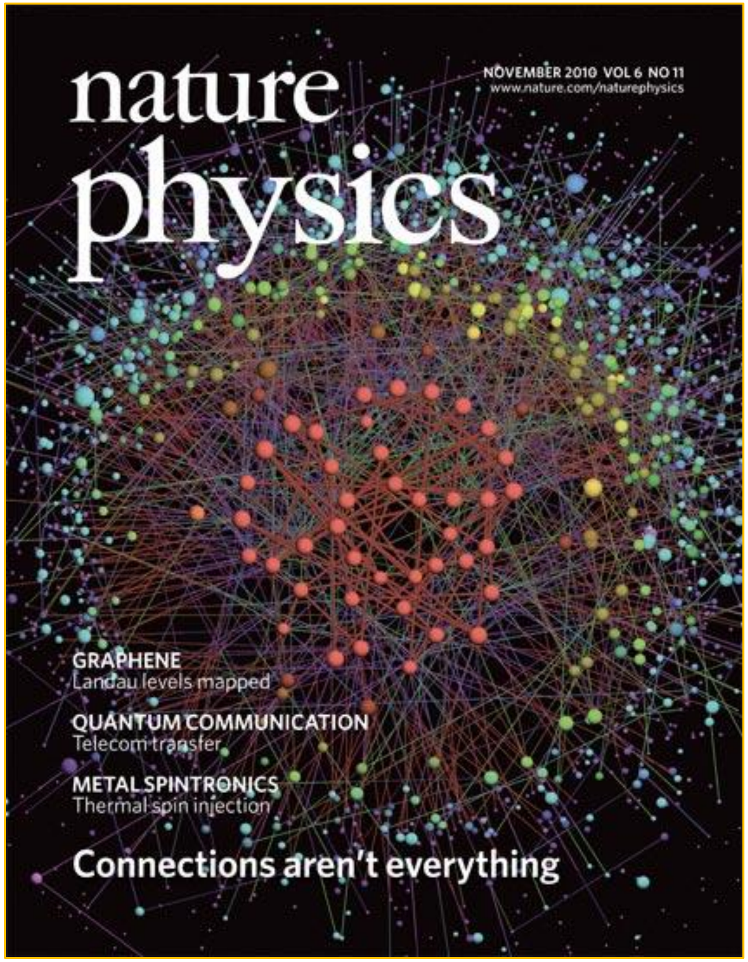


Quantum Random Networks

Zahra Raissi

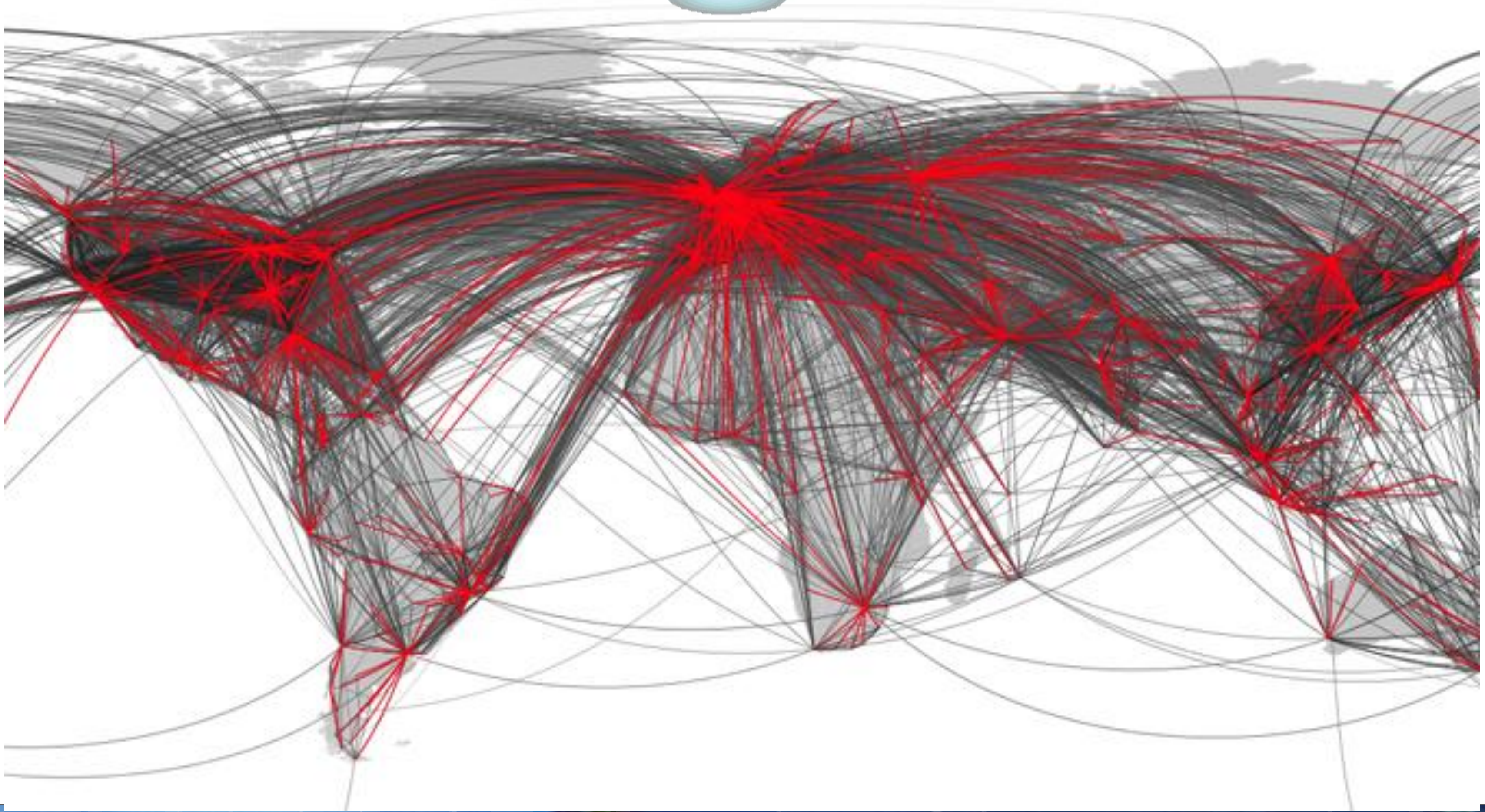
Based on:

- *S. Perseguers, M. Lewenstein, A. Acin, J. I. Cirac "Quantum random networks"*, Nature Physics 6, 539 - 543 (2010)

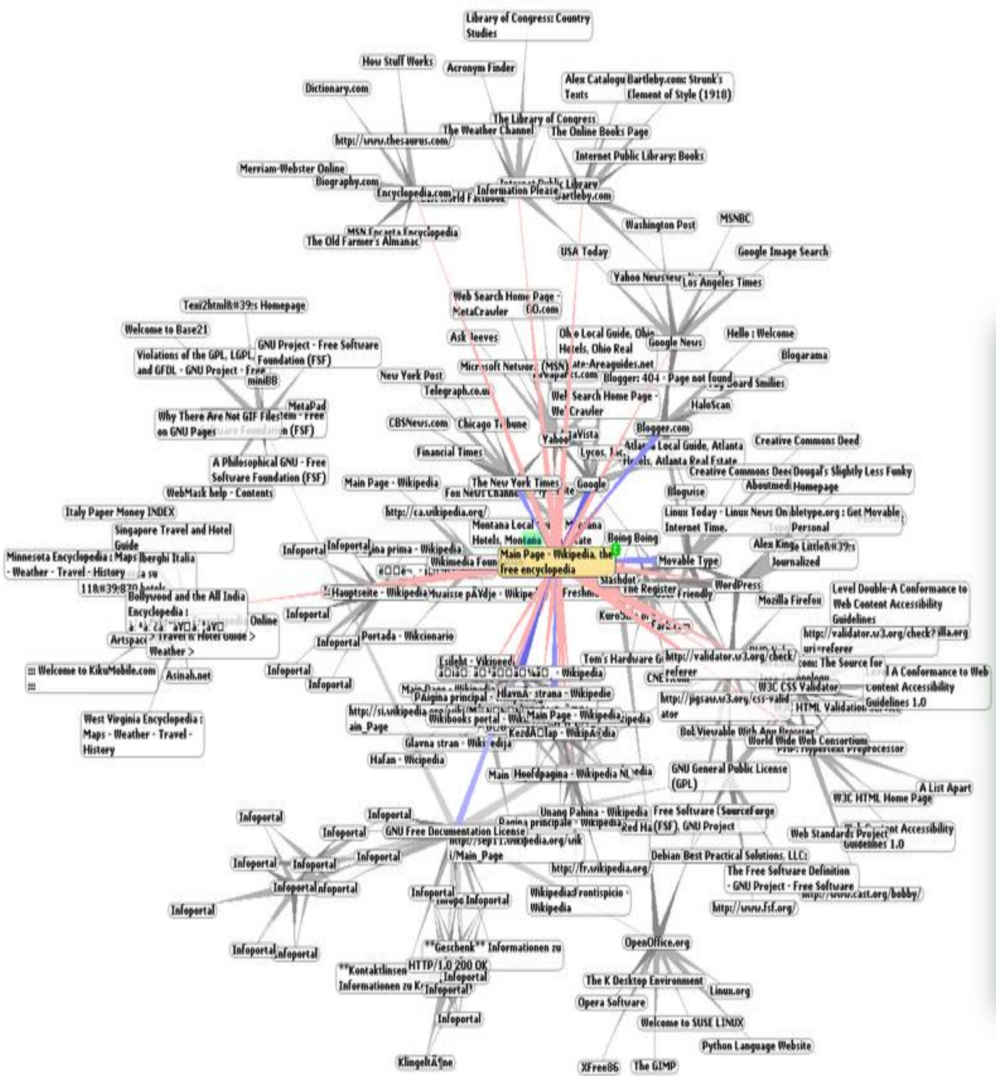
Motivation

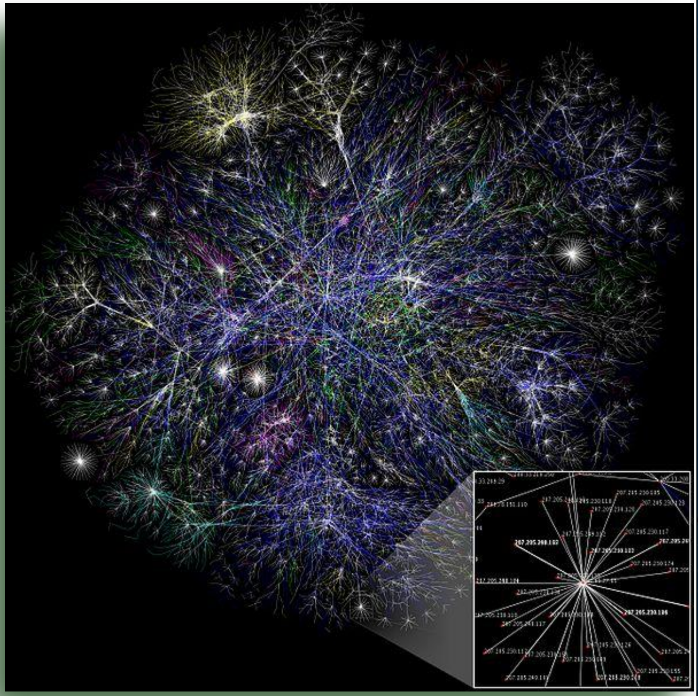
- Classical Network
  - Some Examples
  - Erdös-Rényi model

Air transportation

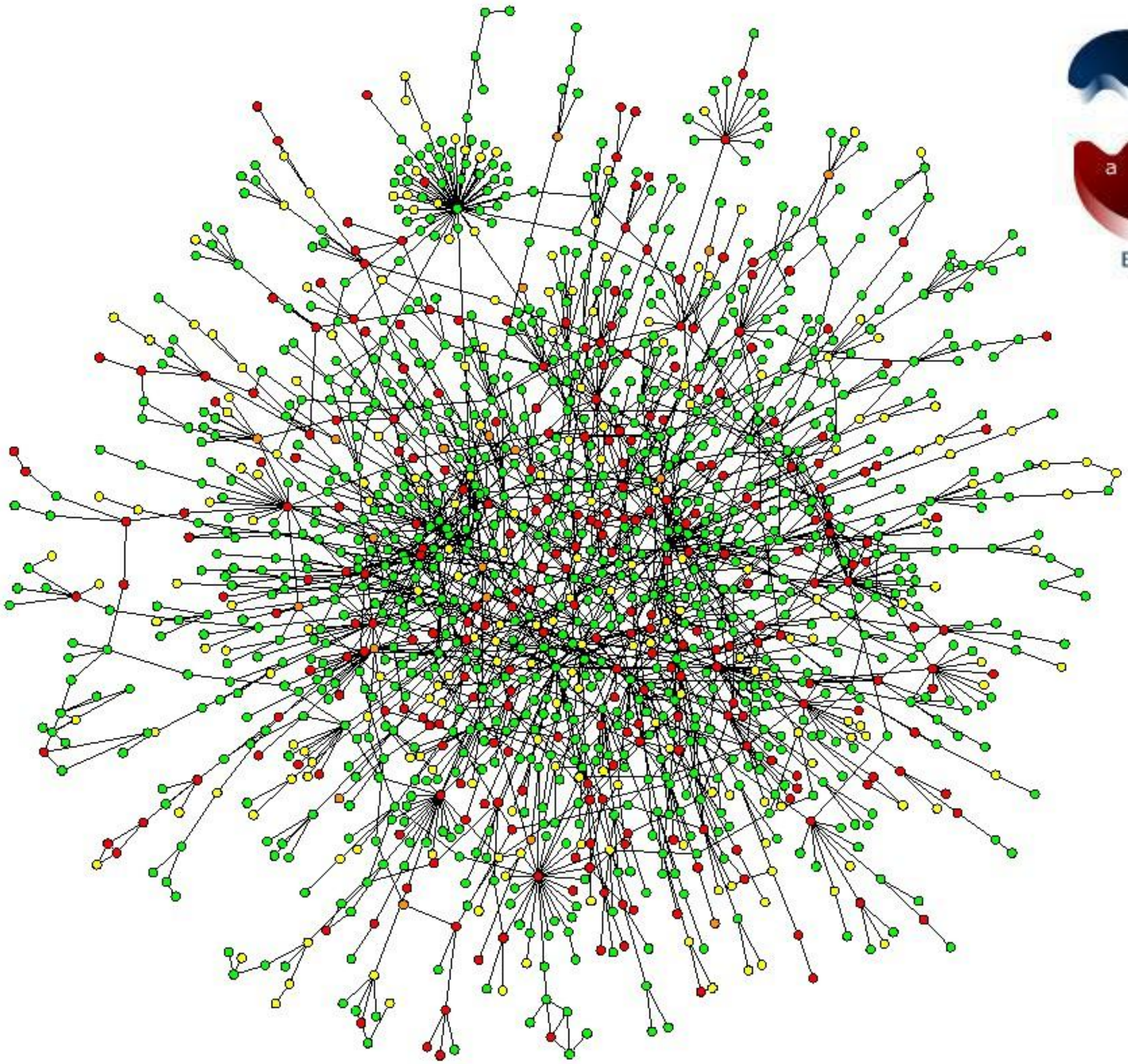


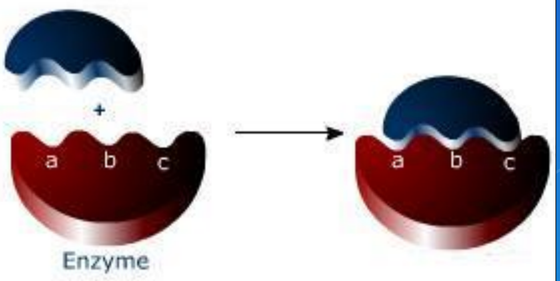
Internet


Library of Congress: Country Studies
How Stuff Works
Acronym Finder
Dictionary.com
The Library of Congress
The Weather Channel
The Online Books Page
http://www.thesaurus.com/
Internet Public Library: Books
Merriam-Webster Online
Biography.com
Encyclopedia.com
Information Please
Bartleby.com
Washington Post
MSNBC
The Old Farmer's Almanac
USA Today
Google Image Search
Los Angeles Times
Welcome to Base21
Ask Jeeves
Google News
Hello : Welcome
Blogarama
New York Post
Telegraph.co.uk
HaloScan
CBSNews.com
Blogger.com
Creative Commons Deed
Financial Times
Main Page - Wikipedia
The New York Times
Google
Blogwise
Italy Paper Money INDEX
http://ca.wikipedia.org/
Journalized
Movable Type
WordPress
Mozilla Firefox
Main Page - Wikipedia, the free encyclopedia
Portada - Wikcionario
Infoportal
Asinah.net
W3C CSS Validator
World Wide Web Consortium
GNU General Public License (GPL)
W3C HTML Home Page
A List Apart
GNU Free Documentation License
http://fr.wikipedia.org/
Debian Best Practical Solutions, LLC:
The Free Software Definition
http://www.fsf.org/
OpenOffice.org
The K Desktop Environment
Linux.org
Opera Software
Welcome to SUSE LINUX
Python Language Website
XFree86
The GIMP

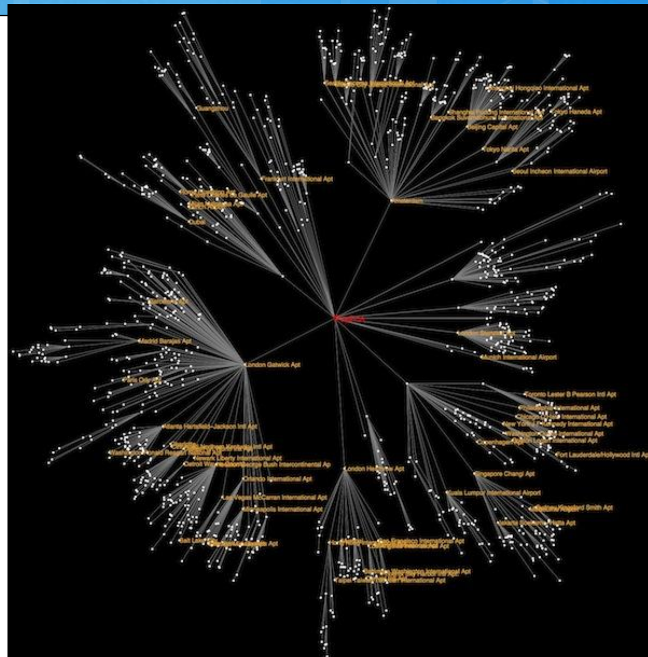


Protein network

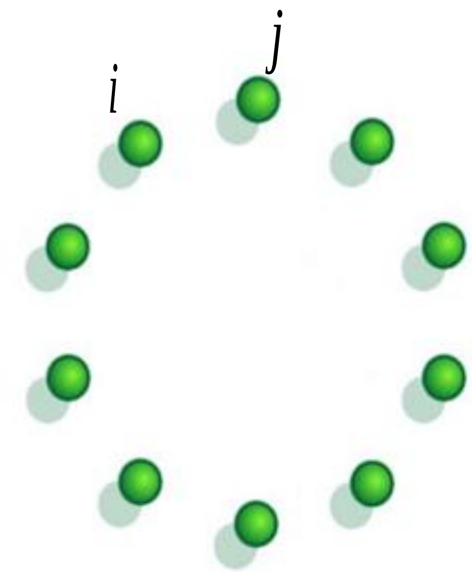




spread of the disease



Erdős and Rényi

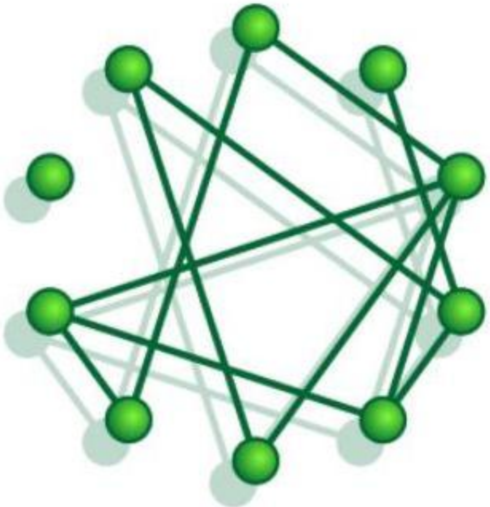


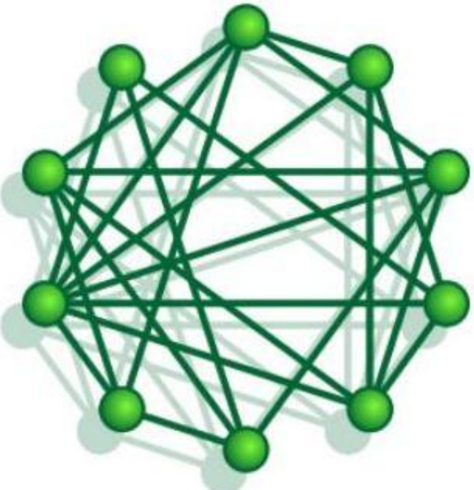
p_{ij}

The simplest and most studied model

$p_{ij} = p$

Erdős and Rényi





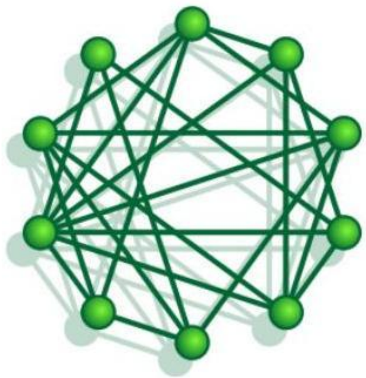
$$p = \frac{1}{6}$$

$$p = \frac{1}{2}$$

Probability in graphs

- The number of links attached to a node?

N*, *p

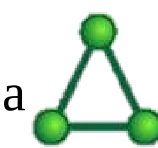


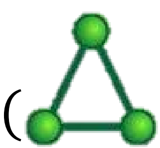
$$prob(m) = \binom{N-1}{m} p^m (1-p)^{N-1-m}$$

Select m nodes from N-1

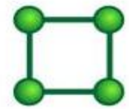
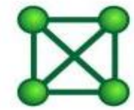
probability of having *m* edges

probability of missing N-1-m edges

- The probability for the appearance a  ?

$prob($  $) = p^3$

Classical Table

$p_c \sim$	$\frac{1}{N^{\frac{3}{2}}}$	$\frac{1}{N^{\frac{4}{3}}}$	$\frac{1}{N}$	$\frac{1}{N^{\frac{2}{3}}}$
			 	

$if \quad p \geq p_c(N)$ → Appearance of subgraph F with n nodes and l

$$p_c(N) = \frac{c}{N^{\frac{n}{l}}}$$

Large N

$p_c \sim$	$\frac{1}{N^{\frac{3}{2}}}$	$\frac{1}{N^{\frac{4}{3}}}$	$\frac{1}{N}$	$\frac{1}{N^{\frac{2}{3}}}$

$$p \geq p_c = \frac{cte}{N^{\frac{4}{3}}}$$

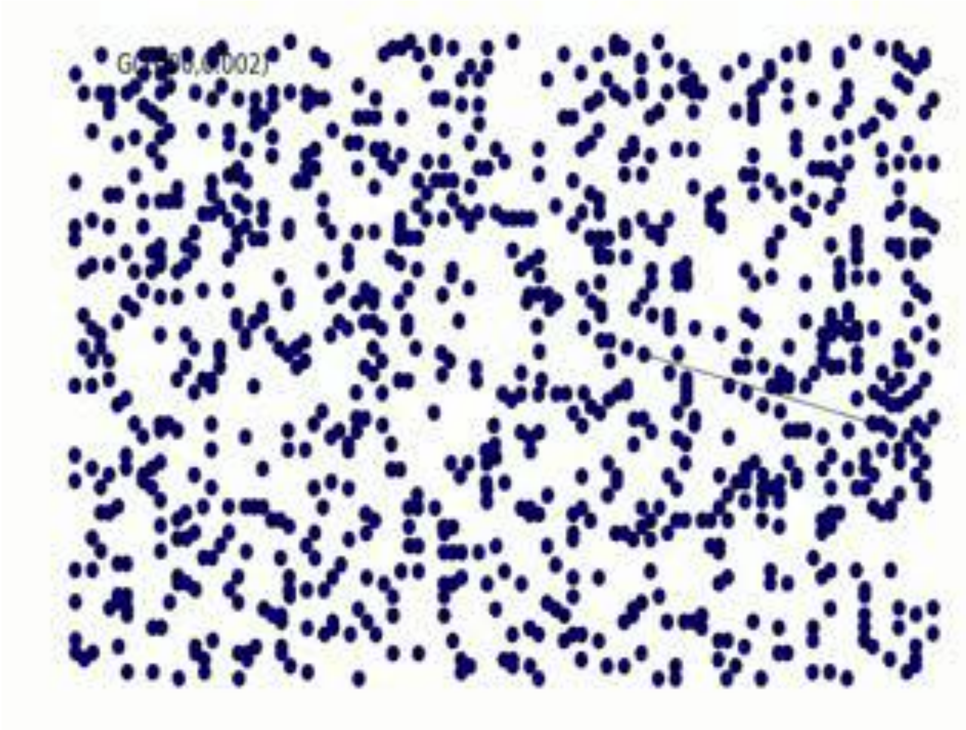
$$\#\,nodes = \binom{N}{4} \sim N^4$$

$$\langle \; \rangle = p^3(N^4) \geq 1 \;\Rightarrow p \geq \frac{1}{N^{\frac{4}{3}}}$$

Erdős and Rényi

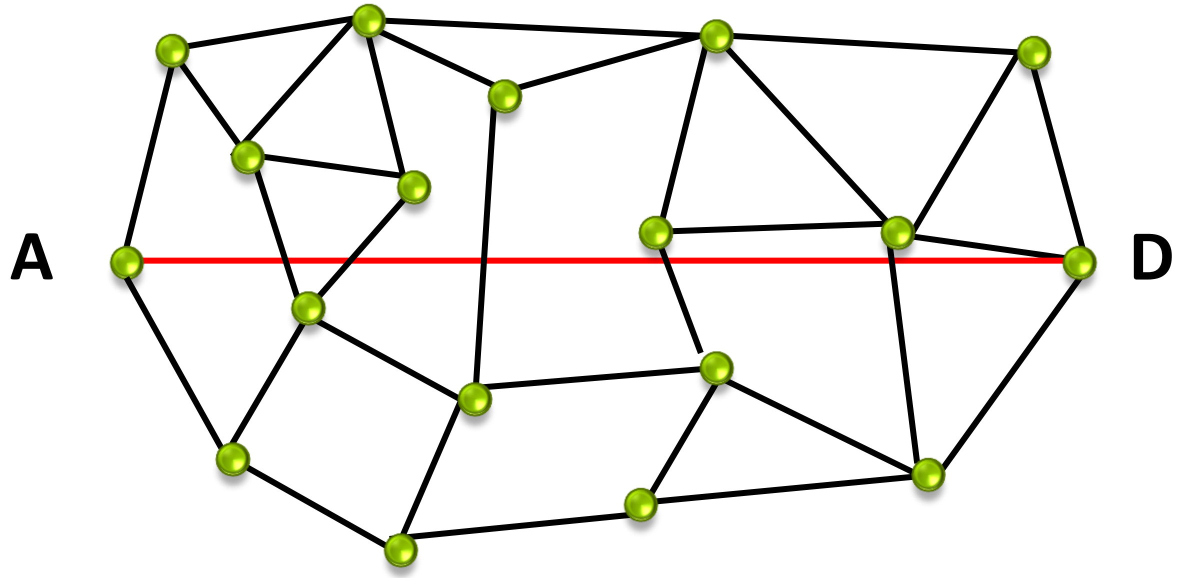
$p_c \sim$	$\frac{1}{N^{\frac{3}{2}}}$	$\frac{1}{N^{\frac{4}{3}}}$	$\frac{1}{N}$	$\frac{1}{N^{\frac{2}{3}}}$

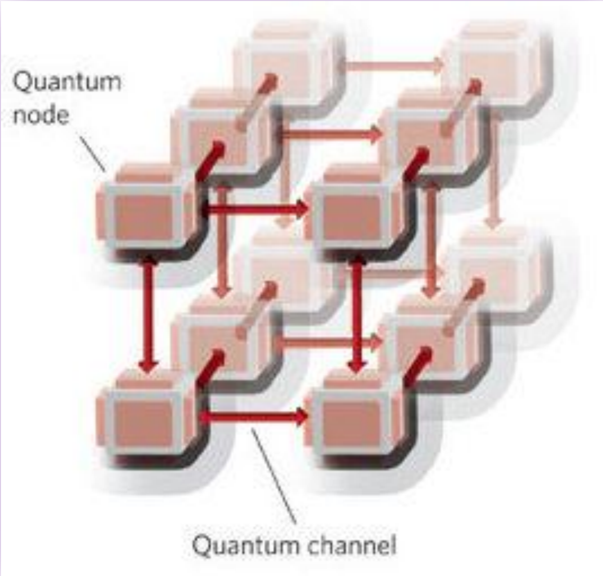
$G(N, p)$



Prof. Albert-László Barabási

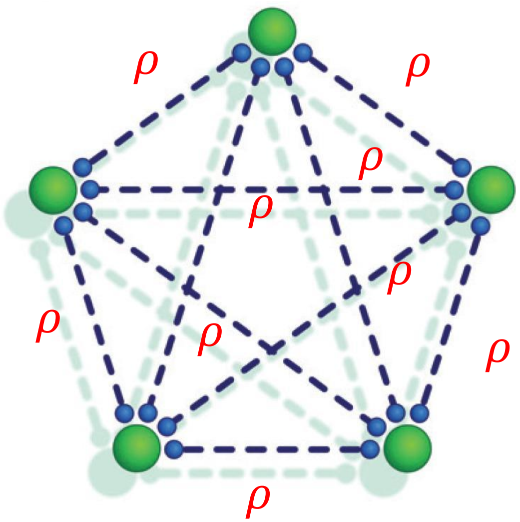
Quantum random graphs

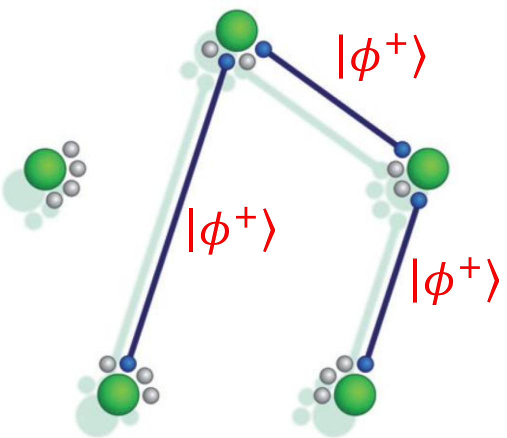




- **Teleportation**
- **Quantum State Transfer**
- **cryptography**
- **Quantum Communication**
- **Distributed Computing**
- **Quantum Information Transfer**

Quantum random graphs





$$p_{ij} \rightarrow \rho_{ij}$$

$$p \rightarrow \rho$$

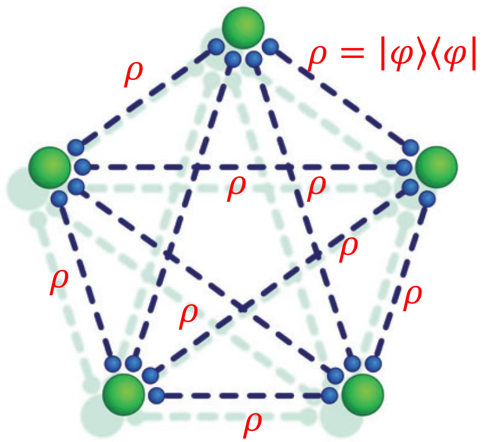
$$\rho = |\varphi\rangle\langle\varphi|$$

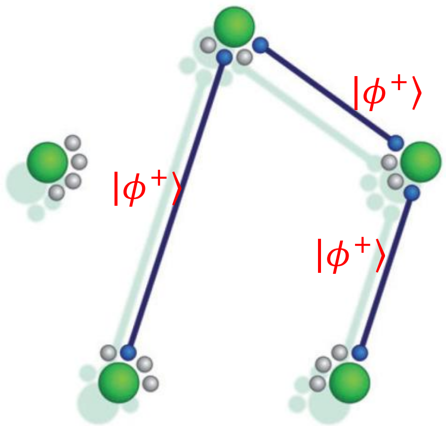
$$|\varphi\rangle = \sqrt{1-\frac{p}{2}}|00\rangle + \sqrt{\frac{p}{2}}|11\rangle \quad where \quad 0 \leq p \leq 1$$

Majorization theory: $\text{prob}\left(|\varphi\rangle \longrightarrow |\phi^+\rangle\right) = 2\left(\frac{p}{2}\right) = p$

Main Results

$|G_{N,p}\rangle =$





$$|\varphi\rangle = \sqrt{1-\tfrac{p}{2}}|00\rangle + \sqrt{\tfrac{p}{2}}|11\rangle$$

Majorization theory:

$$\text{prob}\left(|\varphi\rangle \longrightarrow |\phi^+\rangle\right) = p$$

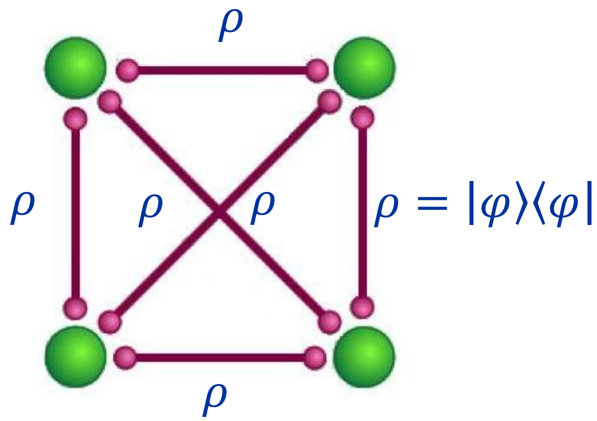
- Min probability requirement?

$$\left|\langle G_{N,p}|0\cdots 0\rangle\right|^2$$

✓ in which case no local quantum operation is able to create entanglement between the nodes.

$$p = \frac{2c}{N^2}$$

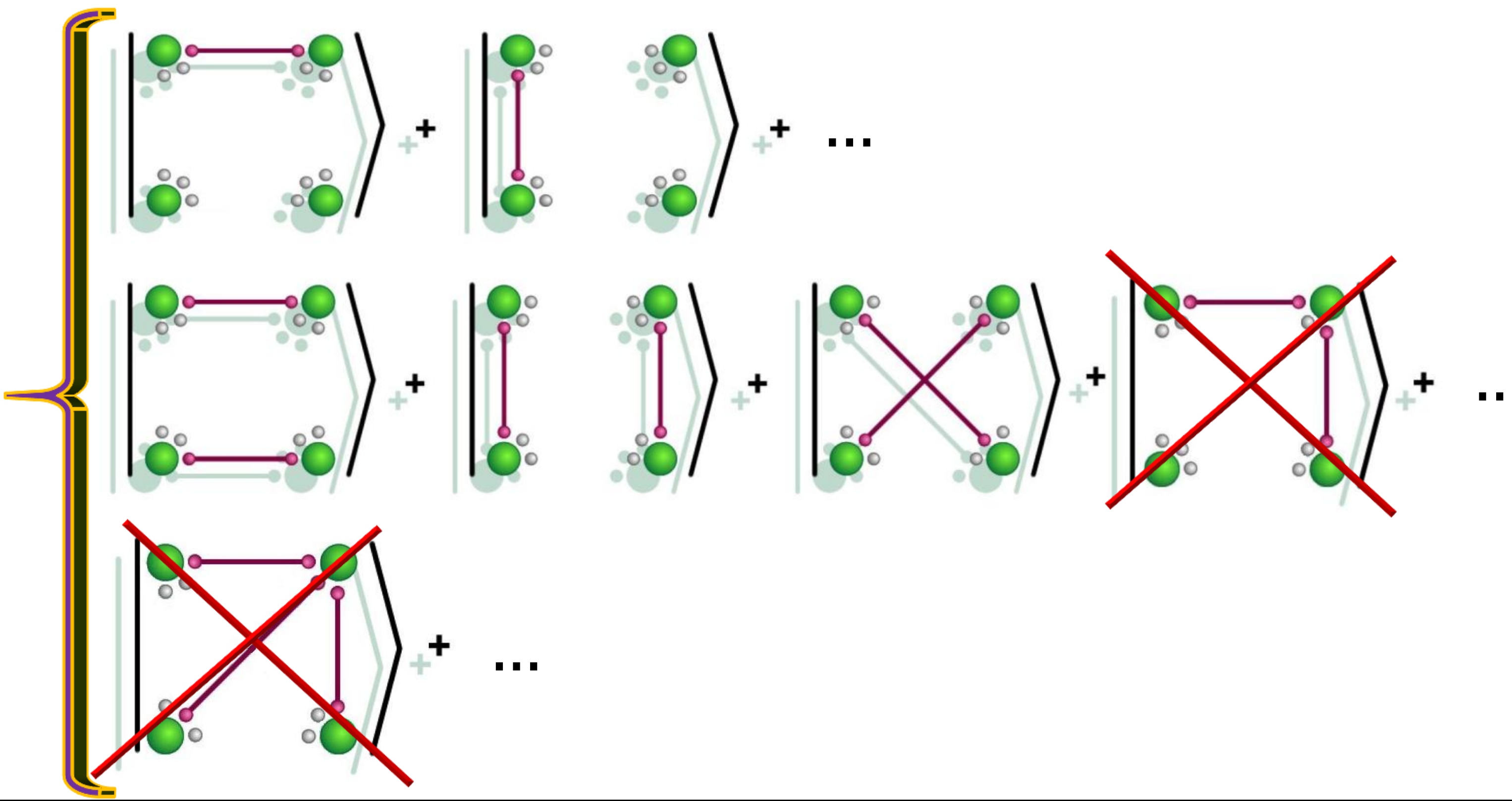
GHZ states of four qutrits



$$|\varphi\rangle = \sqrt{1-\tfrac{p}{2}}|00\rangle + \sqrt{\tfrac{p}{2}}|11\rangle, \qquad p = \frac{2c}{N^2}$$

After LOCC

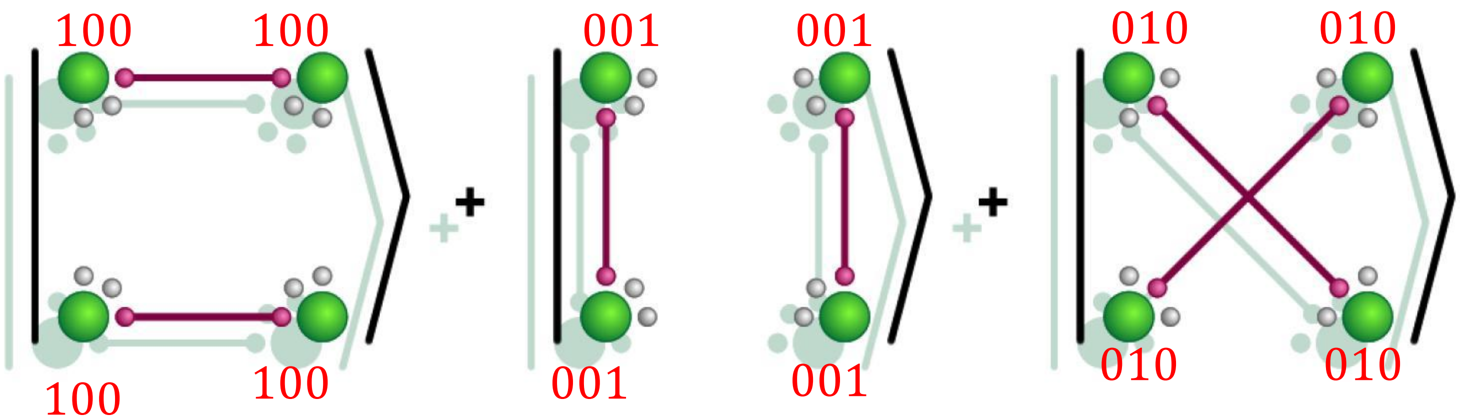
we know prob(m ≥ 2)=0



GHZ states of four qutrits

$$P_m = \sum_{\pi_m} \pi_m \,|\,\underbrace{0\cdots0}_{N-m-1}\,\underbrace{1\cdots1}_{m}\rangle\langle 0\cdots01\cdots1\,|\,\pi_m^{\dagger}$$

$m = 1$



$$|\underbrace{100}_{1}\underbrace{100}_{1}\underbrace{100}_{1}\underbrace{100}_{1}\rangle + |\underbrace{001}_{2}\underbrace{001}_{2}\underbrace{001}_{2}\underbrace{001}_{2}\rangle + |\underbrace{010}_{3}\underbrace{010}_{3}\underbrace{010}_{3}\underbrace{010}_{3}\rangle$$

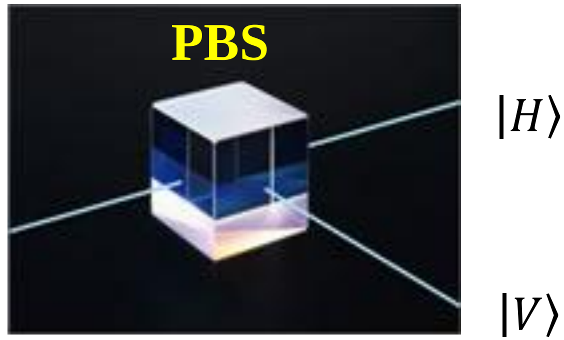
$$|GHZ\rangle \propto |1111\rangle + |2222\rangle + |3333\rangle$$

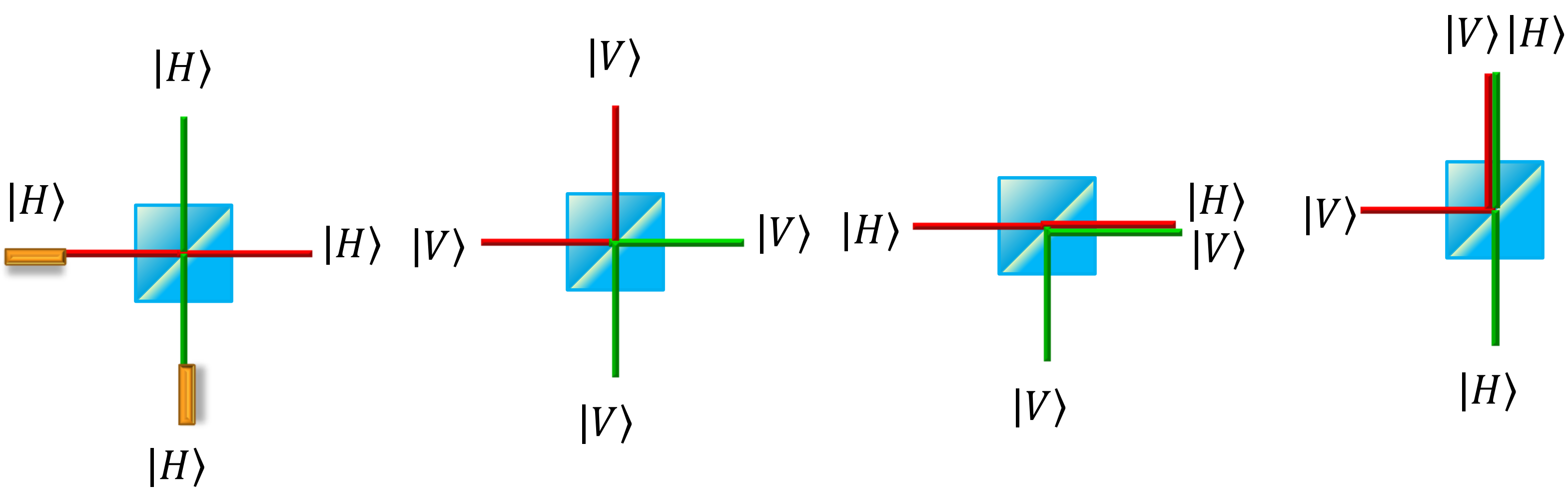
Experiment …?

Create GHZ states

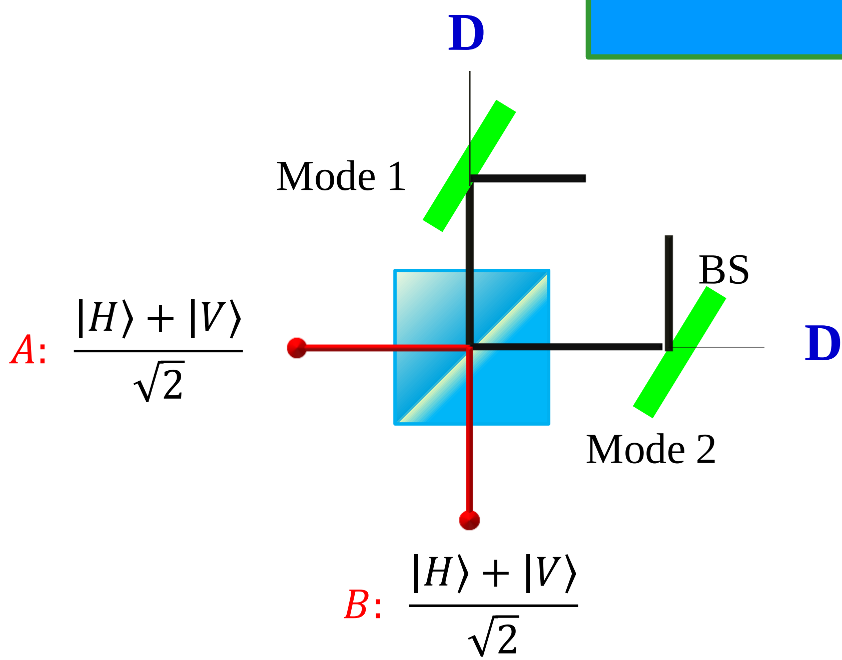
Polarizing Beam Splitter

$$\frac{|H\rangle + |V\rangle}{\sqrt{2}}$$





Bell State

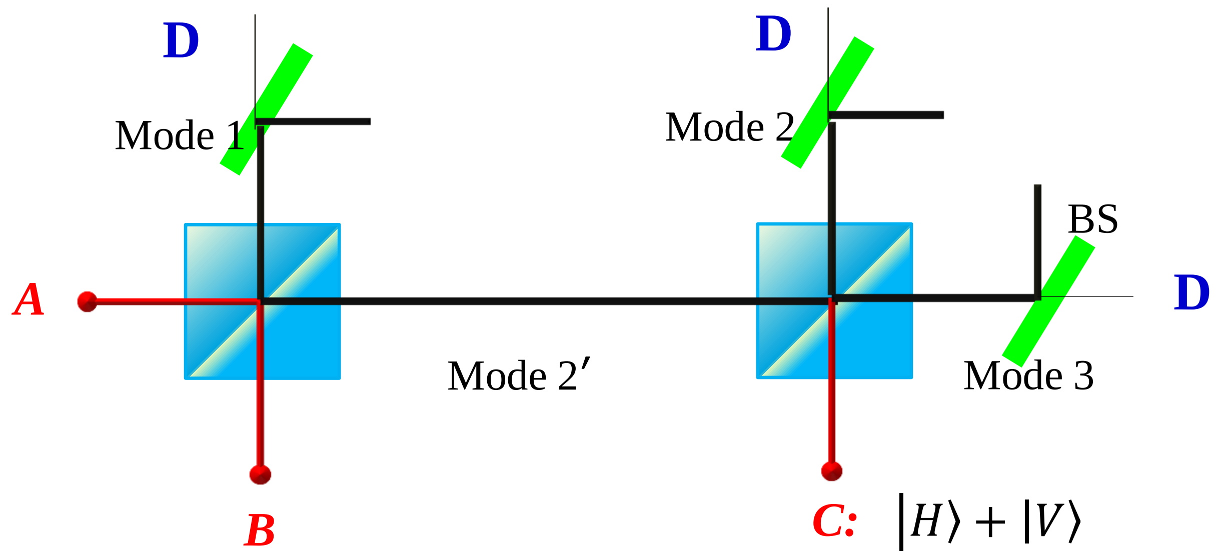


$$|\psi_{in}\rangle = |H_A\rangle\,|H_B\rangle + |H_A\rangle\,|V_B\rangle + |V_A\rangle\,|H_B\rangle + |V_A\rangle\,|V_B\rangle$$

$$|\psi_{PBS}\rangle = |H_2\rangle\,|H_1\rangle + |H_2\rangle\,|V_2\rangle + |V_1\rangle\,|H_1\rangle + |V_1\rangle\,|V_2\rangle$$

$$|\psi_{final}\rangle = |H_1\rangle\,|H_2\rangle + |V_1\rangle\,|V_2\rangle = |\phi^+\rangle$$

Create GHZ states



$$|\phi^+\rangle = \underbrace{|H_1\rangle\,|H_{2'}\rangle}_{I} + \underbrace{|V_1\rangle\,|V_{2'}\rangle}_{II}$$

⇒

I: $|H_1\rangle|H_3\rangle\,|H_2\rangle + |H_1\rangle\,|H_3\rangle\,|V_3\rangle$

II: $|V_1\rangle|V_2\rangle\,|H_2\rangle + |V_1\rangle\,|V_2\rangle\,|V_3\rangle$

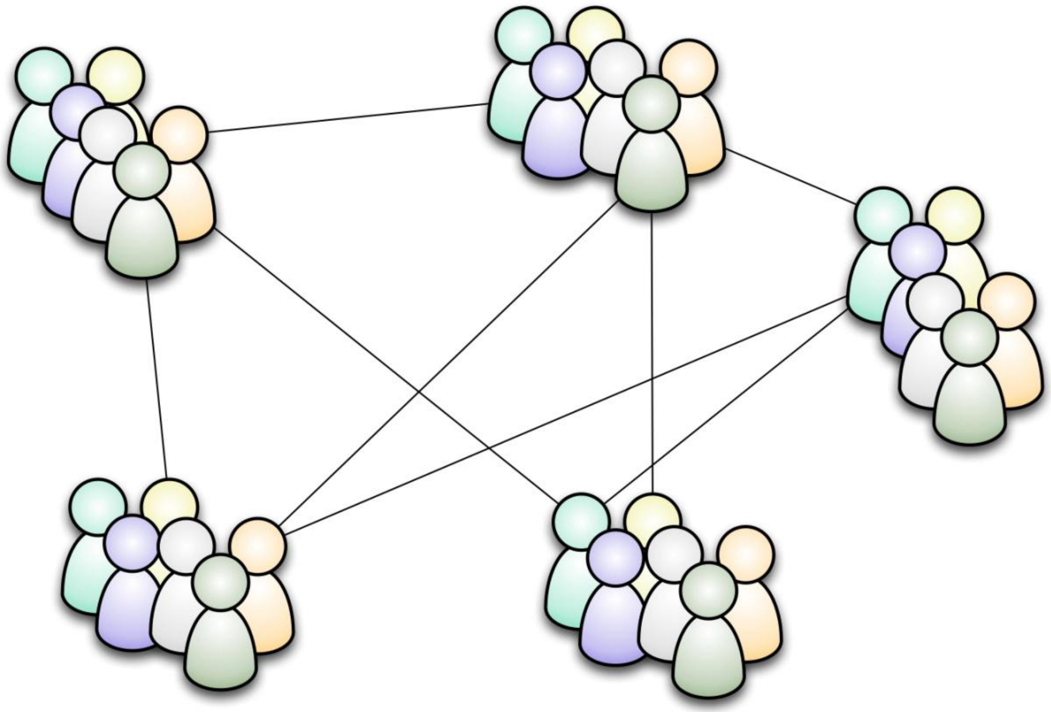
$$|GHZ\rangle = |H\rangle|H\rangle|H\rangle + |V\rangle|V\rangle|V\rangle$$

J. W. Pan and A. Zeilinger, Phys. Rev. A, **57**, 2208 (1998)

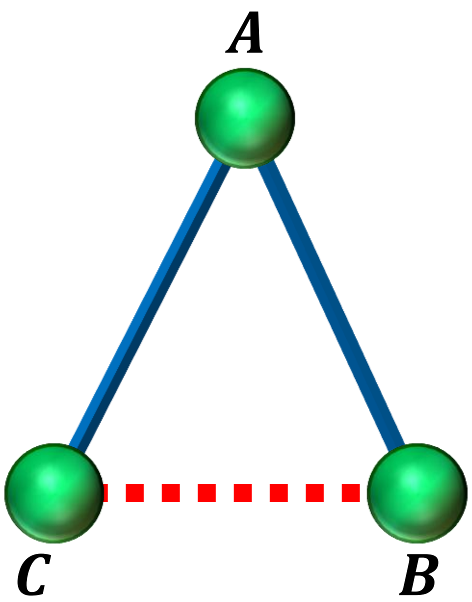
Our Problem

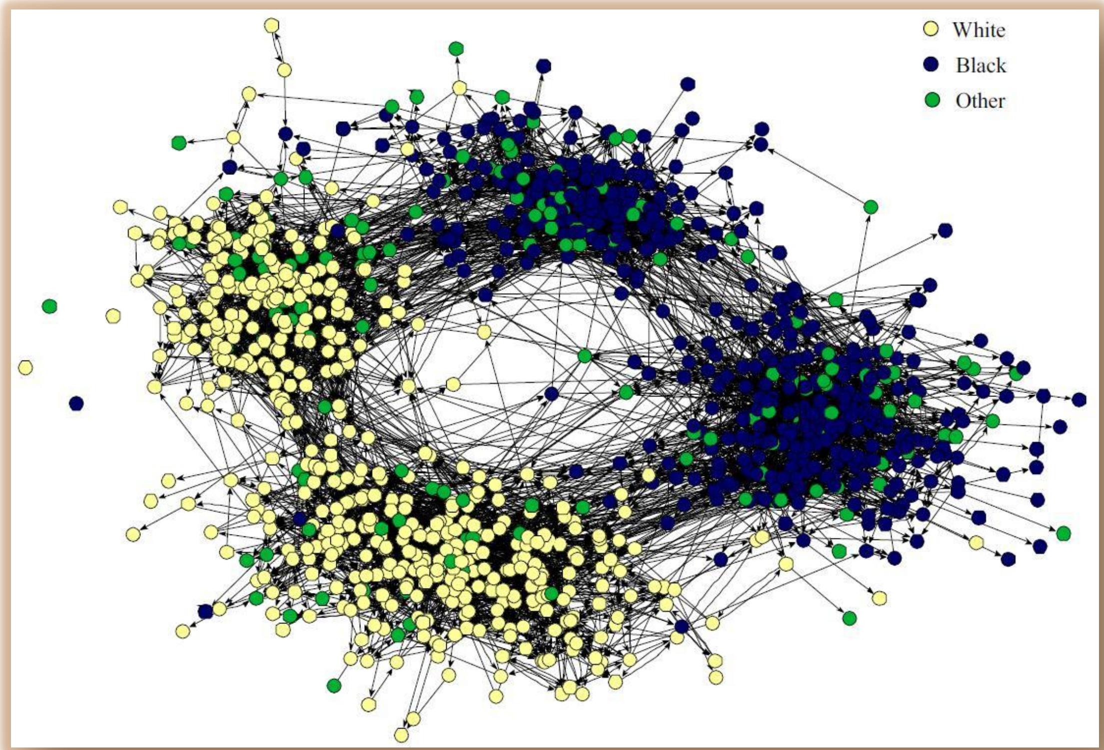
Small World Property

- shortest paths between nodes $\sim\ln(N)$, where N is the number of nodes.



Clustering

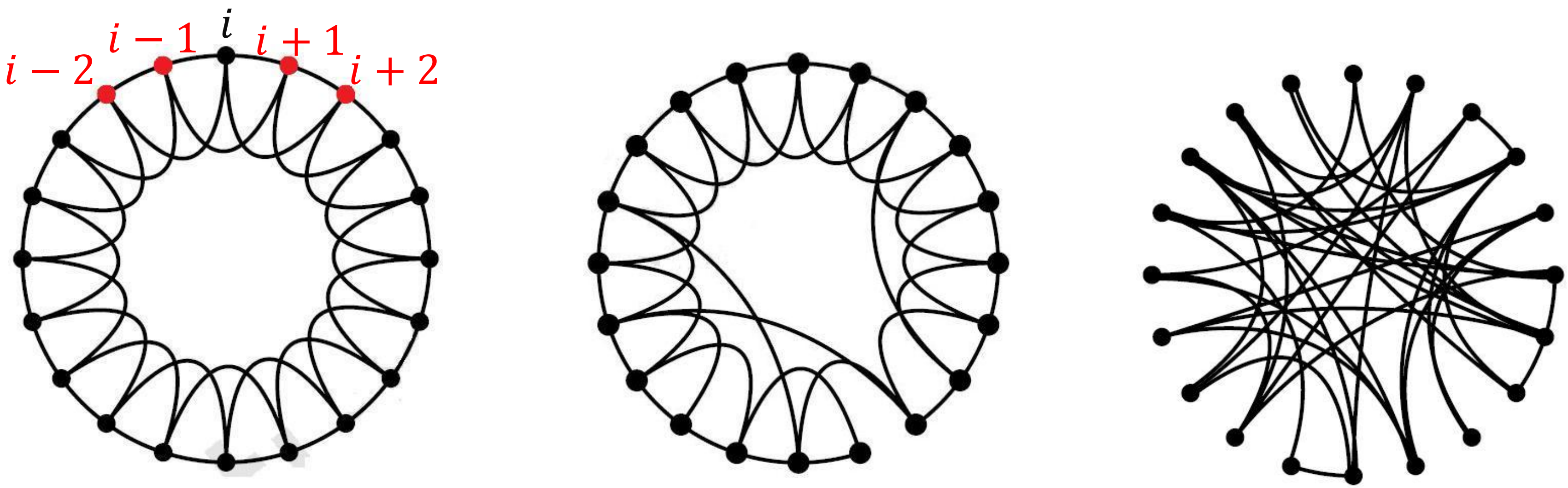




✓ These properties distinguish real networks from simple models such as the Erdös Renyi random graph model

Real Networks

Regular | Small-World | Random



$p = 0$ → increasing randomness → $p = 1$

Regular
- high clustering
- high average path length
  $\bar{l} \propto N$

Small-World
- high clustering
- small average path length
  $\bar{\ell} \propto \ln(N)$

Random
- small clustering
- small average path length
  $\bar{l} \propto \ln(N)$

D. J. Watts and S. H. Strogatz, *Nature*, **393**, 440 (1998).


Thanks for your attention.