Bound entanglement in quantum phase transitions[1]

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Introduction

Entanglement, a key concept in quantum-information & its role in studying many-body systems … **R**elation between quantum phase transitions (QPT) in a system and non-analyticities of bi-[2] or multipartite[3] entanglement of its ground state (GS) … **E**ntanglement measures - existence of free & bound entanglement (BE) … **A**ppearance of BE in many-body systems by varying magnetic fields or interacting with thermal environment …

Objectives

Investigating the role of bound entanglement in quantum phase transitions. **P**resenting a method to construct quantum spin-chain Hamiltonians in which a quantum phase transition can be accompanied by a change in the type of entanglement (of the two-site reduced density matrices of the corresponding ground states) from bound to free or separability. **W**e use the ground-state fidelity $\mathcal{F}$ or its second derivative (fidelity susceptibility - $\mathcal{S}$) and the realignment criterion ($\mathcal{N}_R$) to detect possibly relevant criticalities:

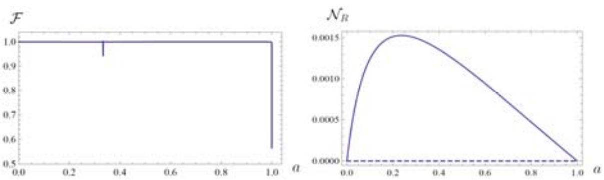
GS fidelity: $\mathcal{F}(a_c,\Delta) = |\langle\Psi(a_c-\Delta)|\Psi(a_c+\Delta)\rangle|$

Realignment: $\mathcal{N}_R(\varrho) = (\|\varrho^R\|_1 - 1)/2 \quad ; \quad \varrho^R_{ab,cd} = \varrho_{ac,bd}$

Results

❖$_1$ **Spin-1 quantum chain** in which **QPT occurs at the transition points from BE to seperability** and its two-site reduced density matrix in GS is the states[4]:

$$\varrho(a) = \frac{1}{1+8a}\begin{pmatrix} a & 0 & 0 & 0 & a & 0 & 0 & 0 & a \\ 0 & a & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & a & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & a & 0 & 0 & 0 & 0 & 0 \\ a & 0 & 0 & 0 & a & 0 & 0 & 0 & a \\ 0 & 0 & 0 & 0 & 0 & a & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1+a}{2} & 0 & \frac{\sqrt{1-a^2}}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & a & 0 \\ a & 0 & 0 & 0 & a & 0 & \frac{\sqrt{1-a^2}}{2} & 0 & \frac{1+a}{2} \end{pmatrix}$$



$J_i(a)$ $\quad$ $g(a)$

$$H = \sum_{i\ \mathrm{odd}}\Big[8 + 4\{\boldsymbol{S}_i\cdot\boldsymbol{S}_{i+1}, S^z_iS^z_{i+1}\}_+ - 4(\boldsymbol{S}_i\cdot\boldsymbol{S}_{i+1})^2 - J_1{S^z_i}^2 + J_2{S^z_{i+1}}^2 + J_3{S^z_i}^2S^z_{i+1} + J_4S^z_i{S^z_{i+1}}^2$$
$$+g^2({S^z_i}^2{S^z_{i+1}}^2 - S^z_iS^z_{i+1}) + 4g(S^z_{i+1} + S^z_i)$$
$$+4(\{S^x,S^y\}_{i_+}\{S^x,S^y\}_{{i+1}_+} + g(({S^z_i}^2 - S^z_i){S^y_{i+1}}^2$$
$$-{S^y_i}^2({S^z_{i+1}}^2 + S^z_{i+1})))\Big]{S^z_{i+2}}^2$$

❖$_2$ **Spin-1 quantum chain** in which **QPT occurs at the transition point from bound entanglement to free entanglement** with the following bipartite state in its GS:

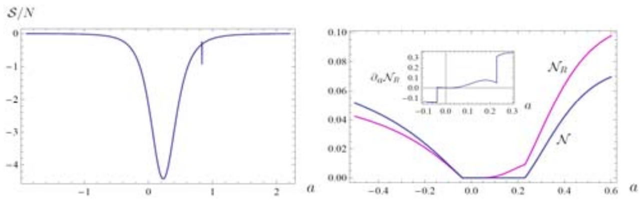
Partial transpose $\quad \approx 0.76$

$$\varrho = p\left[q\,\chi^{T_A} + \tfrac{(1-q)}{9}\,\mathbb{1}\otimes\mathbb{1}\right] + (1-p)\,|\Phi^+\rangle\langle\Phi^+|$$

Maximally entangled state in *d=1*

$$\chi(\alpha,\beta,a) = \tfrac{1}{9}(1-\alpha-\beta-a)\mathbb{1}\otimes\mathbb{1} + \alpha P_{00}$$
$$+\tfrac{\beta}{2}(P_{10}+P_{20}) + \tfrac{a}{3}(P_{01}+P_{11}+P_{21})$$

P_{nm} are projectors onto maximally entangled two-qutrit vector states[5] . q,α,β are functions of a.

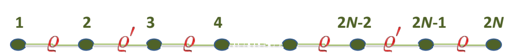


$J_i(a)$

$$H = \sum_{i\ \mathrm{odd}}\Big[J_1(\boldsymbol{S}_i\cdot\boldsymbol{S}_{i+1} + \{\boldsymbol{S}_i\cdot\boldsymbol{S}_{i+1}, S^z_iS^z_{i+1}\}_+ - S^z_iS^z_{i+1})$$
$$+ J_2{S^z_i}^2{S^z_{i+1}}^2 + J_3({S^z_i}^2S^z_{i+1} - S^z_i{S^z_{i+1}}^2 + S^z_i - S^z_{i+1})$$
$$+ {S^z_i}^2 + {S^z_{i+1}}^2\Big](\mathbb{1} - {S^z}^2)_{i+2}$$

Method

Consider a closed chain of 2N identical quantum systems $|\Psi\rangle_{1,\dots,2N}$



$$\varrho_{i,i+1} = \sum_{i=1}^K \lambda_i|v_i\rangle_{i,i+1}\langle v_i| + 0\times\sum_{j=1}^{d^2-K}|w_j\rangle_{i,i+1}\langle w_j|$$
$$\varrho'_{i,i+1} = \sum_{i=1}^{K'} \lambda'_i|v'_i\rangle_{i,i+1}\langle v'_i| + 0\times\sum_{j=1}^{d^2-K'}|w'_j\rangle_{i,i+1}\langle w'_j|$$

Our goal is to construct a Hamiltonian ***H*** for which this state is a GS.
We just need to find a positive ***H*** which kills this state: $\langle\Psi|H|\Psi\rangle = 0$
Suppose:

$$H = \sum_{i\ \mathrm{odd}} H_{i,i+1} + \sum_{i\ \mathrm{even}} H'_{i,i+1}$$
$$\langle\Psi|H|\Psi\rangle = \sum_{i\ \mathrm{odd}}\mathrm{Tr}[H_{i,i+1}\varrho_{i,i+1}] + \sum_{i\ \mathrm{even}}\mathrm{Tr}[H'_{i,i+1}\varrho'_{i,i+1}]$$

Thus, it is sufficient if we construct corresponding $H_{i,i+1}\ (H'_{i,i+1})$ from the null eigenvectors $\{|w_j\rangle\}(\{|w'_j\rangle\})$!

$$H_{i,i+1} = \sum_{j\in\mathcal{J}} h^{[i]}_j|w_j\rangle_{i,i+1}\langle w_j|$$

We can repeat this construction for ***L***-local Hamiltonians by considering ***L***-partite density matrix of the GS. As a general example, we considered the following global state and obtained the Hamiltonian of the system for some bound entangled state ϱ

$$|\Psi\rangle_{1,\dots,2N} = \sum_{i=1}^K\sqrt{\lambda_i}|v_i\rangle_{1,2}|v_i\rangle_{3,4}\cdots|v_i\rangle_{2N-1,2N}$$

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