

# Separation of quantum, spatial quantum, and approximate quantum correlations

Salman Beigi

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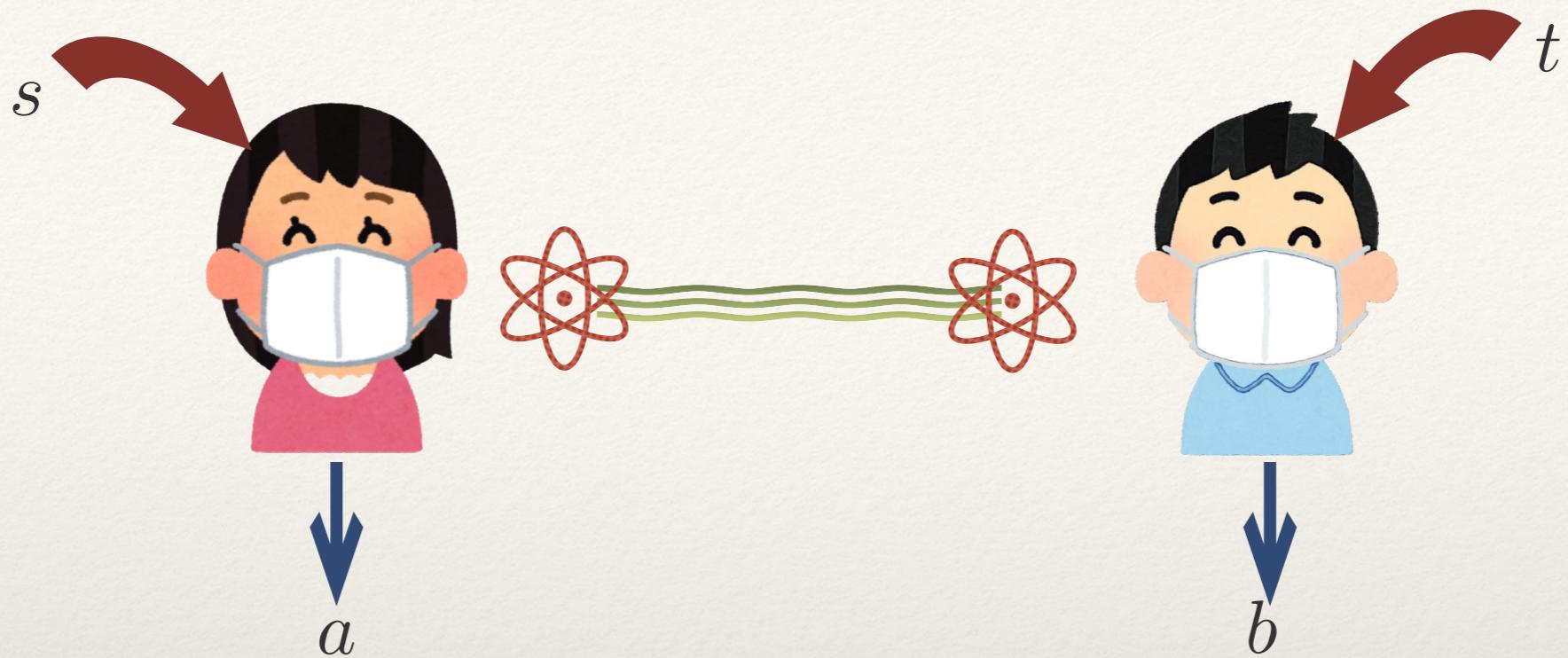


# Two-player one-round games



- ❖ Cooperative game
- ❖ Alice and Bob win if  $(a, b, s, t) \in V$
- ❖ CHSH game:  $a, b, s, t \in \{0, 1\}$ . Winning condition:  $a \oplus b = s.t$

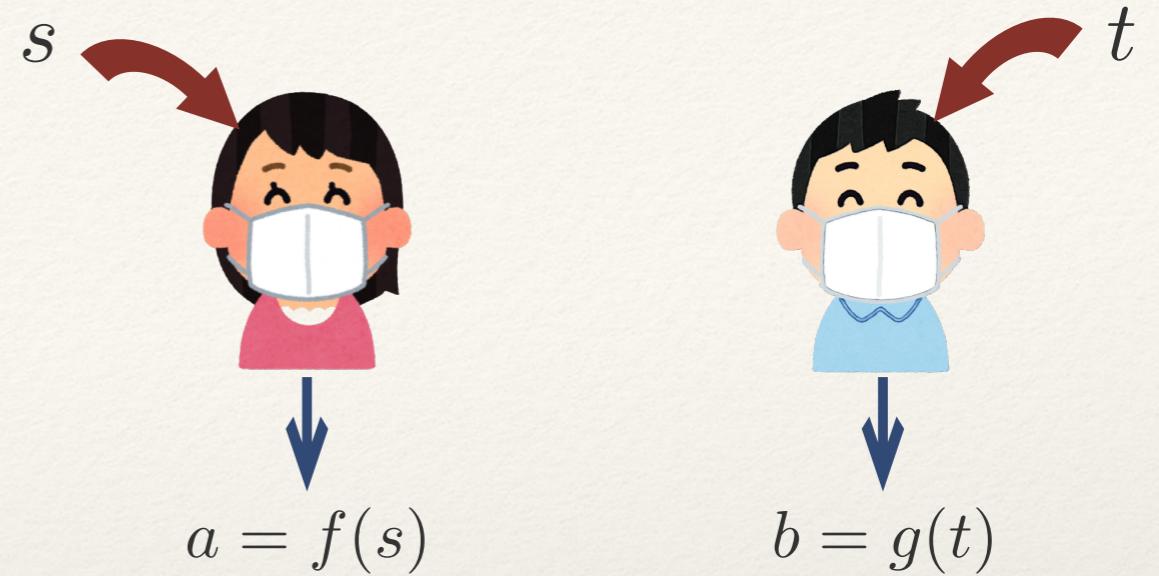
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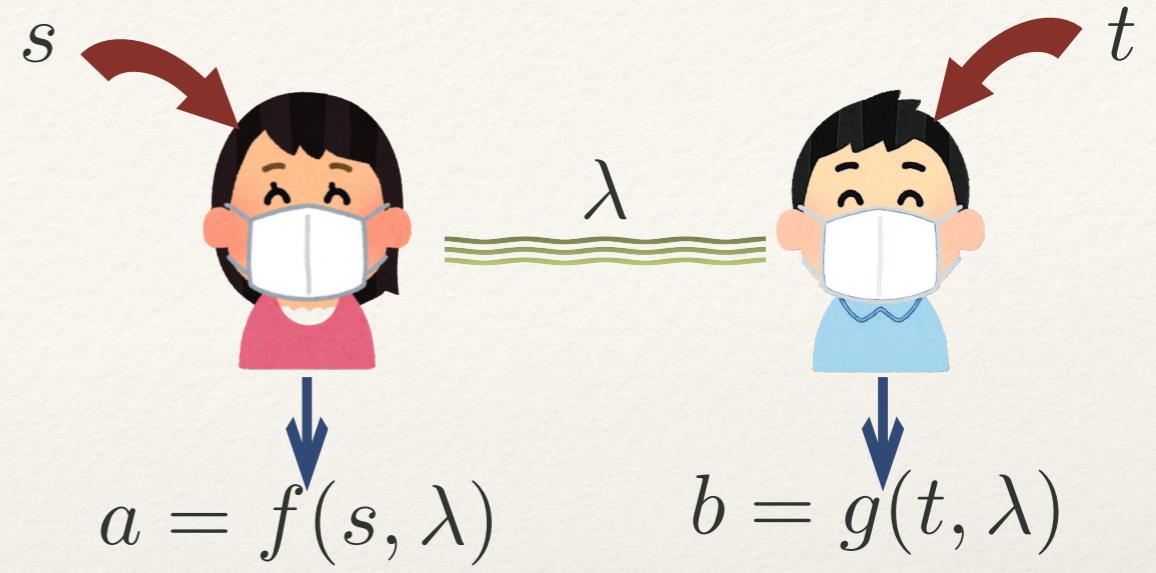
# Classical Strategies

- ❖ Alice and Bob apply **functions**  $f$  and  $g$  on their inputs to produce  $a = f(s)$  and  $b = g(t)$



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- ❖ Alice and Bob apply **functions**  $f$  and  $g$  on their inputs to produce  $a = f(s)$  and  $b = g(t)$
- ❖ If they **share randomness**  $\lambda$  then they compute  $a = f(s, \lambda)$  and  $b = g(t, \lambda)$

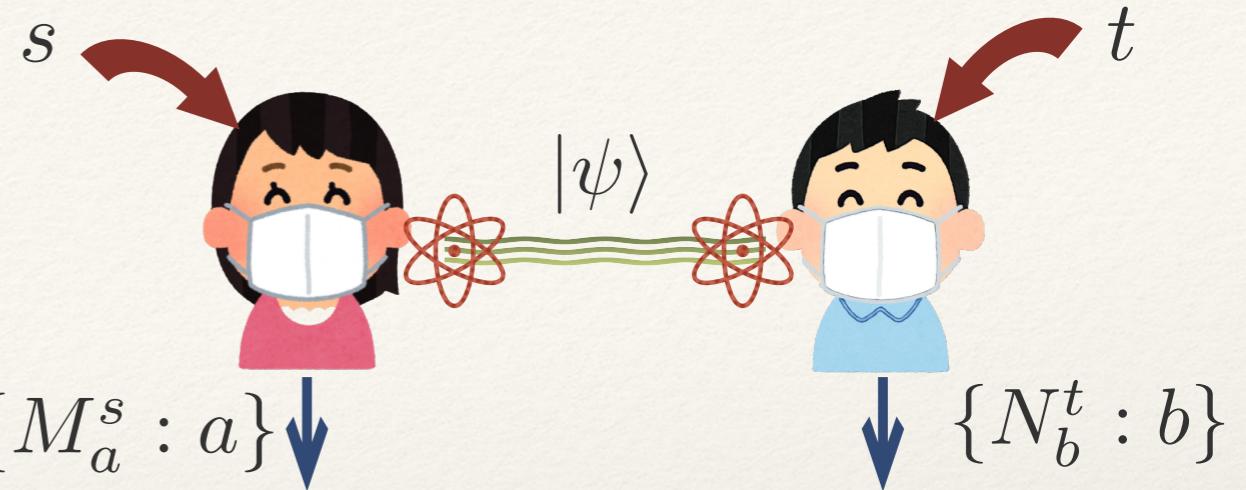


$p(a, b|s, t) =$  probability of outputs  $a$  and  $b$ , when the inputs are  $s$  and  $t$

$$p(a, b|s, t) = \sum_{\lambda} p(\lambda) p(a|s, \lambda) p(b|t, \lambda)$$

# Quantum Strategies

- ❖ Alice and Bob share quantum state  $|\psi\rangle$
- ❖ They apply measurements  $\{M_a^s : a\}$  and  $\{N_b^t : b\}$



$$p(a, b | s, t) = \langle \psi | M_a^s \otimes N_b^t | \psi \rangle$$

Vector in  $\mathcal{H} \otimes \mathcal{H}$

Positive semidefinite  
Operators acting on a  
Hilbert space  $\mathcal{H}$

$$\sum_a M_a^s = I_{\mathcal{H}} \quad \forall s,$$

$$\sum_b N_b^t = I_{\mathcal{H}} \quad \forall t.$$

# Example: CHSH game

$$a, b, s, t \in \{0, 1\}$$

Winning condition:  $a \oplus b = s.t$

## *Classical Strategy*

- ❖ Alice and Bob always output  $a=0$  and  $b=0$ .
- ❖ With probability  $3/4$  we have  $s.t=0$
- ❖ Thus with probability  $3/4$  Alice and Bob win
- ❖ Maximum probability of winning is  $3/4$

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## *Quantum Strategy*

- ❖  $|\psi\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$
- $M_0^0 - M_1^0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$      $M_0^1 - M_1^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$
- $N_0^0 - N_1^0 = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$      $N_0^1 - N_1^1 = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -1 \\ -1 & -1 \end{pmatrix}$
- ❖ For all  $s, t, a, b$  with  $a \oplus b = s.t$  :
- $\langle \psi | M_a^s \otimes N_b^t | \psi \rangle = \frac{1+\sqrt{2}}{2\sqrt{2}}$
- ❖ Wining probability is  

$$\frac{1+\sqrt{2}}{2\sqrt{2}} > 3/4$$

Bell's Nonlocality

# Hierarchy of Quantum Correlations

$$p(a, b|s, t) = \langle \psi | M_a^s \otimes N_b^t | \psi \rangle = \langle \psi, M_a^s \otimes N_b^y \psi \rangle$$

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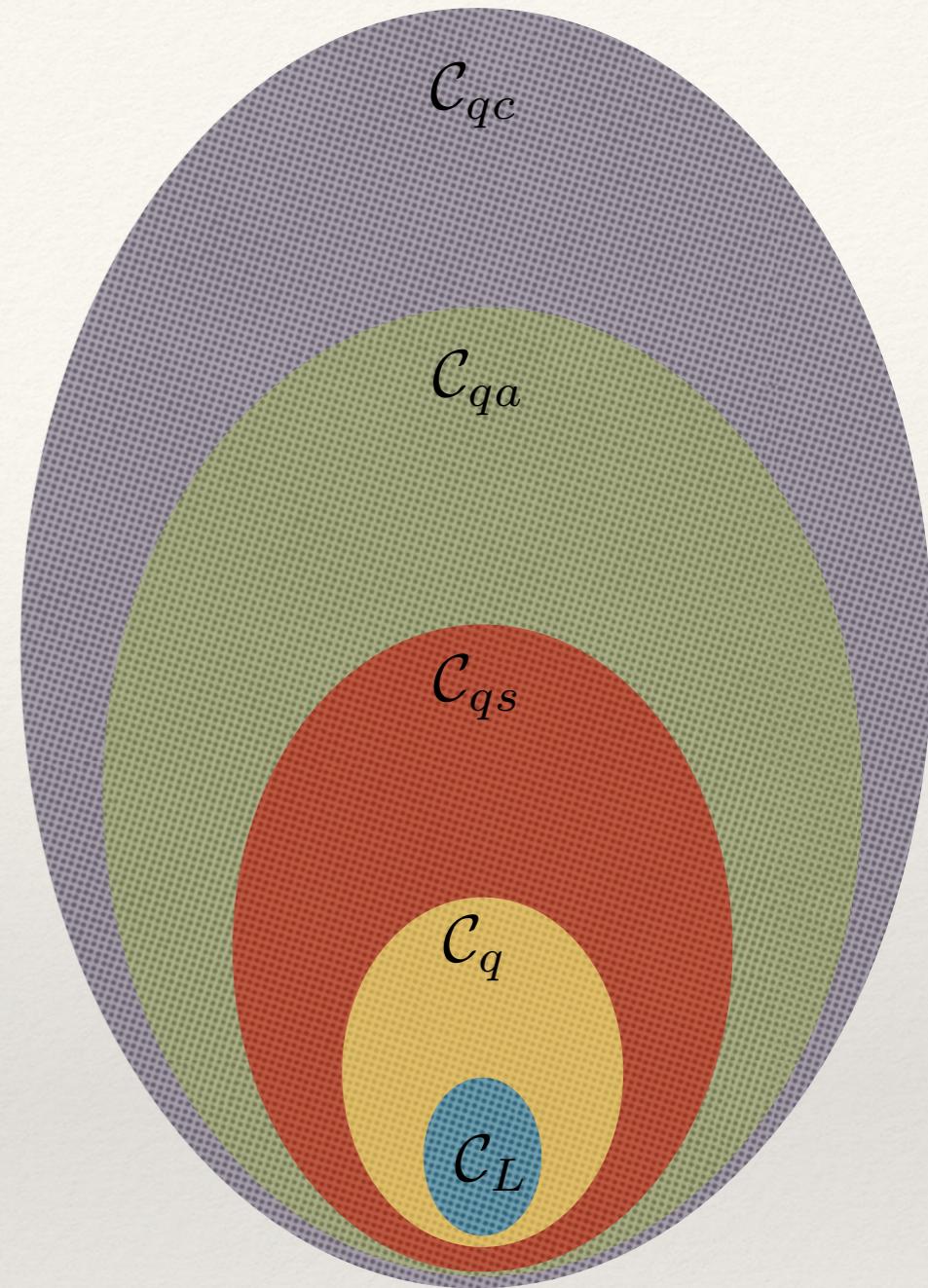
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## Question?

# Hierarchy of Quantum Correlations

$$\mathcal{C}_L \subseteq \mathcal{C}_q \subseteq \mathcal{C}_{qs} \subseteq \mathcal{C}_{qa} \subseteq \mathcal{C}_{qc}$$

- ❖ Bell (1964):  $\mathcal{C}_L \neq \mathcal{C}_q$

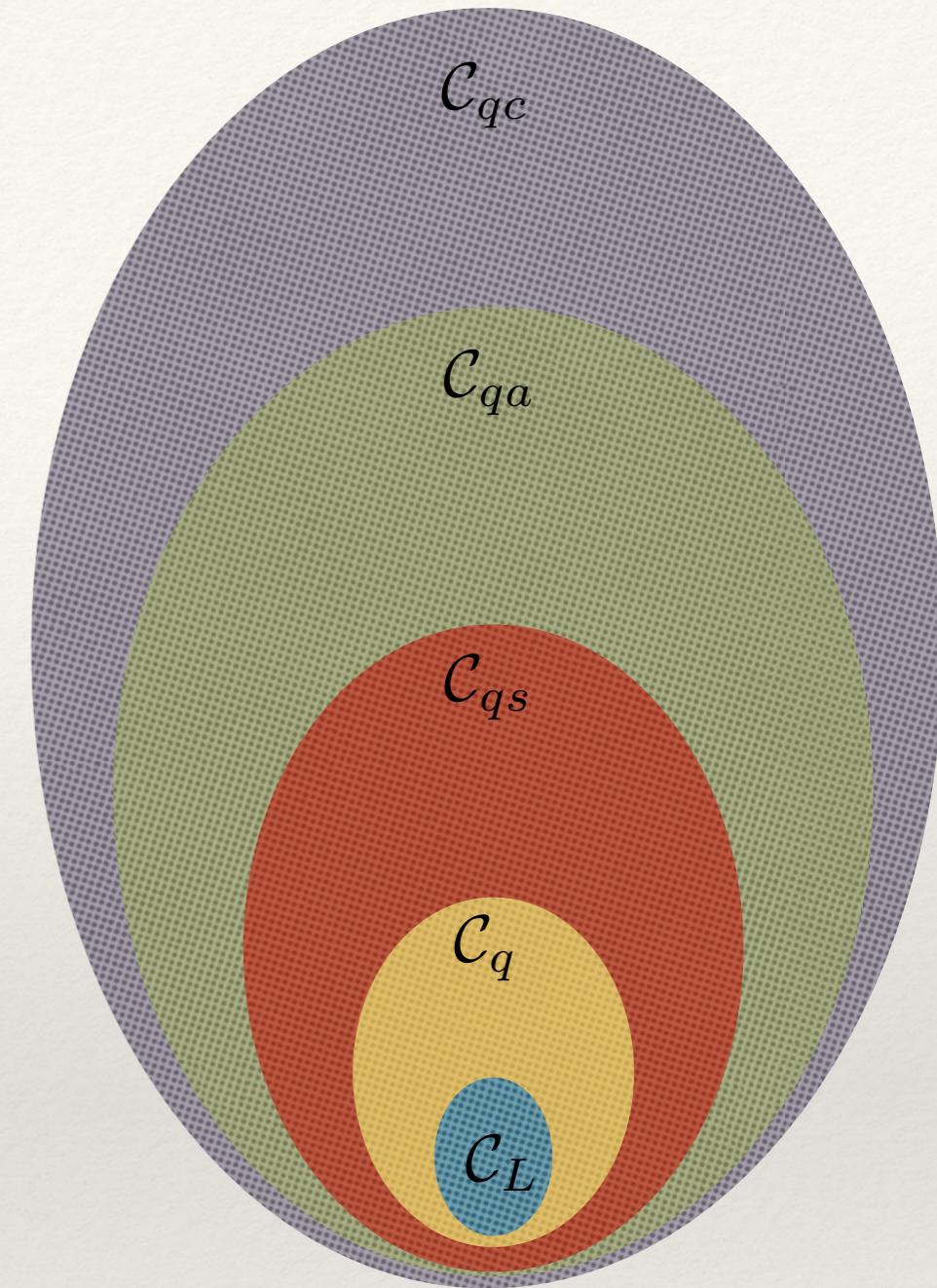


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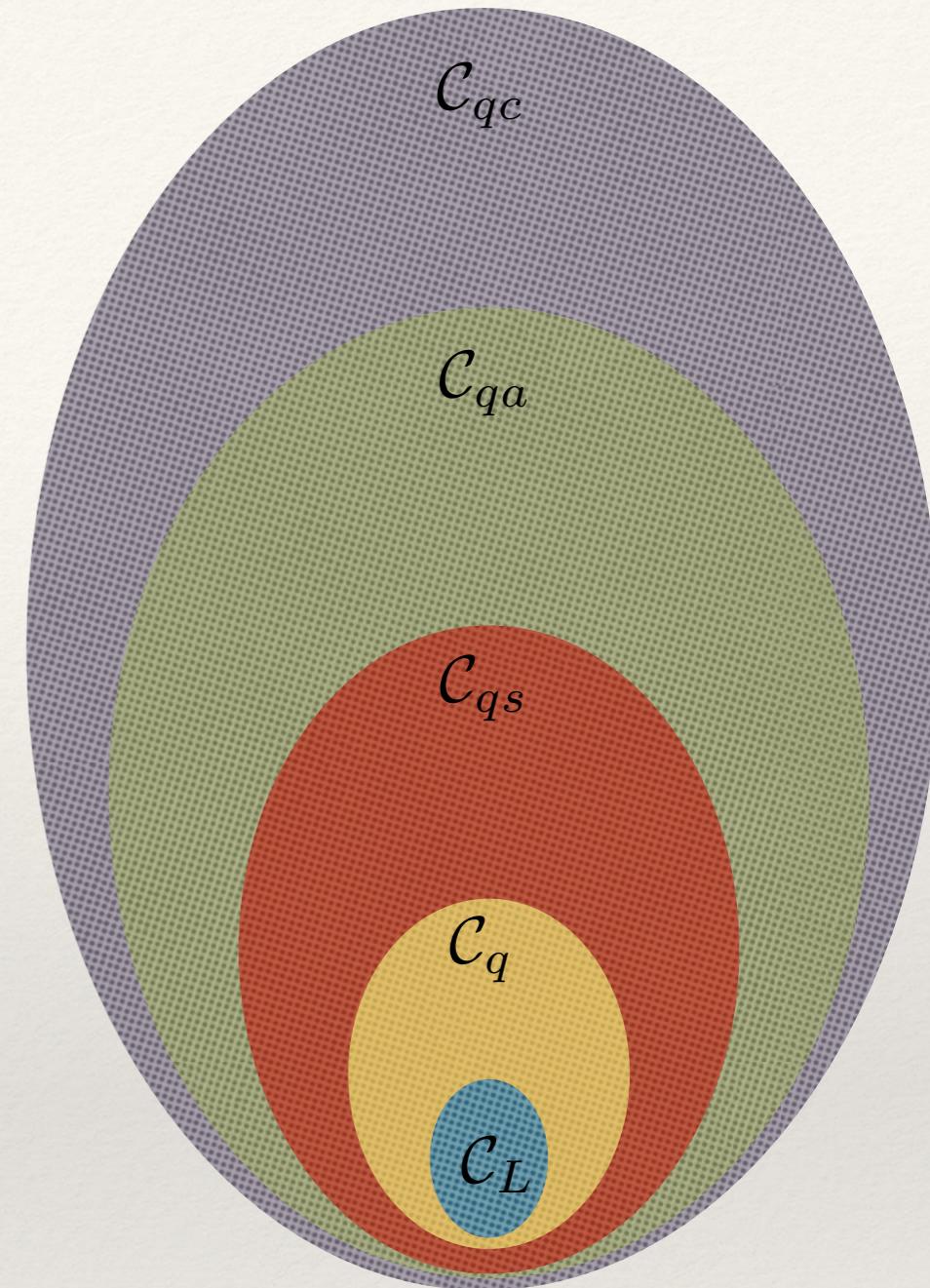
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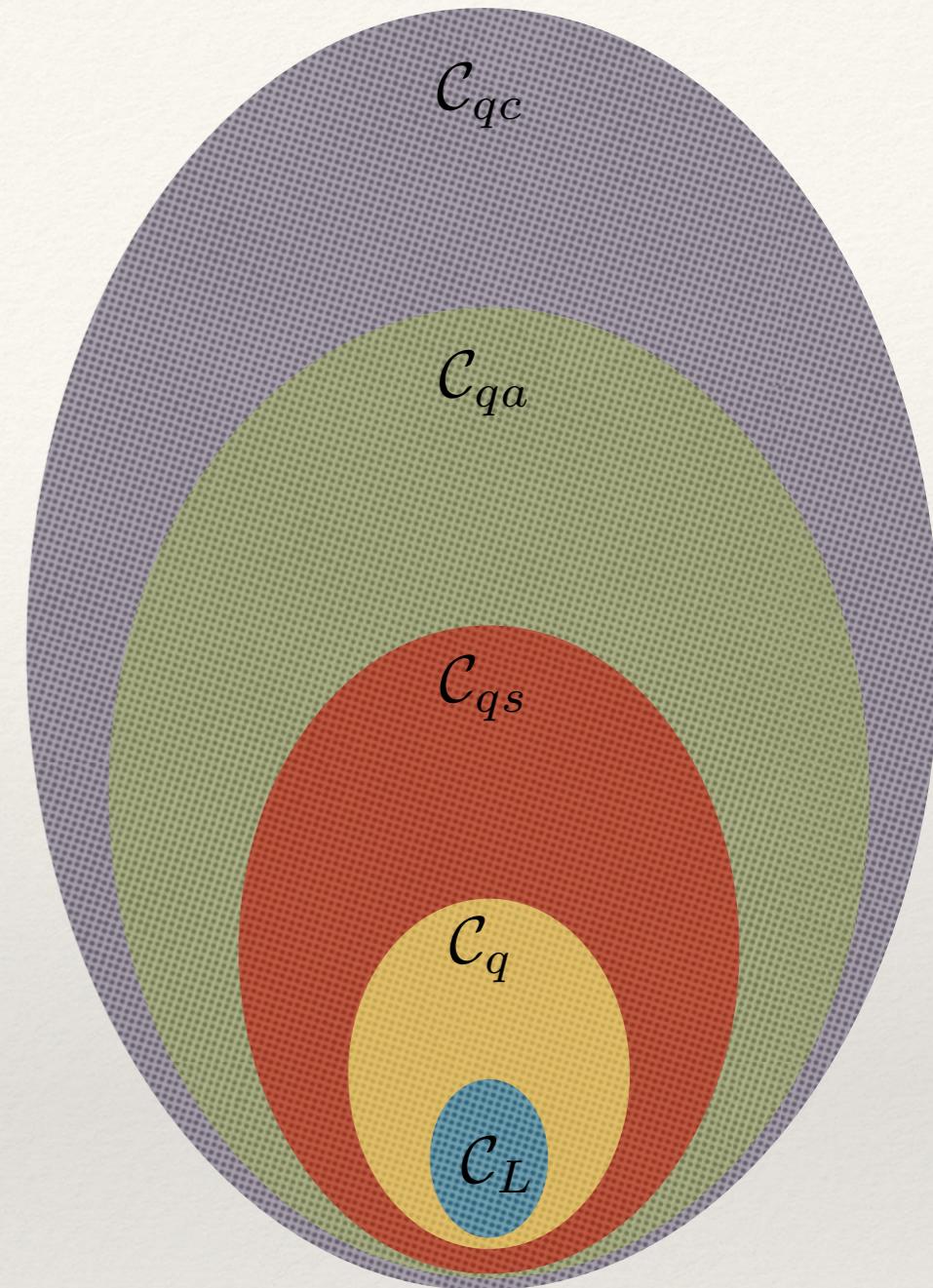
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They indeed prove **MIP\* = RE**  
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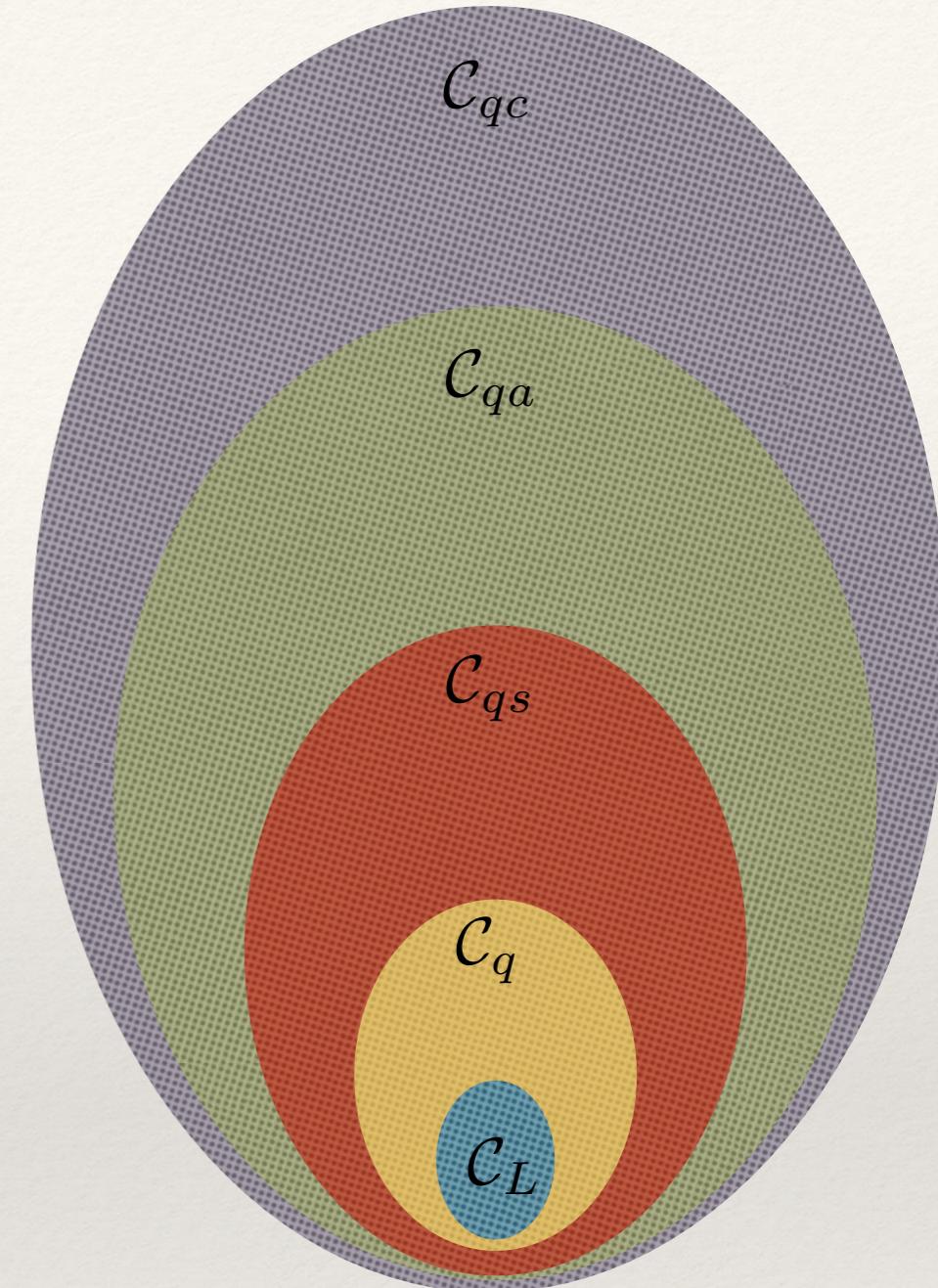
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All inclusions are strict

**Our results:** Improvements in  $\mathcal{C}_q \neq \mathcal{C}_{qs}$  and  $\mathcal{C}_{qa} \neq \mathcal{C}_{qs}$

# Main results:

$\mathcal{C}_*^{(n_A, n_B, m_A, m_B)}$ : set of correlations with  $s \in [n_A], t \in [n_B]$   
and  $a \in [m_A], b \in [m_B]$

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Dykema et al and Musat & Rordam improved to  $(5, 5, 2, 2)$   
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# Proof ideas (I): self-testing

The quantum strategy for winning the CHSH game with probability  $\frac{1+\sqrt{2}}{2\sqrt{2}}$  is essentially **unique**.

If

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Given  $|\psi\rangle$  the tilted-CHSH game can be played in two different ways!

## Proof ideas (II): entanglement embezzlement

There is  $|\Phi_n\rangle$  such that up to local isometries

$$|\Phi_n\rangle \approx^{\epsilon_n} \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle) \otimes |\Phi_n\rangle \quad \epsilon_n = \frac{\ln 2}{\ln n}$$

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$$\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle) \otimes |\Phi_n\rangle = \frac{1}{\sqrt{C_n}} = \sum_{j=1}^n \frac{1}{\sqrt{2j}} (|0j, 0j\rangle + |1j, 1j\rangle)$$

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Isometry:  $\begin{cases} 0j \mapsto 2j - 1 \\ 1j \mapsto 2j \end{cases}$

$$\text{Inner product: } \geq \frac{1}{C_n} \left( \frac{1}{2} + \frac{1}{2} + \frac{1}{4} + \frac{1}{4} + \dots + \frac{1}{n} + \frac{1}{n} \right)$$

$$\geq \frac{\ln(n/2)}{\ln n} = 1 - \frac{\ln 2}{\ln n}$$

# Proof ideas

[Van Dam & Hayden, arXiv:quant-ph/0201041]

- ❖ Embezzling vector:

$$|\Phi_n\rangle = \frac{1}{\sqrt{C_n}} \sum_{j=1}^n \frac{1}{\sqrt{j}} |j, j\rangle$$

- ❖ Target vector:

$$|v\rangle = \frac{1}{\sqrt{2}} (|00\rangle + |11\rangle)$$

Harmonic number  
 $C_n = \sum_{j=1}^n \frac{1}{j}$

$$|\Phi_n\rangle : \frac{1}{\sqrt{C_n}} \left( \frac{1}{\sqrt{1}} \quad \frac{1}{\sqrt{2}} \quad \frac{1}{\sqrt{3}} \quad \dots \frac{1}{\sqrt{n}} \right)$$

$$|v\rangle \otimes |\Phi_n\rangle : \frac{1}{\sqrt{C_n}} \left( \begin{array}{cccc} \frac{1}{\sqrt{2 \cdot 1}} & \frac{1}{\sqrt{2 \cdot 2}} & \frac{1}{\sqrt{2 \cdot 3}} & \dots \frac{1}{\sqrt{2 \cdot n}} \\ \frac{1}{\sqrt{2 \cdot 1}} & \frac{1}{\sqrt{2 \cdot 2}} & \frac{1}{\sqrt{2 \cdot 3}} & \dots \frac{1}{\sqrt{2 \cdot n}} \end{array} \right)$$

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$$\text{Inner product} \geq \frac{1}{C_n} \times \left( \frac{1}{2} + \frac{1}{2} + \frac{1}{4} + \frac{1}{4} + \dots + \frac{1}{n} + \frac{1}{n} \right)$$

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# Summary

- ❖ Introduced the hierarchy of correlations

$$\mathcal{C}_L \subseteq \mathcal{C}_q \subseteq \mathcal{C}_{qs} \subseteq \mathcal{C}_{qa} \subseteq \mathcal{C}_{qc}$$

- ❖ We now know that all the inclusions are strict.
- ❖ **Problem:** find the smallest parameters  $(n_A, n_B, m_A, m_B)$  for which the separations work.
- ❖ We showed  $\mathcal{C}_q \neq \mathcal{C}_{qs}$  for  $(4, 4, 2, 2)$ .
- ❖ **Conjecture:**  $\mathcal{C}_q \neq \mathcal{C}_{qs}$  for  $(3, 3, 2, 2)$ .
- ❖ **Question:** can we use other features in infinite dimensions to prove separations?

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THANK YOU