Matrix product representation for spontaneous quantum spin ferrimagnets[1]

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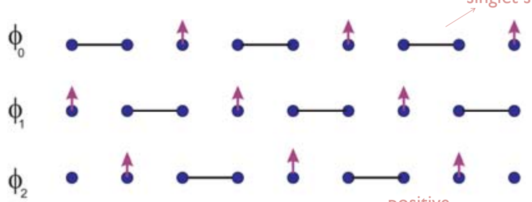
1 Introduction

The importance of finding the exact solution of quantum many-body systems from the physicists point of view in different fields: cond-mat, stat-mech, math-ph, quant-ph & … **T**his provides a theoretical laboratory to study fundamental properties of the system: entanglement – quantum phase transitions – correlations functions & … **M**atrix product formalism is a powerful tool for constructing new quantum spin chains with desirable properties.

2 Results

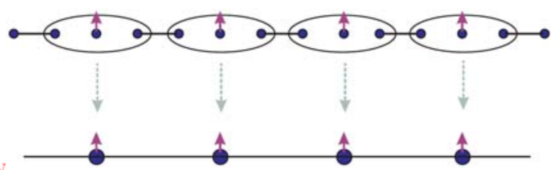
1- Introducing a family of spin-**1/2** chains whose ground states break the rotational & translational symmetry of the Hamiltonian:



$$H_{1/2}=\sum_{i=1}^{3N}\mathbf{s}_i\cdot\mathbf{s}_{i+1}(1-2\mathbf{s}_{i+2}\cdot\mathbf{s}_{i+3})-\frac{4+J}{3}\mathbf{S}_i\cdot\mathbf{S}_i+\frac{1+J}{6}(\mathbf{S}_i\cdot\mathbf{S}_i)^2$$

positive; spin operator of one site; spin operator of a block of 4 sites

2- Constructing a spin-**3/2** quantum chain with nearest neighbor interaction from this spin-**1/2** chain using projection[2]:



$$H_{3/2}=C+\sum_{i=1}^{N}\frac{81}{8}\mathbf{S}_i\cdot\mathbf{S}_{i+1}+\frac{29}{6}(\mathbf{S}_i\cdot\mathbf{S}_{i+1})^2+\frac{2}{3}(\mathbf{S}_i\cdot\mathbf{S}_{i+1})^3$$

$C=\frac{165}{32}N$

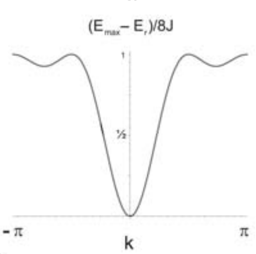
3- Deriving the max energy state & some of the lower energy states of spin-**1/2** chain in closed analytical form:

Top state: $|+,+,+,\cdots,+,+\rangle$

with energy: $E_{max}=6NJ$

Lower states: $|\psi^r\rangle=\sum_{n=1}^{3N}e^{in\frac{2\pi r}{3N}}|n\rangle$, $\quad |n\rangle = |+,+,\cdots,-,\cdots,+,+\rangle$

$$E_r=2J(3N-4+2\cos^3\frac{2\pi r}{3N}+2\cos^2\frac{2\pi r}{3N})$$



4- Finding the matrix product representation for the ground states of these chains. For spin-**1/2** chain:

$$|\phi_0\rangle=tr(A_{i_1}A_{i_2}B_{i_3}A_{i_4}A_{i_5}B_{i_6}\cdots)|i_1,i_2,i_3,i_4,i_5,i_6,\cdots\rangle$$

$$A_+=|+\rangle\langle 0|+|0\rangle\langle -| \qquad B_+=|0\rangle\langle 0| \quad \& \quad B_-=0$$

$$A_-=|-\rangle\langle 0|-|0\rangle\langle +|$$

orthonormal states

For spin-**3/2** chain: $|\mathrm{GS}\rangle=tr(A_{i_1}A_{i_2}A_{i_3}\cdots)|i_1,i_2,i_3,\cdots\rangle$

$$A_{3/2}=A_+B_+A_+ \;(\sigma^+) \quad \& \quad A_{1/2}=\frac{1}{\sqrt{3}}(A_-B_+A_++A_+B_+A_-) \;(-\sigma_z)$$

$$A_{-1/2}=\frac{1}{\sqrt{3}}A_-B_+A_- \;(-\sigma^-) \quad \& \quad A_{-3/2}=0$$

5- Calculating correlation functions of spin-**3/2** quantum chain:

$$\langle S^z\rangle=1/2,\quad \langle S^x\rangle=\langle S^y\rangle=0$$

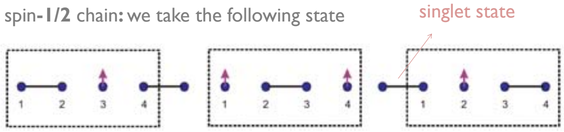
$$\langle (S^z)^2\rangle=5/4-u,\quad \langle (S^x)^2\rangle=\langle (S^y)^2\rangle=5/4+\frac{u}{2}, \quad u = 1/(1+\sqrt{3})$$

$$\langle S^xS^y+S^yS^x\rangle=0,\quad G^a(1,r):=\langle S_1^aS_r^a\rangle-\langle S_1^a\rangle\langle S_r^a\rangle$$

$$G^a(1,r)=A_a(-1)^{r-1}e^{-\frac{r}{\xi_a}}\begin{cases}A_l:=(12+6\sqrt{3})/4, & A_t:=(12+7\sqrt{3})/4\\ \xi_l=1/\ln(2+\sqrt{3}), & \xi_t=1/\ln(1+\sqrt{3})\end{cases}$$

3 Method & remarks

For spin-**1/2** chain: we take the following state



Demanding it to be the GS of : $H=\sum_{k=1}^{3N}h_{k,k+1,k+2,k+3}$ (positive Hamiltonian)

Gives us:

$$\begin{cases}\mathrm{tr}\,[h(|S\rangle\langle S|\otimes|+\rangle\langle +|\otimes I)]=0,\\ h\,[|+\rangle|S\rangle|+\rangle]=0,\\ \mathrm{tr}\,[h(I\otimes|+\rangle\langle +|\otimes|S\rangle\langle S|)]=0.\end{cases}$$

Total spin of each block can be **0** or **1**. Finally: $h=JP_2+P_0$

Writing the projectors in terms of spin operators $\longrightarrow H_{1/2}$.

NB. $-E_{max}$ ($-E_r$) is the exact energy of GS (low-lying excitations or spin waves) of $-H$.

For spin-**3/2** chain**:** we use of this fact that the total spin of two adjacent sites can be **2**, **1** and **0** $\longrightarrow H=\sum_k(P_3)_{k,k+1}\longrightarrow H_{3/2}$.

NB. On a lattice of **3*N*** sites, these ground states have a total spin of ***N*/2**, and hence have ferrimagnetic character.

NB. The ground state of spin-**3/2** chain is exponentially degenerate! If in the spin-**1/2** chain, isolated spins are in arbitrary state $\alpha_i|0\rangle+\beta_i|1\rangle$ MP representation of each state in the ground space of the lower chain is given by:

$$A_{3/2}^{(i)}\equiv\alpha_i\sigma^+\qquad A_{1/2}^{(i)}\equiv\frac{1}{\sqrt{3}}(-\alpha_i\sigma_z+\beta_i\sigma^+)$$

$$A_{-1/2}^{(i)}\equiv\frac{1}{\sqrt{3}}(-\beta_i\sigma_z+\alpha_i\sigma^-)\qquad A_{-3/2}^{(i)}\equiv-\beta_i\sigma^-$$

NB. The novel point of our construction is that we constructed a MP representation, not for a state which is invariant under rotation as has been hitherto considered, but for a state which is a member of multiplet and transforms to other states under symmetry.

NB. Similarity of our spin-**3/2** model with that of alternating spin-**1** & spin-**1/2** systems[3] and the possibility of generalizing our models to higher spins[4].

4 Acknowledgment

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5 References

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