Quantum Information Theory Meets Classical Statistical Mechanics

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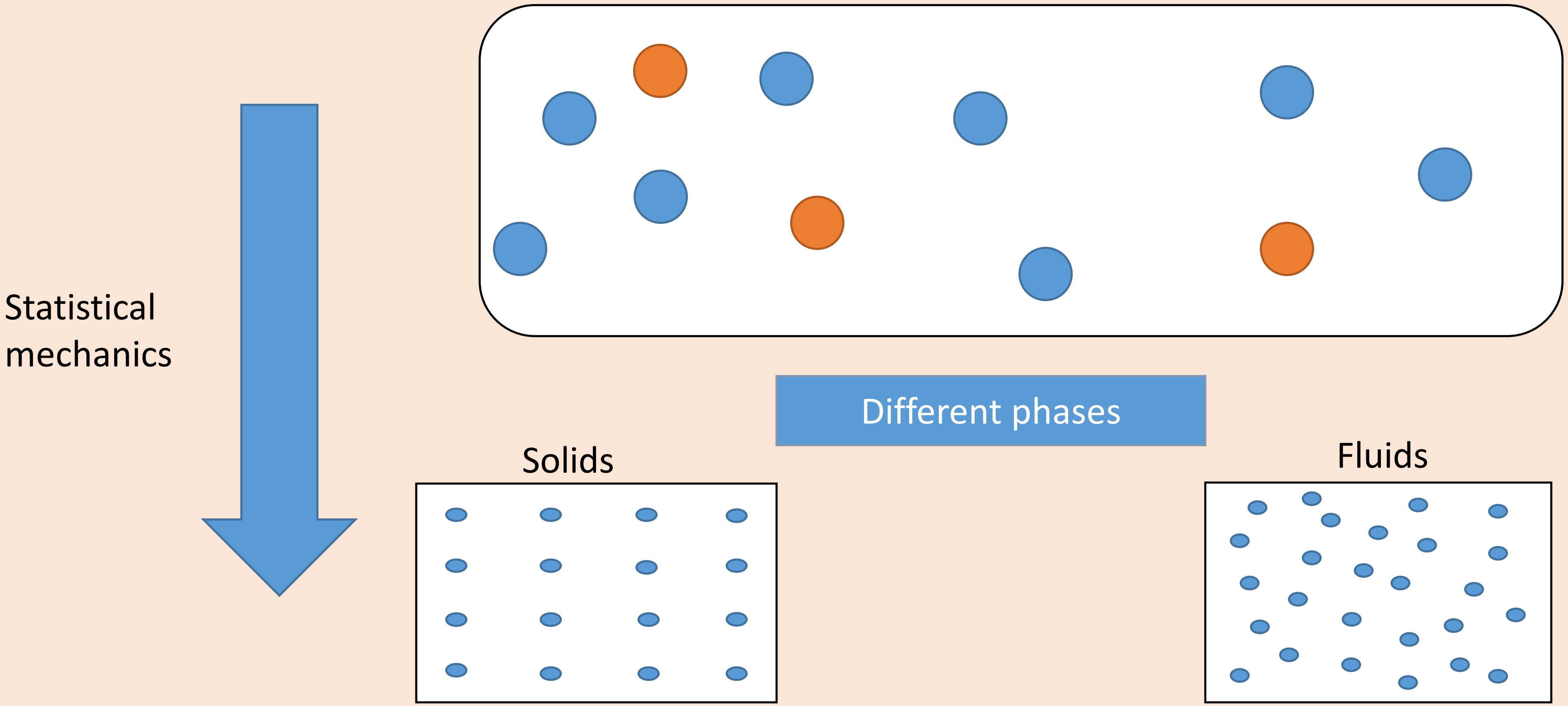
Outlines

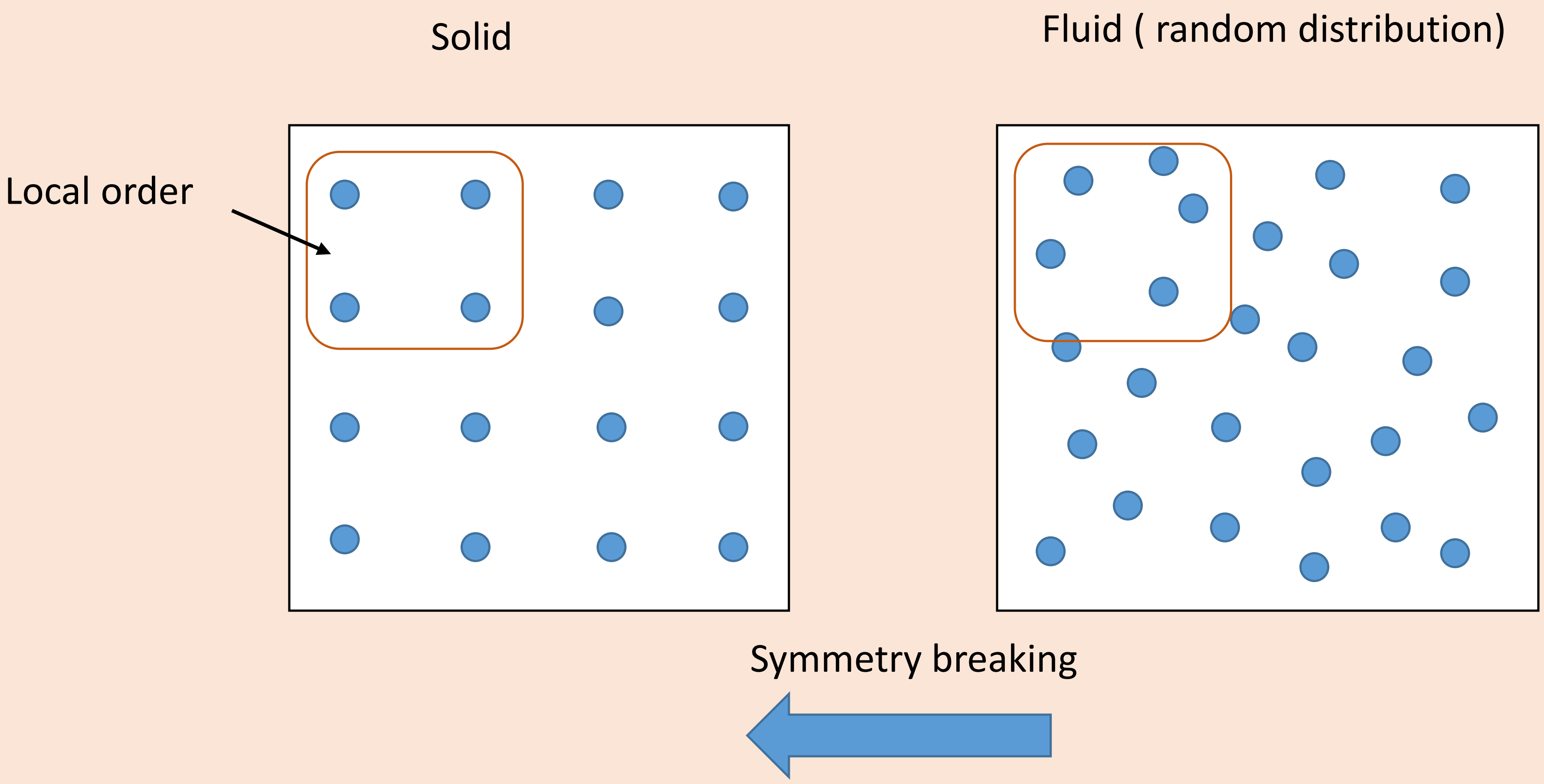
- Different phases of matter: from statistical mechanics to quantum information theory
- Encoding classical spin models in quantum wave functions
- Studying different quantum phases by classical statistical mechanics

Different phases of matter: emergence of different macroscopic behaviors

Fundamental particles of all matter are the same



Landau symmetry breaking theory:



Symmetry breaking theory in quantum physics:

Symmetric quantum Hamiltonian: $$H = -J \sum Z_i Z_j - h \sum X_i$$

$$\left(\frac{h}{J}\right)_c$$

$$|\mathrm{GHZ}\rangle = \frac{1}{2}(|\uparrow\uparrow \cdots \uparrow\rangle + |\downarrow\downarrow \cdots \downarrow\rangle)$$

$$(|\uparrow\rangle + |\downarrow\rangle)^{\otimes N}$$

Symmetry breaking in ground state:

$$\begin{cases} |\uparrow\uparrow \ldots . \uparrow\rangle \\ |\downarrow\downarrow \ldots . \downarrow\rangle \end{cases}$$

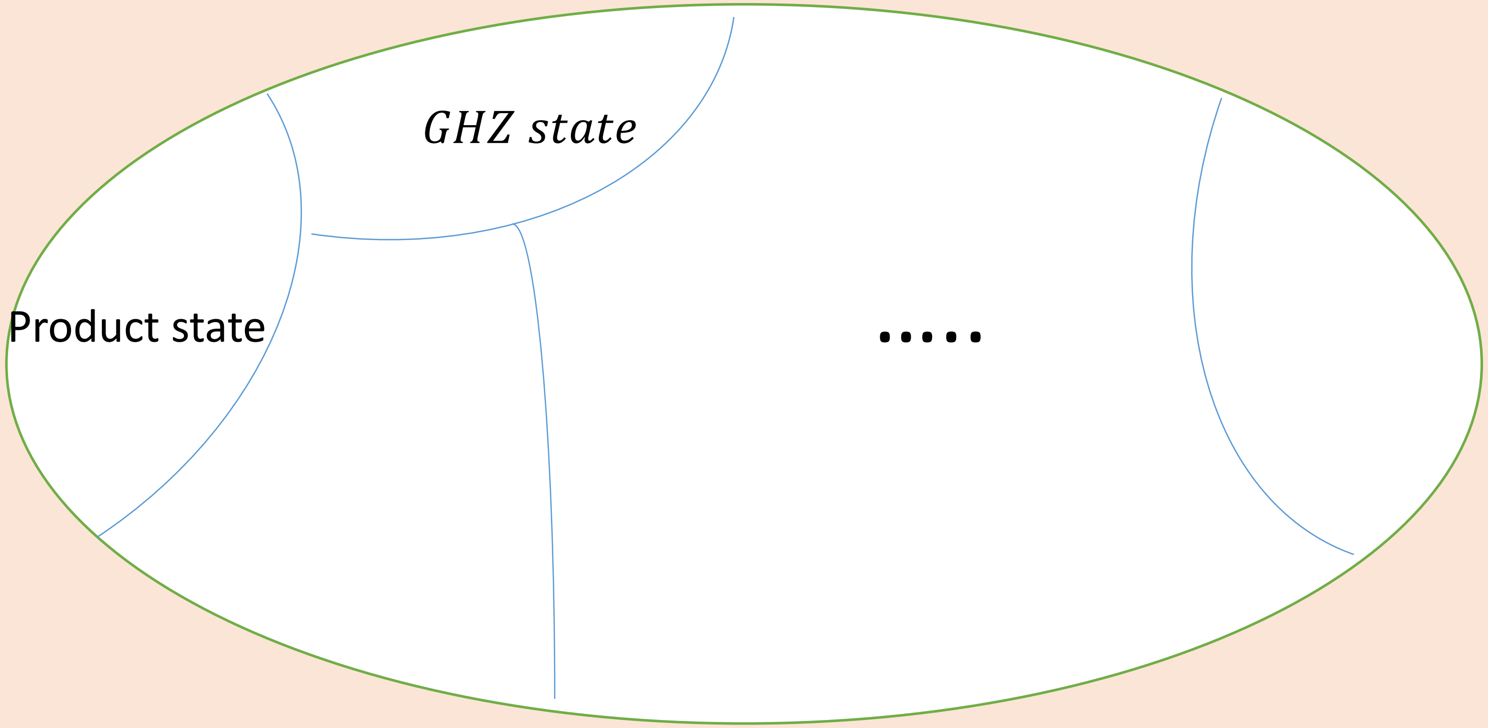
In thermodynamic limit, the symmetry is broken.

Different quantum phases in view of quantum information theory (QIT):

All accessible information of a quantum system are encoded in wave function

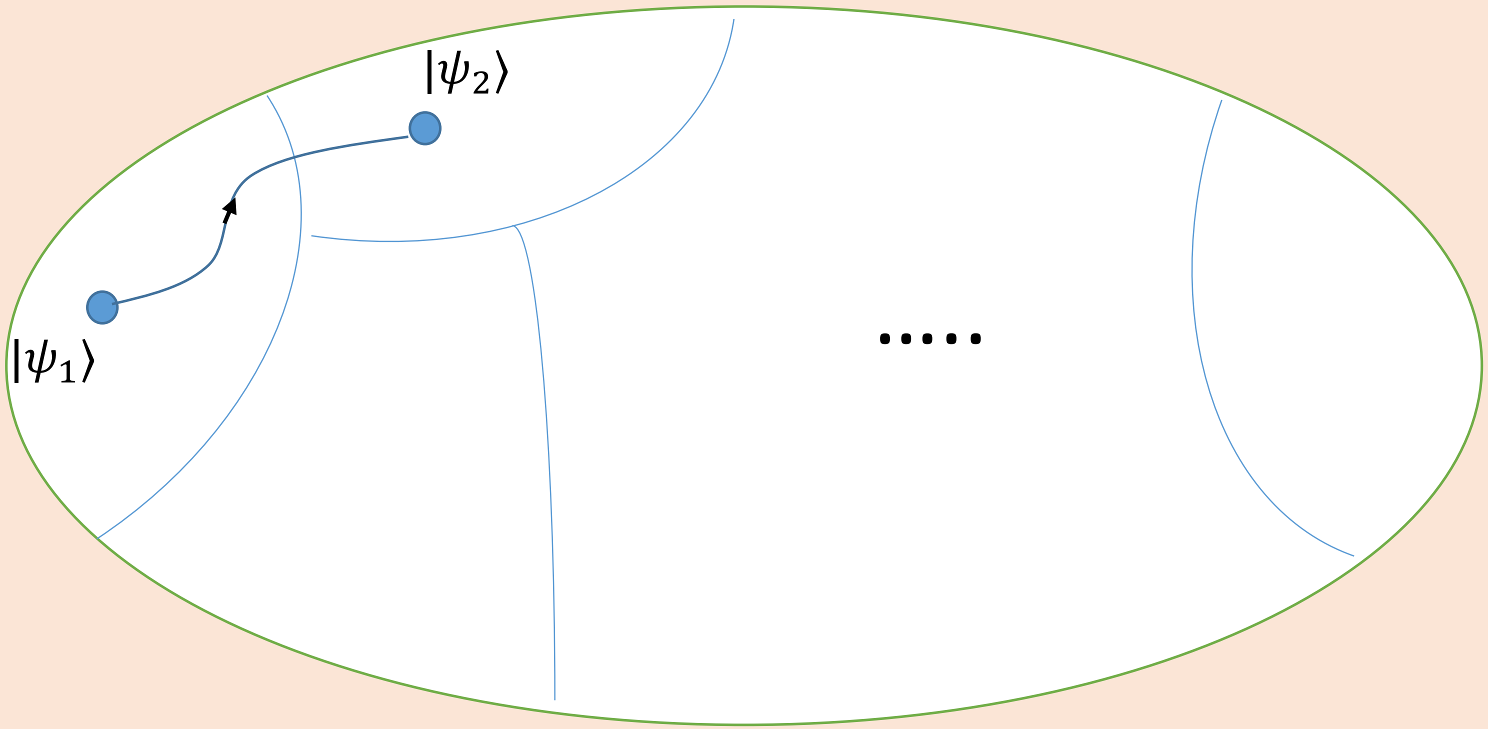
$$|\psi\rangle = a_0|00\ldots0\rangle + a_1|10\ldots0\rangle + \cdots a_{2^N}|11\ldots1\rangle$$

Different phases in the space of quantum states



We can ignore the microscopic Hamiltonian

Phase transition in view of QIT:

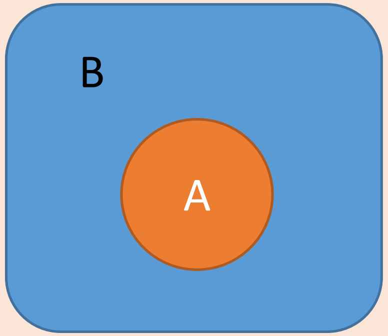


$$|GS\rangle = a_0(\lambda)|00\ldots 0\rangle + a_1(\lambda)|10\ldots 0\rangle + \cdots a_{2^N}(\lambda)|11\ldots 1\rangle$$

If you like $H = \lambda H_1 + (1-\lambda) H_2$

characterizing phase transition in the wave function:

Physical system



- Bipartite Entanglement

$$E(\lambda) = S(\rho_A) \quad ; \quad \rho_A = tr_B(|GS(\lambda)\rangle\langle GS(\lambda)|)$$

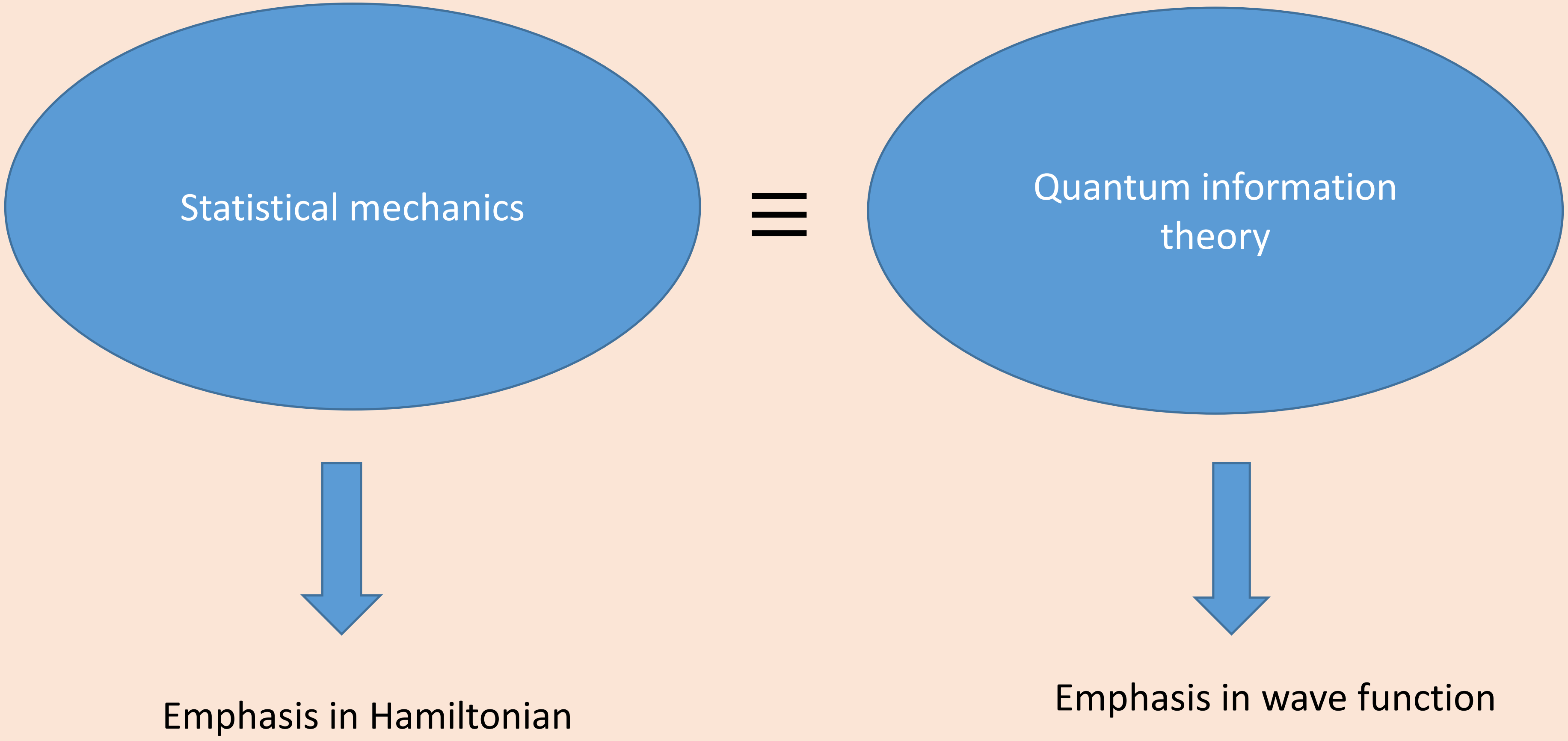
- Ground state fidelity

$$F(\lambda) = \langle GS(\lambda)|GS(\lambda + d\lambda)\rangle$$

Phase transition is characterized by a singularity in entanglement or ground state fidelity.

1. L. Amico, R. Fazio, A. Osterloh, V. Vedral, Entanglement in many-body systems, Reviews of modern physics, 80(2), 517 (2008)
2. S. Chen, L. Wang, Y. Hao, Y. Wang, Intrinsic relation between ground-state fidelity and the characterization of a quantum phase transition, Physical Review A, 77(3), 032111 (2008).

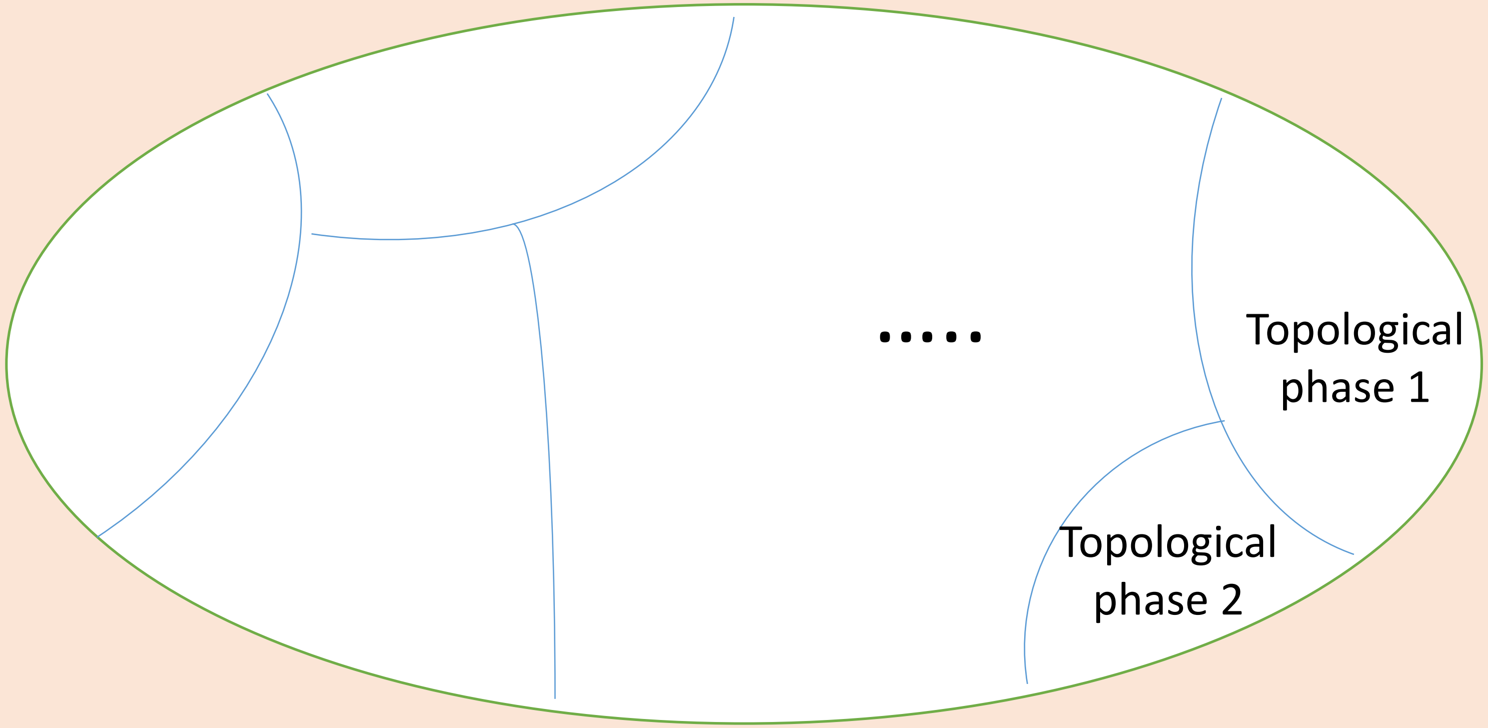
Symmetry breaking phase transitions



However, symmetry breaking theory is essentially a classical theory.

Challenge of quantum topological phases of matter:

There are different phases with the same symmetry: symmetry breaking theory and classical statistical mechanics do not work

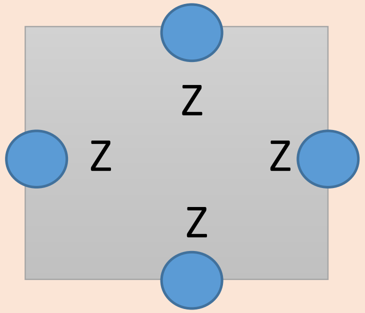


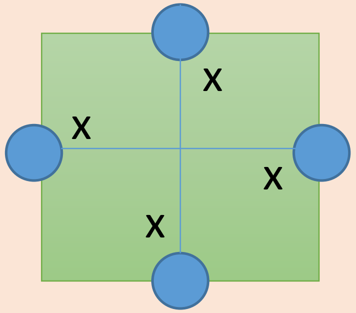
Topological order is essentially a property of wave function

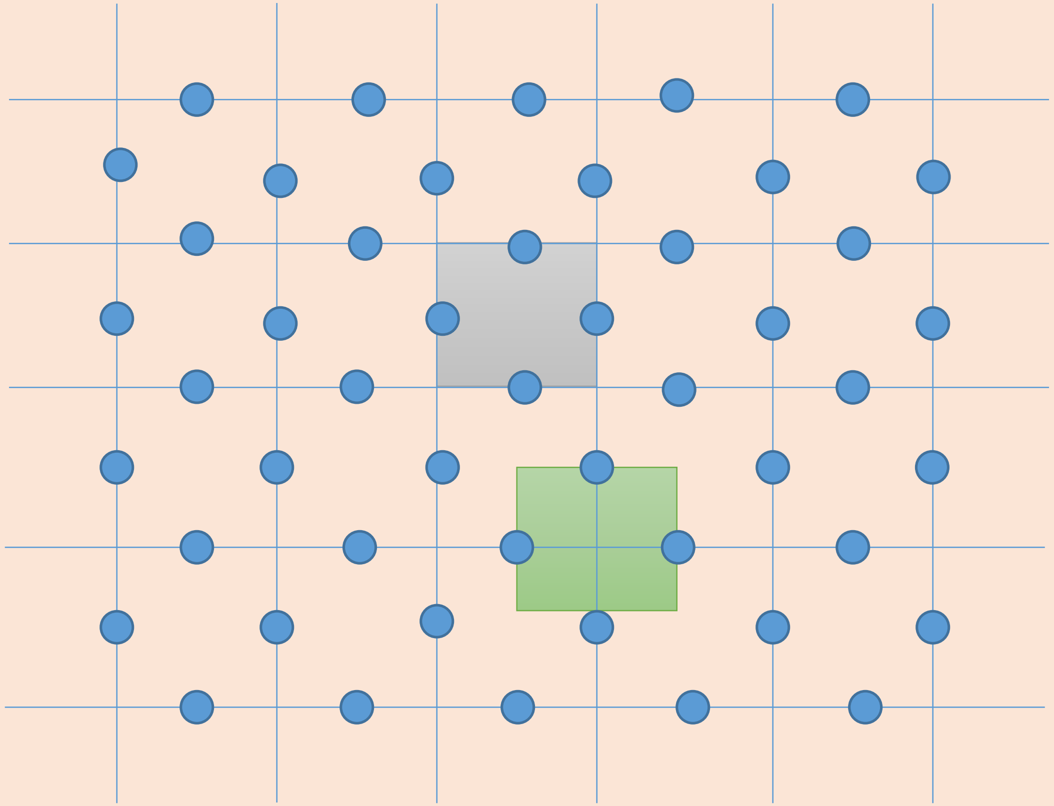
Quantum information theory

X.-G. Wen, Topological orders and Chern-Simons theory in strongly correlated quantum liquid, Int. J. Mod. Phys. B 5, 1641 (1991).

A well-known example: Toric code model

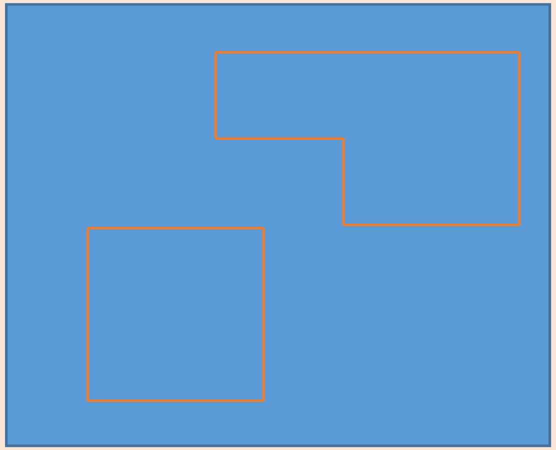
$B_p =$ 

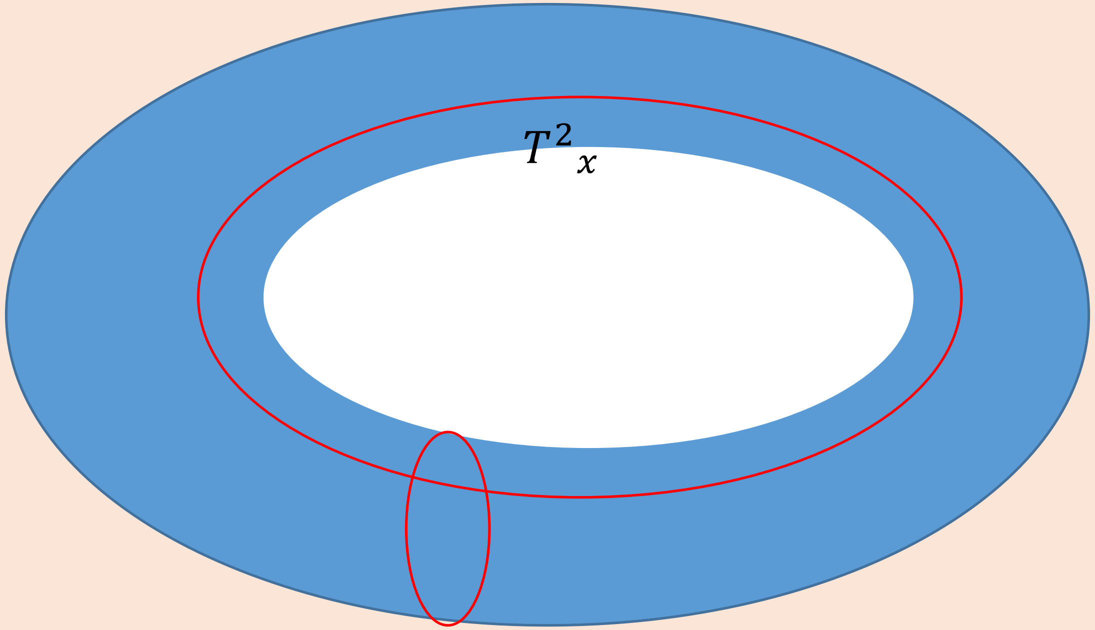
$A_s =$ 



$$H = -\sum B_p - \sum A_s$$

A. Yu. Kitaev , Annals Phys. 303 (2003) 2-30

Ground state: $|TC> = \sum |$  $>$



Topological degeneracy:

There is no role for symmetry

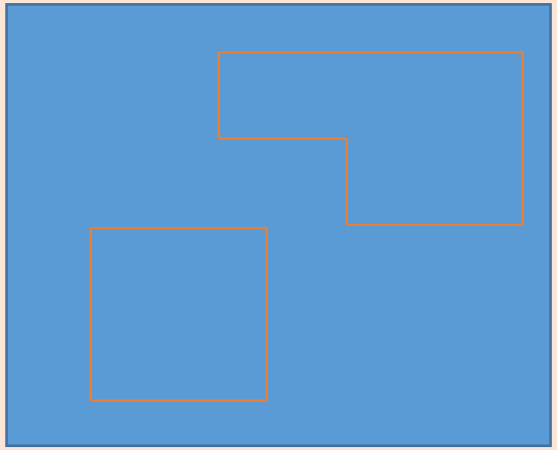
$|\psi_{10}> = T^1{}_x|TC\rangle$

$|\psi_{01}> = T^2{}_x|TC\rangle$

$|\psi_{11}> = T^1{}_x T^2{}_x|TC\rangle$

Topological order is a property of wave function.

$$|TC> = \sum |$$

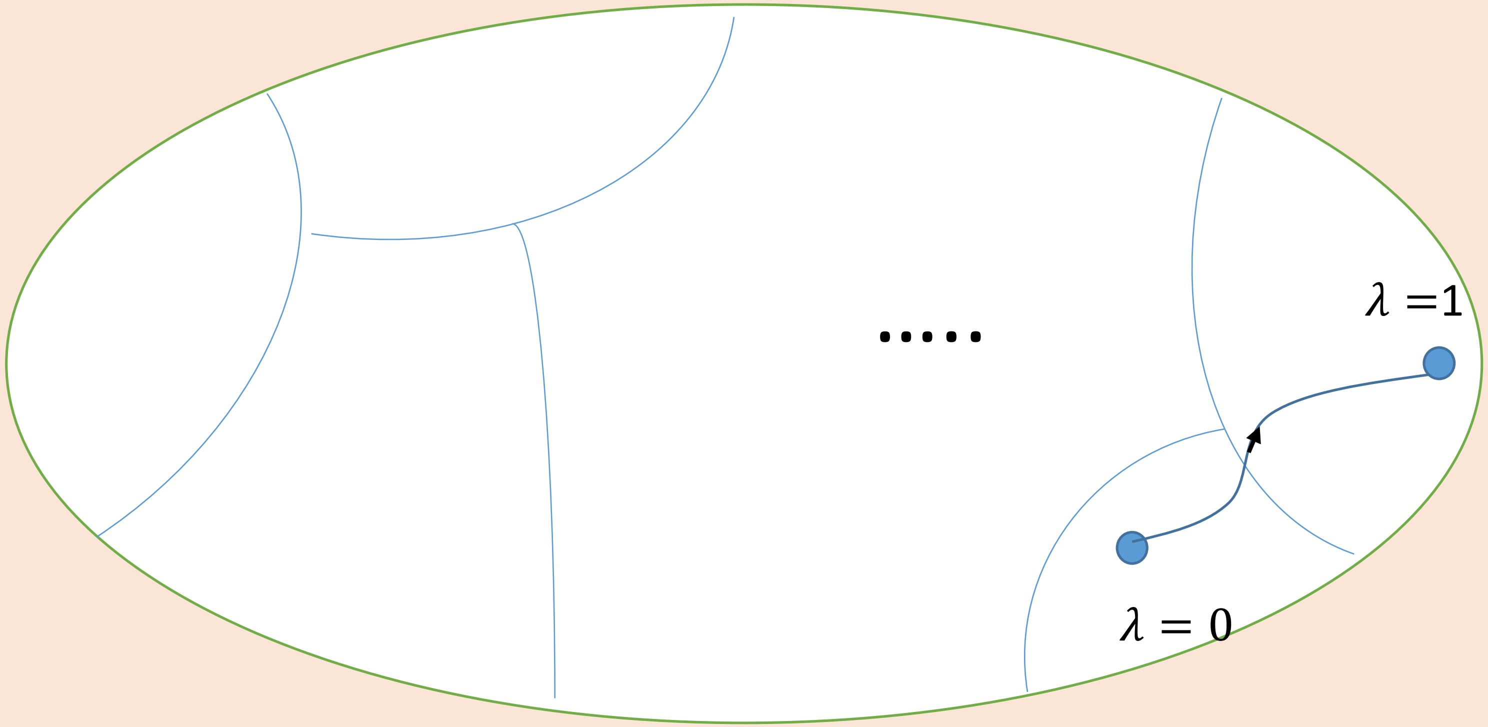


$$>$$

$$a_i = \begin{cases} 1 & loop\ configurations \\ 0 & other\ configurations \end{cases}$$

Point: In a topological phase, a topological constraint determines amplitudes of the wave function

Topological Phase transition:



$$|\psi\rangle = a_0(\lambda)|00\ldots0\rangle + a_1(\lambda)|10\ldots0\rangle + \cdots a_{2^N}(\lambda)|11\ldots1\rangle$$

Can classical statistical mechanics help us here?

An interesting idea: mapping to classical statistical mechanics:

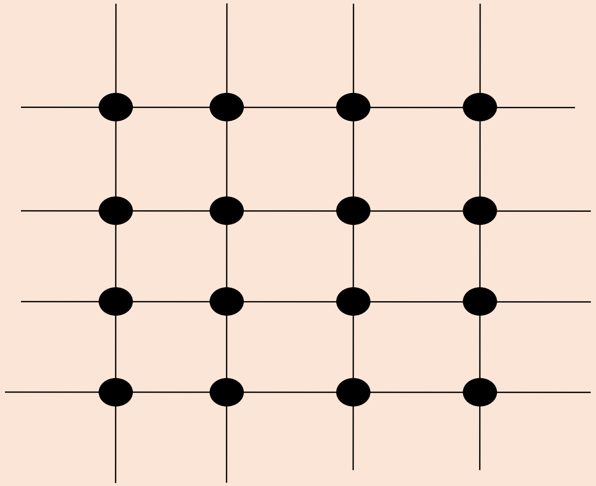
$$|\psi\rangle = a_0(\beta)|00\ldots0\rangle + a_1(\beta)|10\ldots0\rangle + \cdots a_{2^N}(\beta)|11\ldots1\rangle$$

The main idea: encoding Boltzmann weights of a classical spin model in amplitudes of the wave function

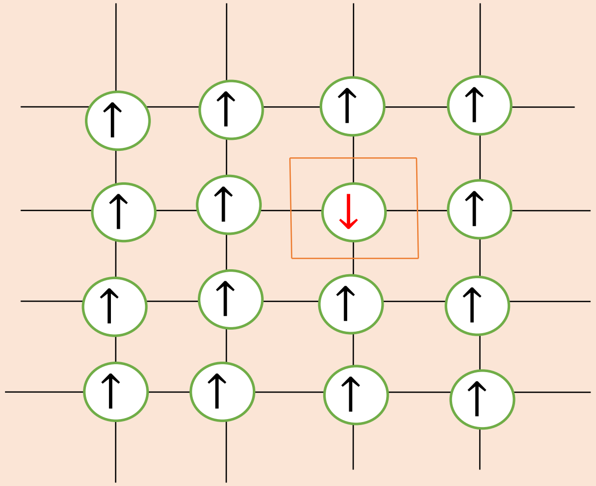
$$a_i(\beta) = e^{-\beta E_i}$$

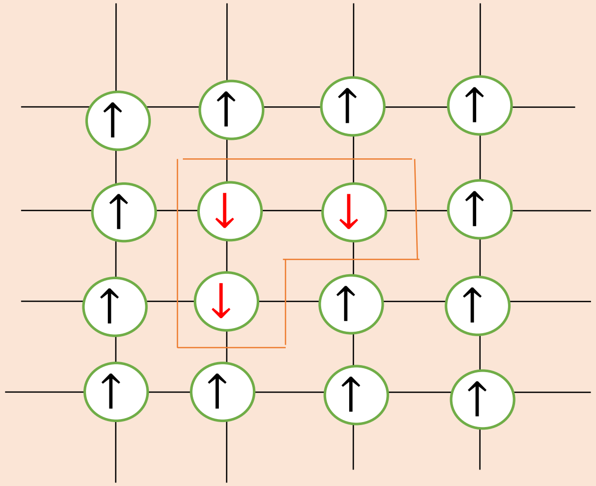
C. Castelnovo, C. Chamon, C. Mudry, P. Pujol, From quantum mechanics to classical statistical physics: Generalized Rokhsar-Kivelson Hamiltonian and the stochastic matrix form decomposition, Ann. Phys. 318(2), 316-344 (2005).

Example: mapping to classical Ising model



$$H = -\sum S_i S_j$$



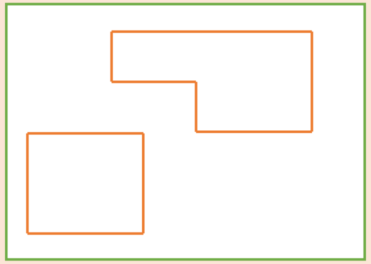


Different spin configurations can be represented by loop configurations.

C. Castelnovo and C. Chamon, Quantum topological phase transition at the microscopic level, Phys. Rev. B 77, 054433 (2008).

Encoding Boltzmann weights in toric code state:

$$|\psi(\beta)> = \sum e^{-\beta E_l}|$$



$$>$$

$$\beta = \frac{1}{k_B T}$$

$|TC\rangle$ Quantum topological phase transition $|00 \ldots .0\rangle$

Topological phase β_c Trivial phase

Classical phase transition

Paramagnetic phase β_c Ferromagnetic phase

Thermal fluctuations in Boltzmann weights = quantum fluctuations in amplitudes

Ground state fidelity and heat capacity:

$$F(\beta) = \langle \psi(\beta) | \psi(\beta + d\beta) \rangle = 1 - \frac{\mathrm{C_v}}{8\beta^2}\,\mathrm{d}\beta^2$$

D. F. Abasto, A. Hamma, P. Zanardi, Fidelity analysis of topological quantum phase transitions, Physical Review A, 78(1), 010301 (2008).

It is shown that there is always such a correspondence for all classical spin models with bi-value spins.

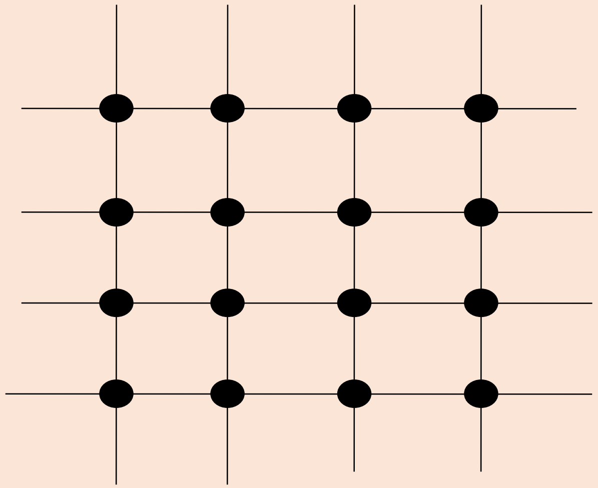
Classical phase transition in a classical spin model → Topological phase transition in a corresponding quantum model

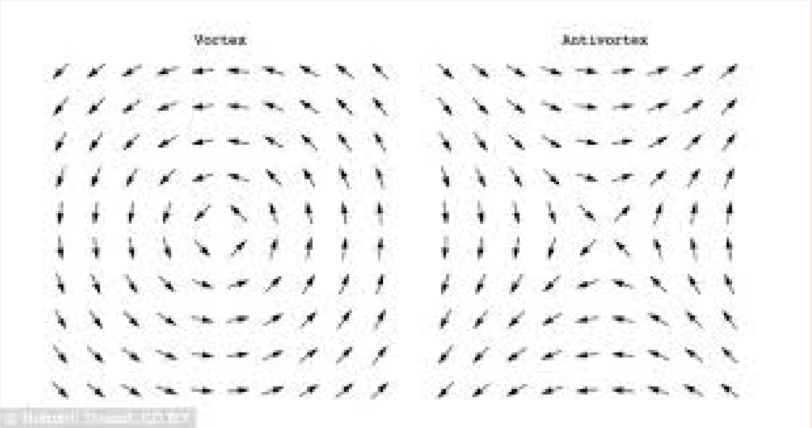
M. H. Zarei, A. Montakhab, classical criticality establishes quantum topological order, Phys. Rev. B 101, 205118 (2020).

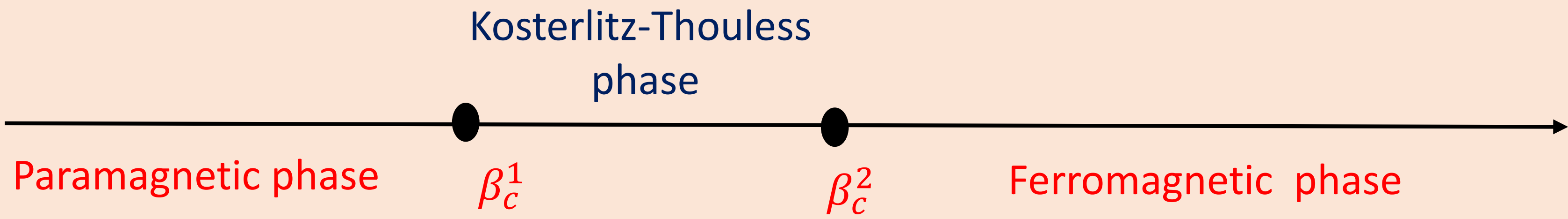
D-state spins: Clock model

$$H = -\sum_{\langle i,j \rangle} Cos(\theta_i - \theta_j)$$

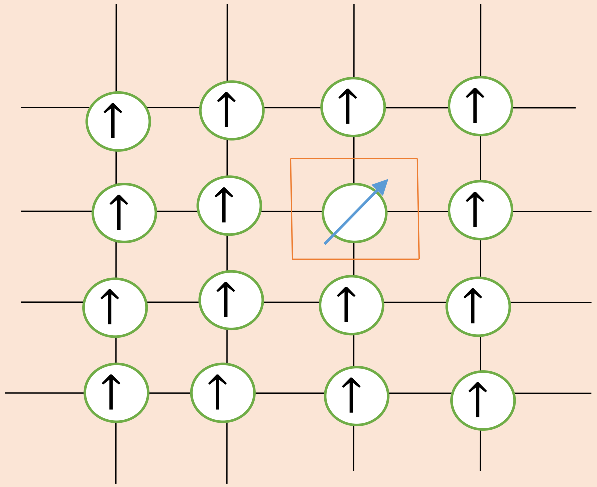
$$\theta = \frac{2\pi m}{d} \quad , \qquad m = 0,1,2,\ldots,d-1$$



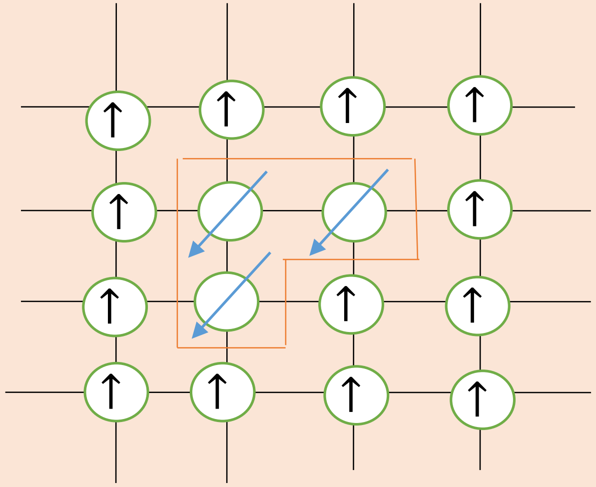




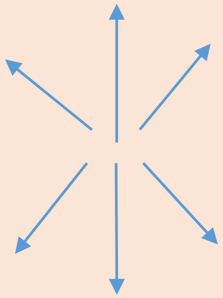
Loop representation for different configurations:



Weighted loop $w = 1$



Weighted loop $w = 4$



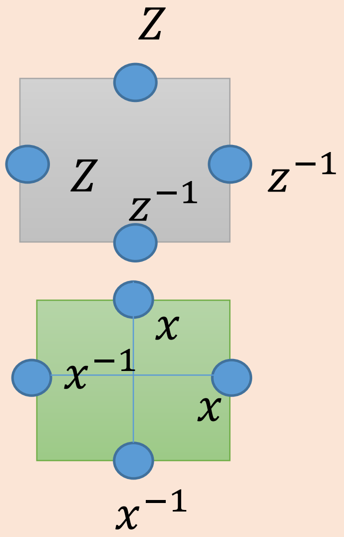
$d = 6$

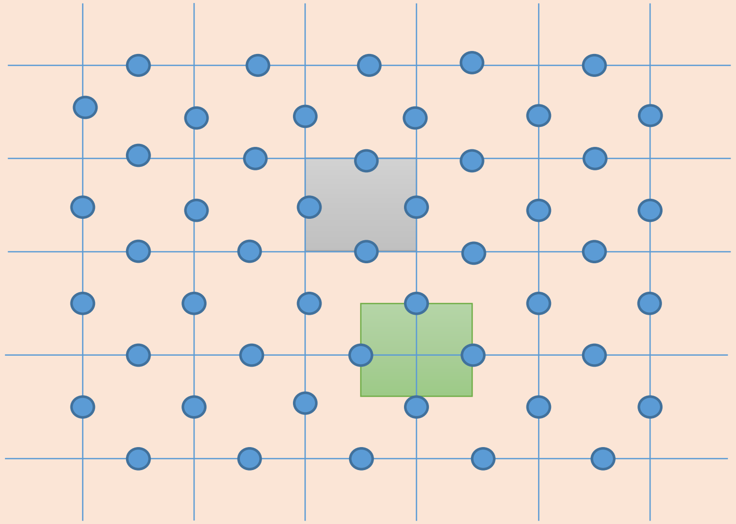
$$w = m_{out} - m_{in}$$

Mapping to Z_d Toric code model

$B_p =$

$A_s =$



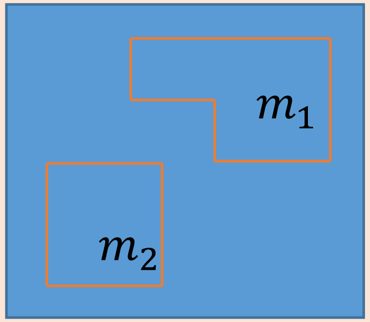


X , Z are generalized Pauli operators:
$\{|0\rangle, |1\rangle, \ldots, |d-1\rangle\}$

$$Z|m\rangle = e^{\frac{2\pi m i}{d}}|m\rangle$$

$$X|m\rangle = |m+1\rangle$$

Z_d toric code state: $|TC_d> = \sum |$



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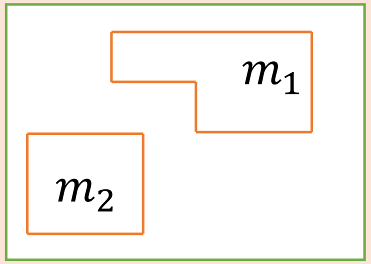
Weighted loop configurations

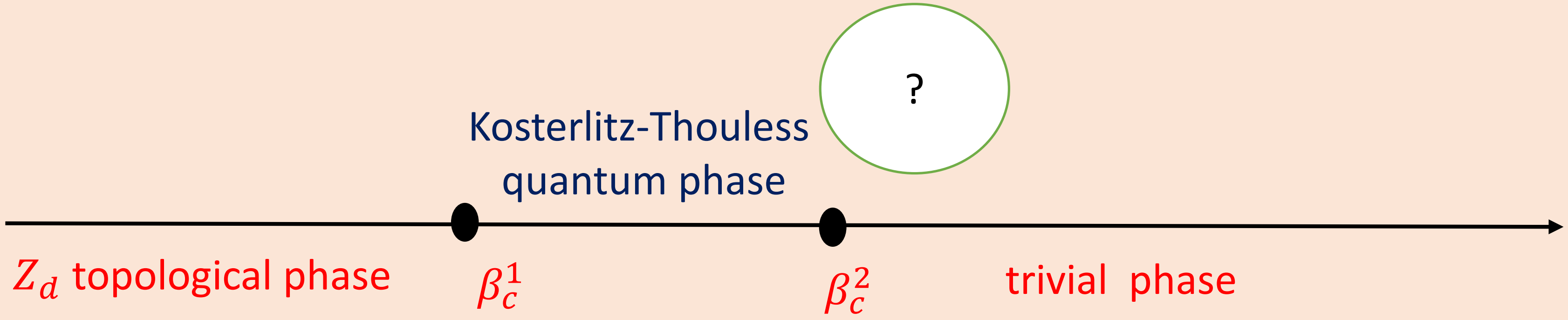
Encoding Boltzmann weights of clock model:

$$|\psi(\beta)> = \sum e^{-\beta E_{clock}} | \quad \text{[loop configuration]} \quad >$$



Weighted loop
configurations

Identifying a KT quantum phase:



M. H. Zarei, Kosterlitz-Thouless phase and Z_d topological quantum phase, arxiv:2004.1469

Outlooks:

- Boltzmann weights of a classical spin model can be encoded in amplitudes of a quantum state
- Phase transition in a classical spin model is mapped to a topological phase transition in a quantum model
- Identifying a new topological quantum phase
- Other interesting applications:

1. Phase transition in random-bond spin models and error threshold of topological quantum codes. M. H. Zarei, A. Ramezanpour, Phys. Rev. A 100, 062313 (2019)

2. Measurement based quantum computing and complete models in statistical mechanics.

V. Karimipour. M. H. Zarei, Phys.Rev. A 86, 052303 (2012).

Thanks