



# Temperature effects on quantum cloning of states and entanglement<sup>1</sup>

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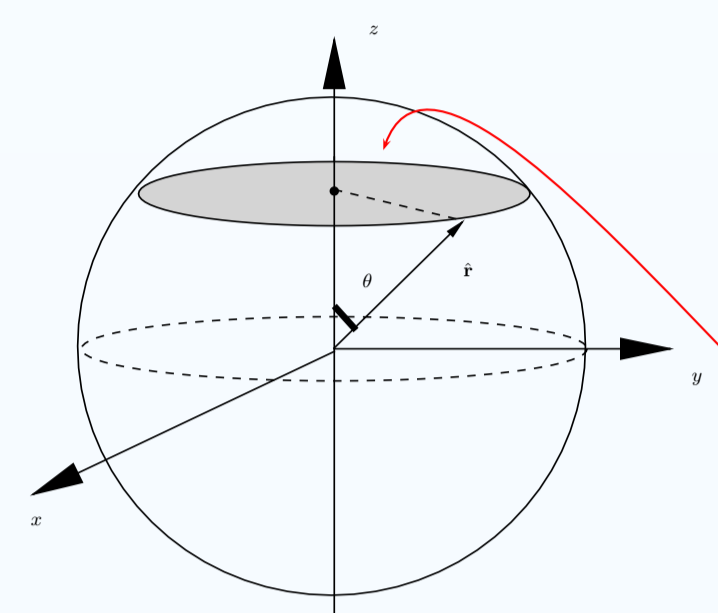
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## Introduction

- No-cloning theorem of { quantum states<sup>2</sup>, entanglement<sup>3</sup> }
- Quantum computation and cloning
- Approximate quantum cloners:

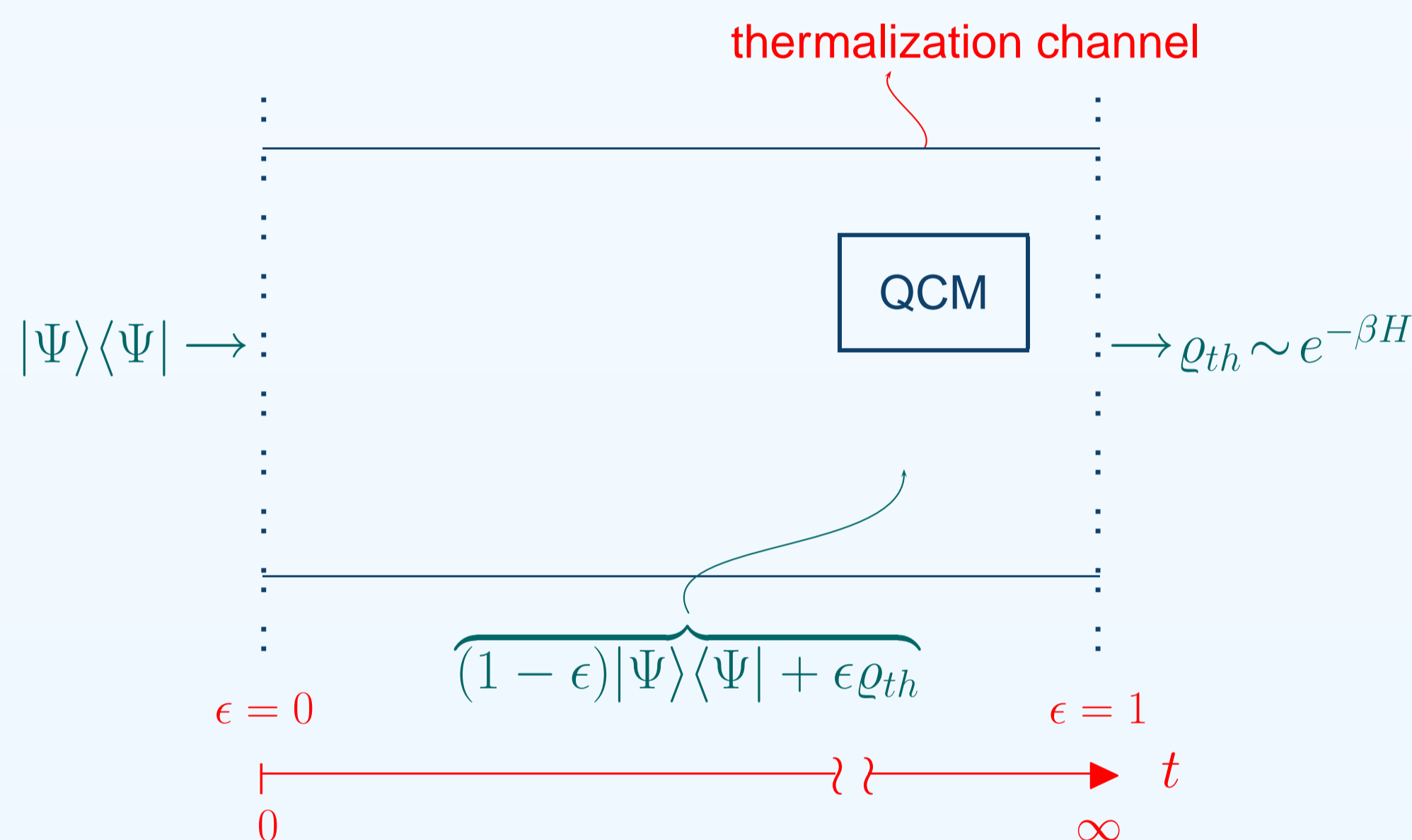


- universal cloning (UC) machines<sup>2</sup>
- phase-covariant cloning (PCC) machines of
  - equatorial qubits<sup>2</sup>
  - orbital states<sup>4</sup>
- entanglement cloners<sup>3</sup>

## Our model

**Our aim** is to investigate temperature effects on the performance of quantum cloning machines.

**Decoherence**<sup>5</sup>: Interaction with thermal environment  $\Rightarrow$  **thermalization**: pure states  $\rightsquigarrow$  mixed states:



["QCM" stands for "Quantum Cloning Machine"].

We assume that:  $\begin{cases} t_{env.} \ll \tau_c, T_{diss.} = \min\{T_1, T_2, T_O\} \\ \tau_c \lesssim T_{diss.} \end{cases}$

- $T_1$  and  $T_2$ : time-scales with respect to energy and phase relaxation processes, respectively.
- $T_O$ : time-scale dictated by all other relaxation sources.
- $\tau_c$ : time-scale of the cloning process

## Summary of the results

In the following sections, we show that:

- ▶ when only the blank copy and the ancilla state are affected, a redefinition of cloning transformations removes thermal effects.
- ▶ this thermalization may reduce performance of a quantum cloner even below classical cloners.
- ▶ there exist some instances in which the quality of cloning for phase-covariant cloners is less than that of universal cloners.
- ▶ an optimal entanglement cloner preserves its higher performance (than the other schemes of entanglement broadcasting) even when thermal noise comes into play.

## Dissipative hardware

### (1) Dissipative (mixed) ancilla

The possibility of optimal cloning with any pure state<sup>2</sup>  $\Rightarrow$  optimal fidelity with mixed ancilla is achievable.

### (2) Dissipative (mixed) ancilla + blank

Attaching some new auxiliary system  $M \rightarrow$  redefinition of the cloning transformation<sup>6</sup>  $\Rightarrow$  optimal fidelity again

$\Rightarrow$  optimal cloning with thermally diluted machinery

## Duplicating a thermally diluted qubit

The following matrix transformation can represent optimal universal and phase-covariant clonings:

$$\begin{pmatrix} a & b \\ b^* & 1-a \end{pmatrix}_a \rightarrow \begin{pmatrix} \mu^2 a + \nu^2 & 2\mu\nu b \\ 2\mu\nu b^* & \mu^2(1-a) + \nu^2 \end{pmatrix}_{a(a')}$$

$$\text{Here } \begin{cases} \text{UC} \rightarrow \nu^2 = \frac{1}{6} \\ \text{PCC} \rightarrow \nu^2 = \frac{1}{4} \left(1 - \frac{1}{\sqrt{1+2\tan^4\theta}}\right) \end{cases}$$

Our Hamiltonian model is  $H = \omega_0 \sigma_z / 2$ . We also define  $\eta = \omega_0 \beta / 2$ :

$$\therefore \begin{cases} \omega_0 > 0, T \uparrow \Rightarrow \eta \downarrow: \text{opposed behavior} \\ \omega_0 < 0, T \uparrow \Rightarrow \eta \uparrow: \text{same behavior} \end{cases}$$

**Bures fidelity**:  $F(\rho, \sigma) = \left( \text{Tr} \sqrt{\rho^{1/2} \sigma \rho^{1/2}} \right)^2 \Rightarrow F(|\Psi\rangle, \rho^{out})$ :

$$F(\theta, \epsilon, \eta) = \mu^2 [1 - \epsilon + \epsilon(e^{-\eta} \cos^2 \frac{\theta}{2} + e^{\eta} \sin^2 \frac{\theta}{2}) / Z] + (\mu\nu - \mu^2/2)(1 - \epsilon) \sin^2 \theta + \nu^2,$$

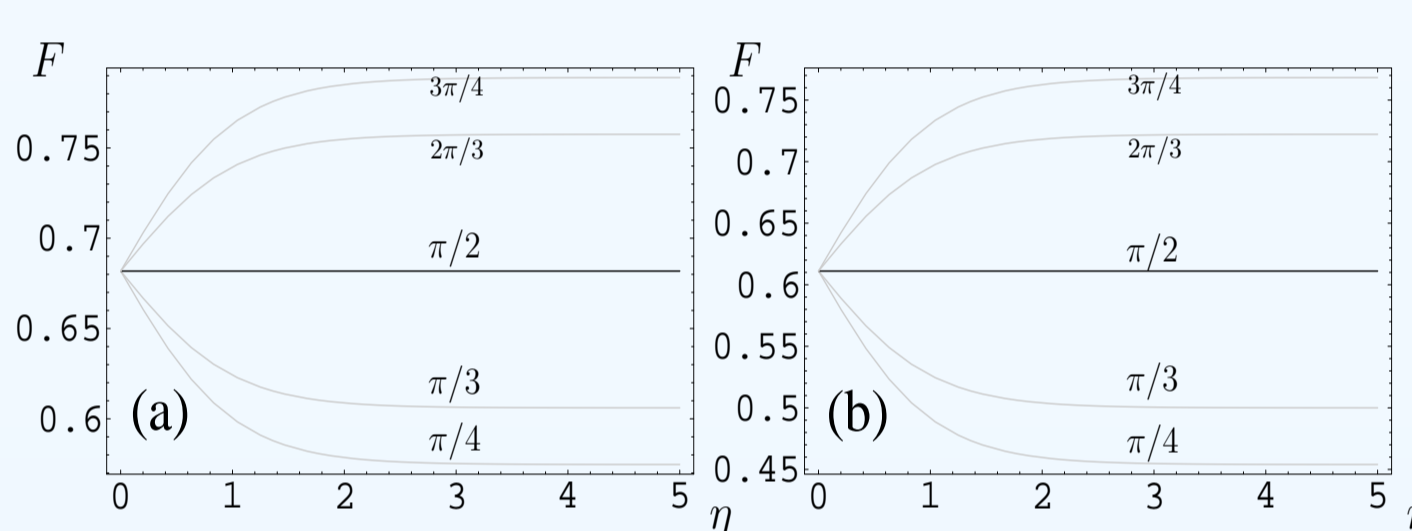
$$\partial_\eta F = \frac{-\mu^2 \epsilon}{2 \cosh^2 \eta} \cos \theta \Rightarrow \forall \epsilon: \begin{cases} \theta < \frac{\pi}{2} \Rightarrow F(\eta) \downarrow \\ \theta = \frac{\pi}{2} \Rightarrow F(\eta) = \text{constant} \\ \theta > \frac{\pi}{2} \Rightarrow F(\eta) \uparrow \end{cases}$$

And in high temperature limit ( $\eta \rightarrow 0$ )

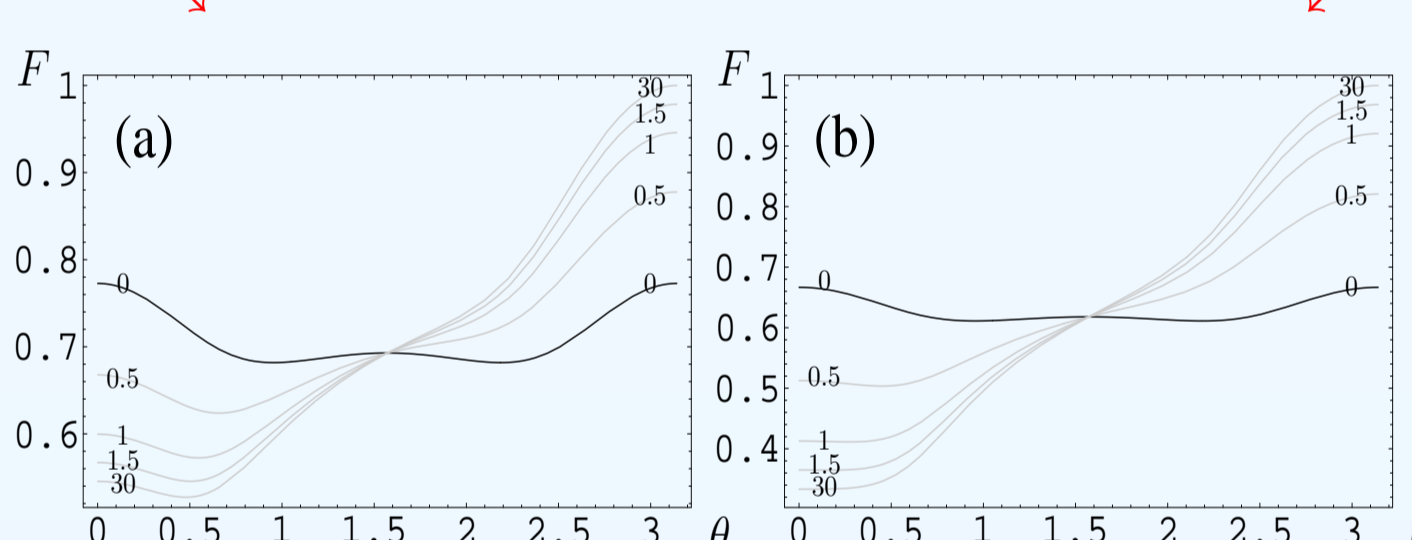
$$\partial_\epsilon F = -\mu(\nu \sin^2 \theta + \frac{\mu}{2} \cos^2 \theta) \Rightarrow \forall \theta: F(\epsilon) \downarrow$$

positive

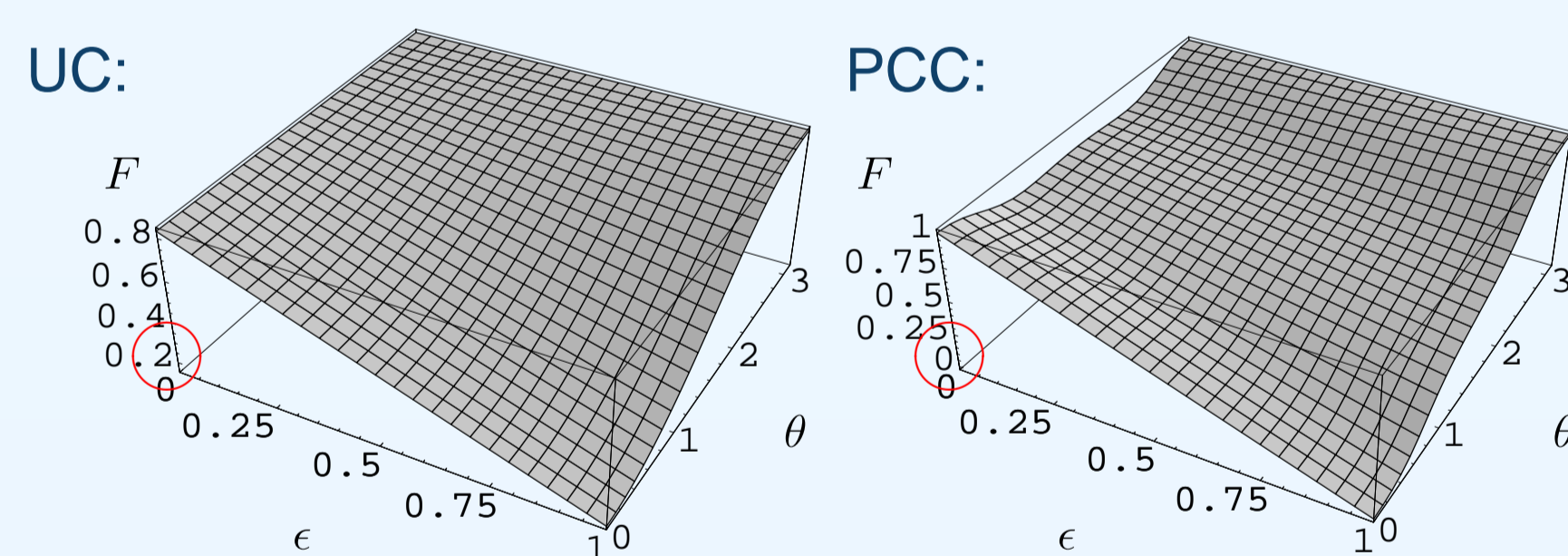
### Universal cloning



### Phase-covariant cloning



At low temperature and  $\omega_0 > 0$  ( $\eta \rightarrow \infty$ ):



universal:  $\forall \theta \in [0, \pi) \Rightarrow F(\epsilon) \downarrow$

phase-covariant: for  $\theta_s \gtrsim 2.52$  and less than  $\pi$  rad  $\Rightarrow F(\epsilon) \uparrow$

**Important point:**

for some  $(\epsilon, \theta, \eta)$  we see that  $F^{UC} > F^{PCC}$

In the case of universal cloning:

$$\epsilon < \frac{\cosh \eta}{e^{-\eta} \sin^2 \frac{\theta}{2} + e^{\eta} \cos^2 \frac{\theta}{2}} \Rightarrow F_{qua.} > F_{class.} = \frac{1}{2}$$

otherwise a classical cloner is better than quantum cloner.

$$\Rightarrow \begin{cases} \eta > 0 \ \& \ \theta \geq \pi/2 \\ \eta < 0 \ \& \ \theta \leq \pi/2 \end{cases} \Rightarrow F_{qua.} > F_{class.}$$

## Cloning of thermal entanglement

**Clean state**:  $|\Psi_\alpha^-\rangle_{ab} = \alpha|01\rangle_{ab} - \sqrt{1-\alpha^2}|10\rangle_{ab}$ :  $\alpha \in \mathbb{R}$  &  $|\alpha| \leq 1$

**Hamiltonian model**:  $H = J(\sigma_x^a \sigma_x^b + \sigma_y^a \sigma_y^b)$

**Via**: local cloning: (i) with two optimal UC machines<sup>7</sup>

- (ii) with UC machine of 4-level quantum states<sup>2</sup>
- (iii) by an optimal entanglement cloner<sup>3</sup>

▶ When  $\epsilon = 0$  (no external noise), we could write the state of

each clone in cases (ii) & (iii), and nonlocal copies of case (i) in the following compact form

$$\rho^{out} = \frac{1-M}{4}(|00\rangle\langle 00| + |11\rangle\langle 11|) + [\frac{1+M}{4} + L(2\alpha^2 - 1)]|01\rangle\langle 01| + [\frac{1+M}{4} - L(2\alpha^2 - 1)]|10\rangle\langle 10| - M\alpha\sqrt{1-\alpha^2}(|01\rangle\langle 10| + |10\rangle\langle 01|),$$

machine parameter

where

$$M_i = (2/3)^2, \quad M_{ii} = 3/5, \quad M_{iii} = 6A^2 + 4AC$$

$$A = \frac{1}{3}\sqrt{\frac{1}{2} + \frac{1}{\sqrt{13}}}, \quad C = \frac{A}{2}(\sqrt{13} - 3), \quad L = \frac{3}{26}(1 + 2M + \sqrt{1 + 4M - 9M^2}).$$

▶ When temperature comes into play, the above equation takes the following general form

$$\rho^{out} = \frac{(M\epsilon + \frac{1-M}{4})}{Z}(|00\rangle\langle 00| + |11\rangle\langle 11|) + [M(\frac{1-\epsilon}{2} + \frac{\epsilon \cosh \gamma}{Z}) + \frac{1-M}{4} + L(1-\epsilon)(2\alpha^2 - 1)]|01\rangle\langle 01| + [M(\frac{1-\epsilon}{2} + \frac{\epsilon \cosh \gamma}{Z}) + \frac{1-M}{4} - L(1-\epsilon)(2\alpha^2 - 1)]|10\rangle\langle 10| - M[(1-\epsilon)\alpha\sqrt{1-\alpha^2} + \frac{\epsilon}{Z} \sinh \gamma](|01\rangle\langle 10| + |10\rangle\langle 01|),$$

in which  $Z = 2(1 + \cosh \gamma)$  and  $\gamma = 2\beta J$ .

$$\therefore \begin{cases} \epsilon = 1 \ \& \ \forall \gamma \\ \alpha = \pm \frac{1}{\sqrt{2}} \ \& \ \forall \epsilon \ \& \ \forall \gamma \end{cases} \Rightarrow \rho^{out} = M \rho^{in} + (\frac{1-M}{4})I$$

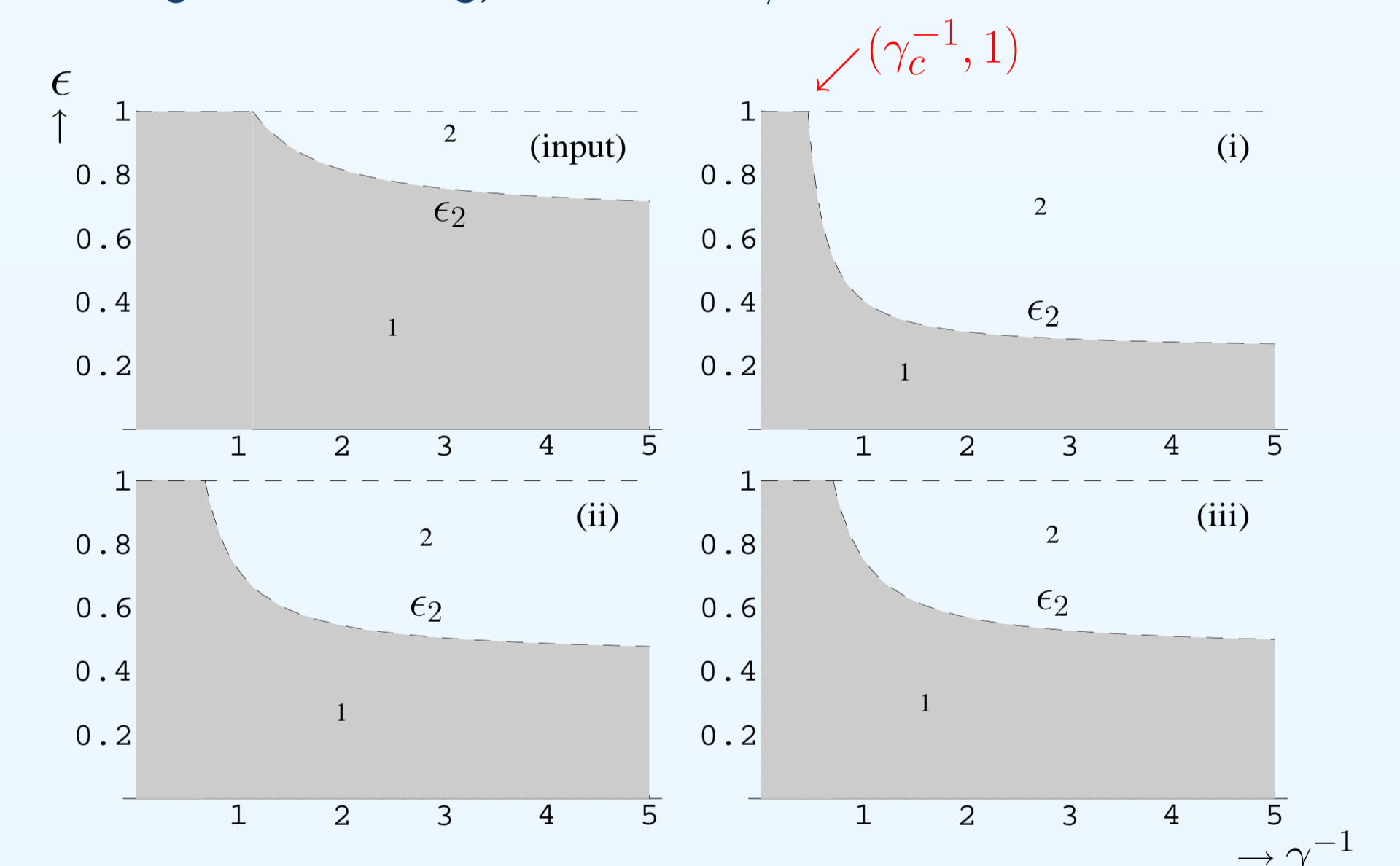
Applying positive partial transposition criterion<sup>8</sup> results in some temperature and state-dependent regions over which the output cloned pairs are inseparable. For example, at room temperature, our clones are entangled when (for more details see<sup>1</sup>)

$$0 \leq \epsilon < (1 - \frac{1}{3M}) \ \& \ |\alpha^2 - \frac{1}{2}| < \frac{\sqrt{(3M(1-\epsilon) - 1)(M(1-\epsilon) + 1)}}{4M(1-\epsilon)}$$

**Remark1.**– For some “ $T$ ”  $\in$  intermediate (high & low) temperatures,  $\exists$  intervals of  $\alpha^2(\epsilon)$  in which the cloned pairs are separable:  $M \uparrow \Rightarrow$  the length of these intervals  $\downarrow$ . Recall:  $M_{iii} > M_{ii} > M_i$ .

**Remark2.**– For a given  $\alpha^2(\epsilon) \in$  moderate (two limits of) temperatures, the range of  $\epsilon(\alpha^2)$  in which the clones are entangled  $\uparrow$  when  $M \uparrow$ .

Entanglement phase diagrams of input and output states (achieved from three different schemes of entanglement cloning/broadcasting), when  $\alpha = 1/\sqrt{2}$  are



The regions labeled by 1 (2) indicate (no-) entanglement regions. Here, we also have  $\gamma_c = \ln[(M+1+2\sqrt{M^2+M})/(3M-1)]$  &  $\epsilon_2 = [(M-1+4M\delta)(1+\cosh \gamma)]/[2M[1-\sinh \gamma + 2\delta(1+\cosh \gamma)]]$ .

$\Rightarrow$  The advantage of optimal entanglement cloner  $M_{iii}$  over other studied scenarios in the sense of robustness against thermal perturbations.

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## References

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