Remote preparation of quantum states (RSP)

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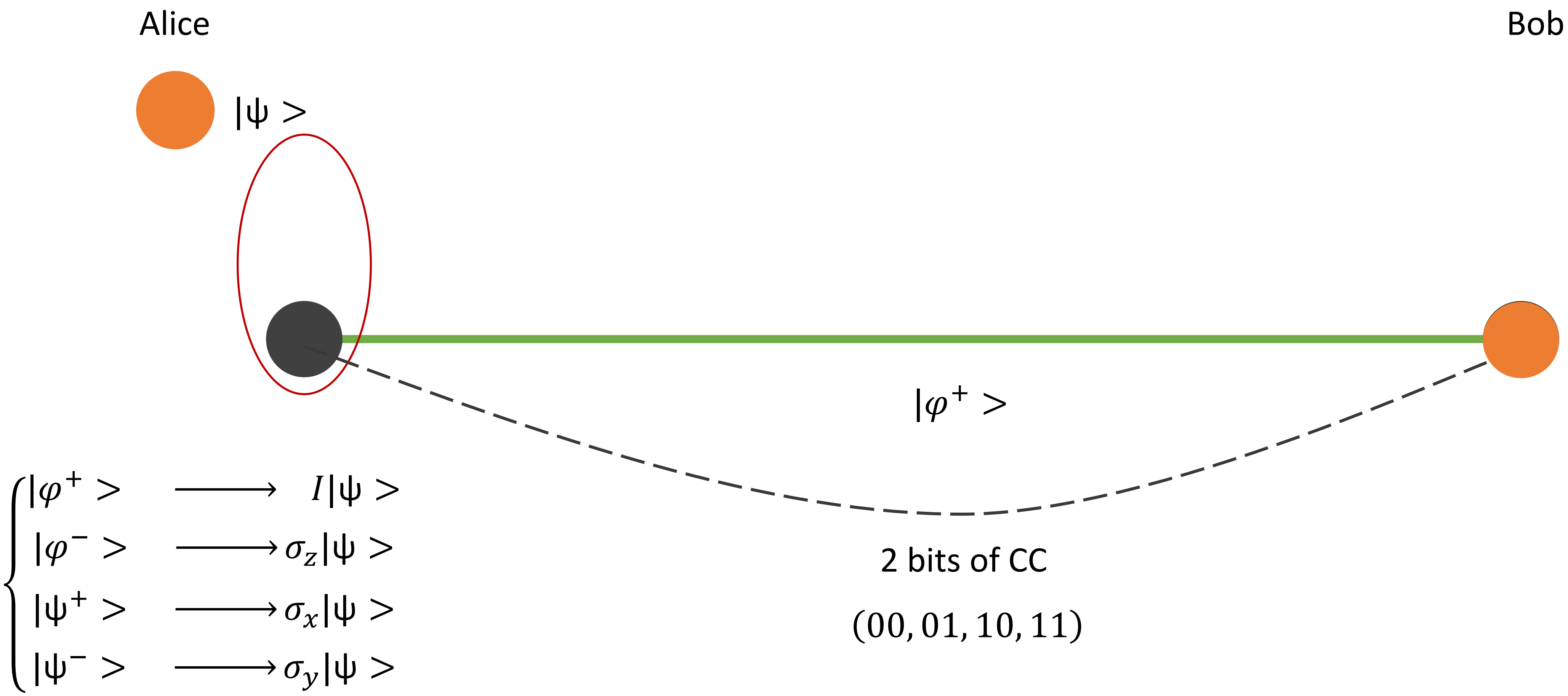
Outline

- Quantum Teleportation
- Remote State Preparation
- Remote State Preparation using quantum discord

Quantum teleportation



Is it possible to teleport a quantum state using less resources ?

YES

Remote State Preparation

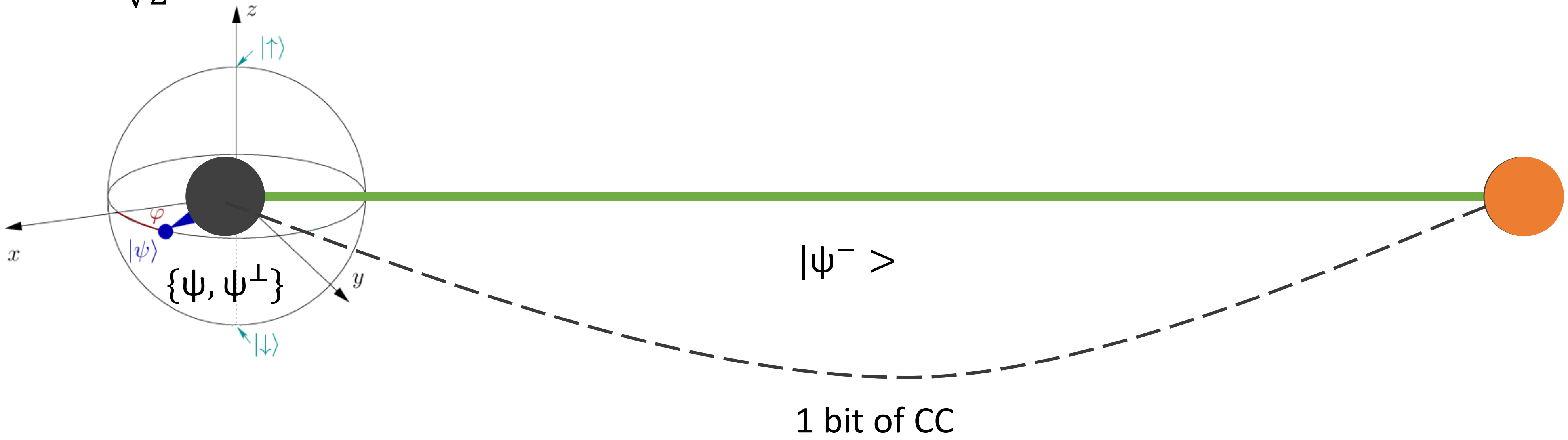
Remote State Preparation

Alice

Bob

$|\psi> = \alpha|0> + \beta|1>$

$|\psi> = \frac{1}{\sqrt{2}}(|0> + e^{i\varphi}|1>)$



$$\begin{cases} |\psi^{\perp}> \longrightarrow |\psi> \\ |\psi> \longrightarrow |\psi^{\perp}> \end{cases}$$

A. K. Pati 2008 quant-ph/9907022

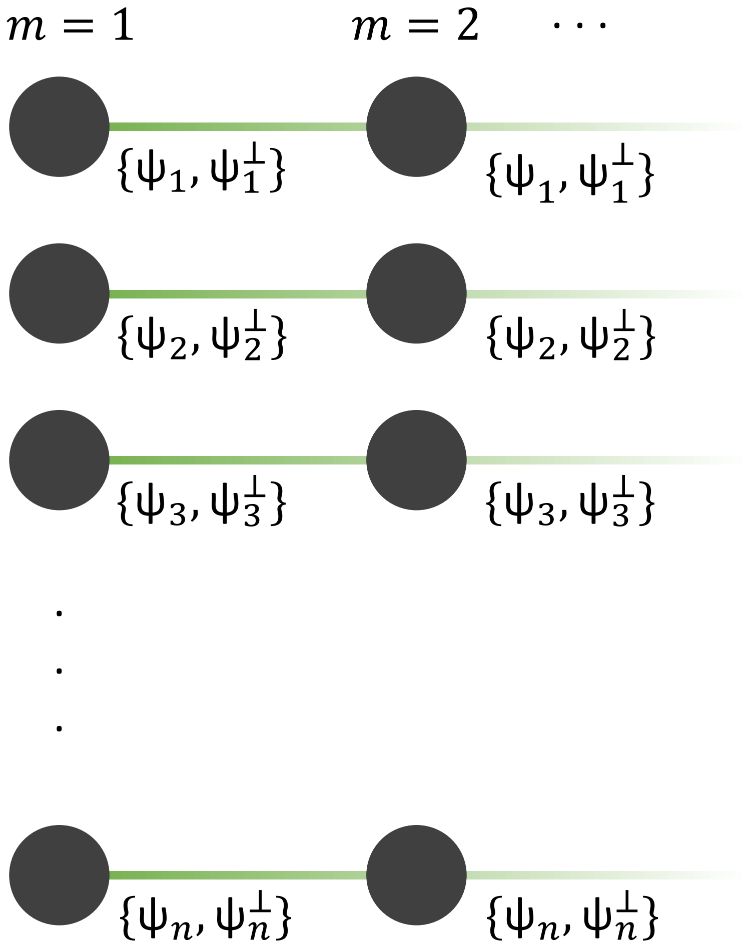
General case

Alice

Unlimited source of $|\psi^- >$

Bob

$\psi_1, \psi_2, \ldots, \psi_n$



$$\Pr\{success\} = \frac{1}{2^n}$$

$$\Pr\{failure\} = (1 - \frac{1}{2^n})^m \leq e^{-m/2^n} \leq \epsilon$$

$$m \;\geq\; 2^{n + \log \ln 1/\epsilon}$$

$$if\ \epsilon = e^{-n} \;\rightarrow m \;\geq\; 2^{n + \log n}$$

Alice --→ Bob

$$b = \log_2 2^{n + \log n}\ \ bits\ of\ CC$$

$$b/_n = 1 + {\log n}/_n \cong 1$$

Bennett, DiVincenzo, Smolin, Terhal 2008 quant-ph/0006044

What about ebits?

Alice ---→ Bob

CC

$\psi_1, \psi_2, \ldots, \psi_n$

Bob is completely ignorant of the qubit states

$S(\rho) = 0.81$

$\rho^{\otimes n} = (\frac{1}{2} I)^{\otimes n}$

$m = 1$ $\quad$ $m = 2$ $\quad \cdots$

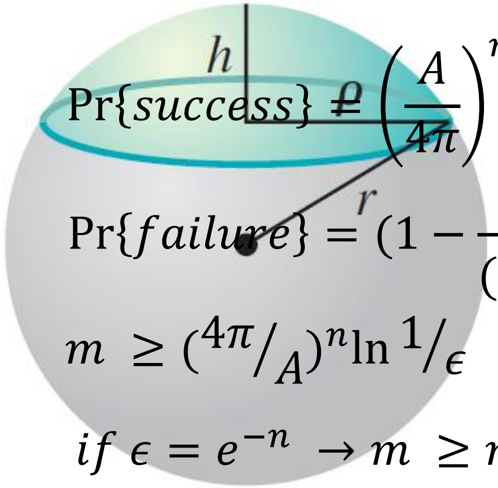
$R(1,1)$ $\quad R(1,2)$

$R(2,1)$ $\quad R(2,2)$

$R(3,1)$ $\quad R(3,2)$

$\vdots$ $\quad \vdots$

$R(n,1)$ $\quad R(n,2)$



$\Pr\{success\} = \left(\frac{A}{4\pi}\right)^n$

$\Pr\{failure\} = (1 - \frac{1}{(4\pi/A)^n})^m \leq e^{-m/(4\pi/A)^n} \leq \epsilon$

$m \geq (4\pi/A)^n \ln 1/\epsilon$

$if\ \epsilon = e^{-n} \rightarrow m \geq n(4\pi/A)^n$

Alice ---→ Bob

$b = \log_2 n(4\pi/A)^n\ bits\ of\ CC$

Bennett, DiVincenzo, Smolin, Terhal 2008 quant-ph/0006044

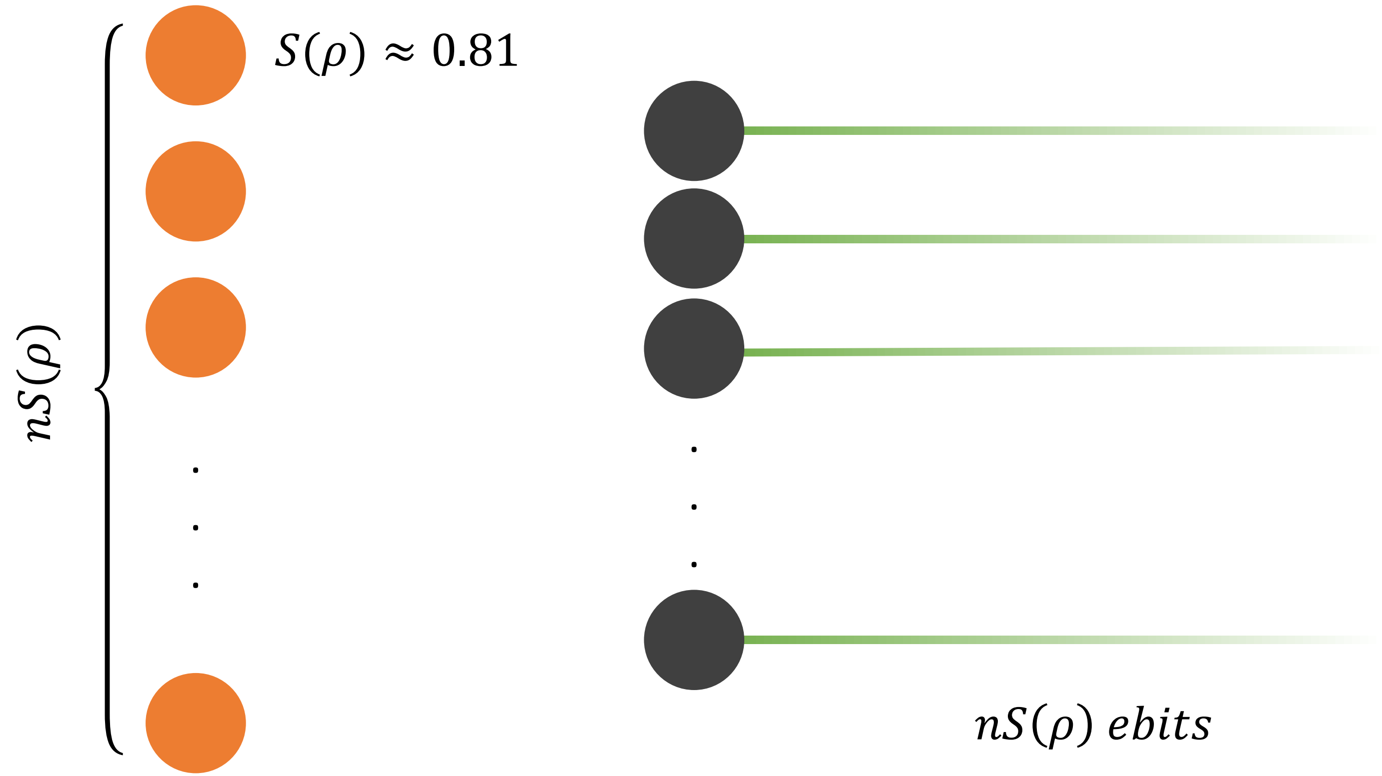
What about ebits?

Alice → Bob

Quantum teleportation

$2nS(\rho)$ *cbits and* $nS(\rho)$ ebits

$\psi_1, \psi_2, \dots, \psi_n$



$b \approx \log_2 n(4\pi/A)^n + 2nS(\rho)$ *bits of CC*

$e = nS(\rho)$ ebits

$e/n = S(\rho) < 1$

Bennett, DiVincenzo, Smolin, Terhal 2008 quant-ph/0006044

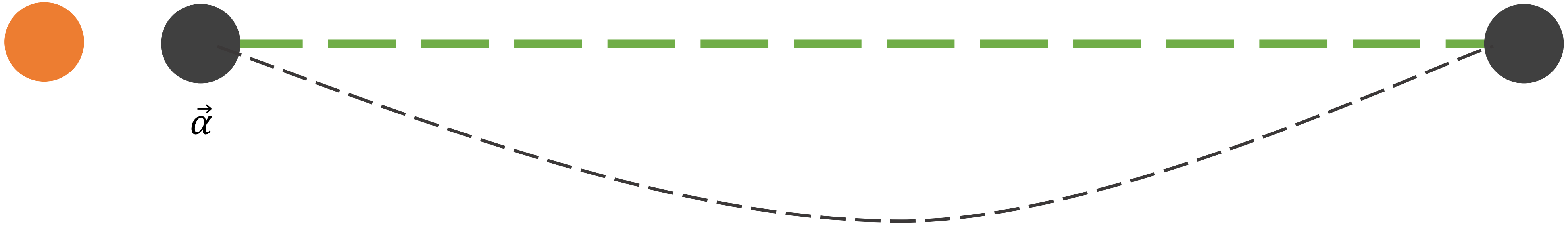
How about separable states? Are they useful?

Alice

Bob

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle + e^{i\varphi}|1\rangle\right)$$

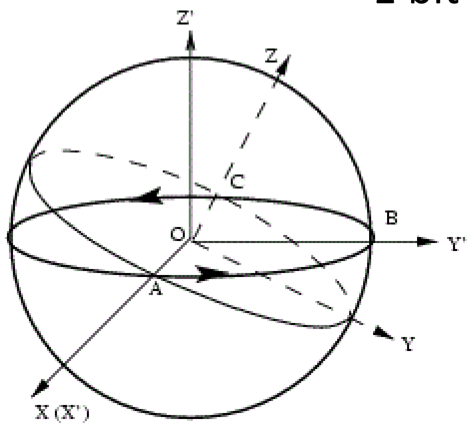
$\vec{s} \perp \vec{\beta} = \hat{z}$ $\quad \vec{a}$ $\quad \vec{b}$



$\vec{\alpha}$

$$\begin{cases} \vec{b}_+ = \dfrac{\left(\vec{b} + E^T\vec{\alpha}\right)}{(1 + \vec{\alpha}.\vec{a})} = \pm\vec{s} \\ \vec{b}_- = \dfrac{\left(\vec{b} - E^T\vec{\alpha}\right)}{(1 - \vec{\alpha}.\vec{a})} = \pm\vec{s} \end{cases}$$

$$E_{kl} = tr[(\sigma_k \otimes \sigma_l)\rho]$$



$$Fidelity = ?$$

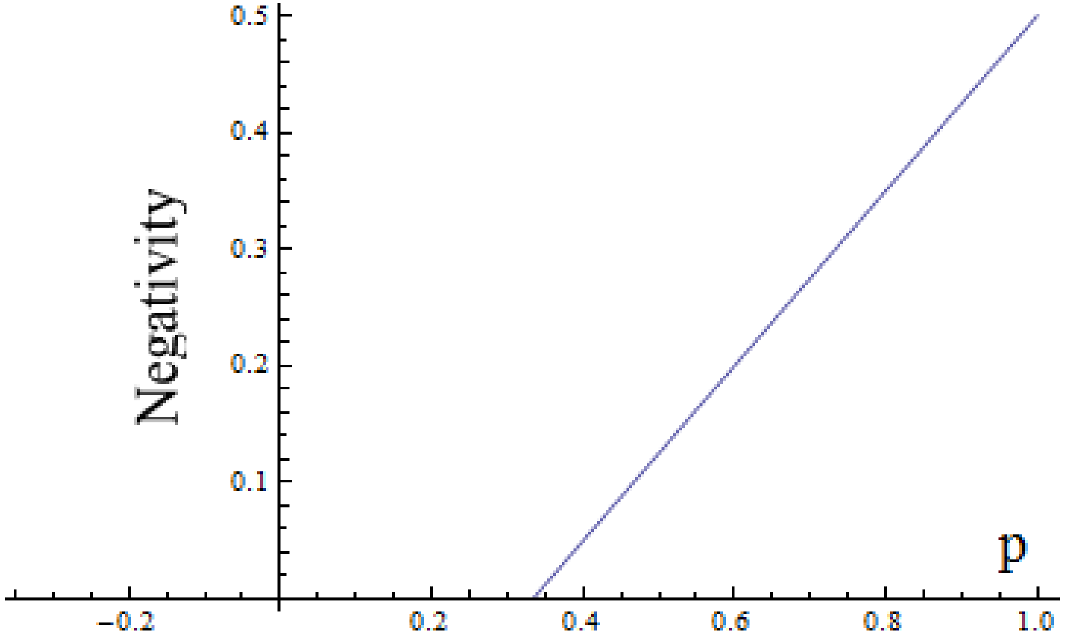
$$\mathcal{F} = \min_{\vec{\beta}} < \mathcal{P}_{opt} > = {}^{1}\!/_{2}\,(E_2^2 + E_3^2)$$

$$E^T E$$

Dakic, Vedral, Zeilinger 2012 quant-ph/1203.1629

Werner States :

$$\rho_w^{(d)} = p|\phi_+^d\rangle\langle\phi_+^d| + \frac{1-p}{d^2} I\otimes I, \qquad p \in \mathbb{R}$$



$\mathcal{F}_w(p=1) = 1$

$\mathcal{F}_w(p = 1/3) = 1/9$

$-1/3 \leq p \leq 1/3 \quad \Rightarrow \quad \rho_w^{(d)}$ *is separable,*

$1/3 < p \leq 1 \quad \Rightarrow \quad \rho_w^{(d)}$ *is entangled.*

$$\rho_E = \frac{1-k}{4}|\psi^+\rangle\langle\psi^+| + \frac{1+3k}{4}|\psi^- >< \psi^-| + \frac{1-2t-k}{4}|00 >< 00| + \frac{1+2t-k}{4}|11 >< 11|$$

$k = 1/5, t = 2/5 \qquad \mathcal{F}_E = 1/25$

Dakic, Vedral, Zeilinger 2012 quant-ph/1203.1629

Geometric measure of quantum discord

$$D_A^{(2)}(\rho) = \frac{1}{2}(\|\vec{a}\|^2 + \left\|\vec{E}\right\|^2 - k_{max})$$

$$K = \vec{a}\vec{a}^T + EE^T \quad \|A\|^2 = trA^T A$$

$$\vec{a}, \vec{b} = 0$$

$$D(\rho) = \frac{1}{2}(E_2^2 + E_3^2) = \mathcal{F}$$

Dakic, Vedral, Zeilinger 2012 quant-ph/1203.1629