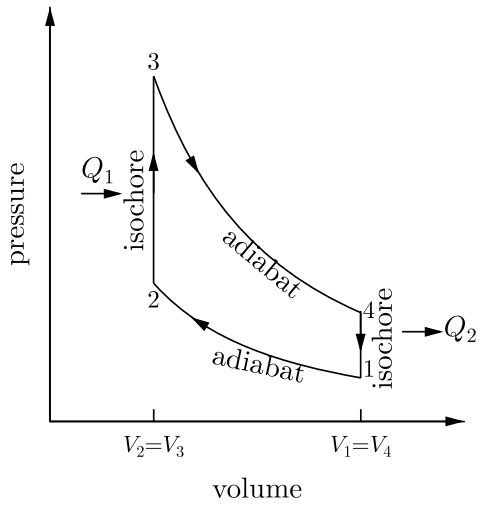
آزمون میان‌ترم درس ترمودینامیک و مکانیک آماری ۱ (آزمونِ در خانه)

توجه: ۱. نام و نام خانوادگی و شماره دانشجویی‌تان را در صفحه‌ی اول پاسخ‌نامه بنویسید. ۲. لطفاً تلاش کنید پاسخ‌ها را با خطی خوانا و نگارشی ساده و قابلِ‌فهم بنویسید. طبیعی است که اگر پاسخی خوانا نباشد تصحیح نمی‌شود، و اعتراضی از این بابت پذیرفته‌نمی‌شود. ۳. فرض‌ها و نتیجه‌های فرعی را که در حل هر مساله به کار می‌برید به‌روشنی بیان کنید. ۴. همه‌ی مساله‌ها هم‌نمره‌اند. تنها به ۸ مساله از ۱۱ مساله پاسخ دهید. ۵. استفاده از کتاب‌های درسی و درس‌نامه‌ها برای یادآوری مطالب یا روابطِ اصلی آزاد است، اما نباید پاسخ مساله‌ای را از آن‌ها رونویسی کرد. اگر از برخی از نتایج منابعی جز کتاب درسیِ اصلی استفاده می‌کنید، به آن‌ها ارجاعِ مناسب بدهید. فرض بر این است که در پاسخ‌نامه‌تان نتیجه‌ی تفکر، خلاقیت، و محاسبات خودتان را می‌نویسید، نه رونوشتی از نتایجِ دیگران یا حاصلِ مشورت با آن‌ها. ۶. لطفاً نسخه‌ای الکترونیکی از پاسخ‌نامه‌تان را (ترجیحاً به‌صورتِ یک فایل pdf یا نسخه‌ی اِسکن‌شده‌ای با پسوند jpg و تا حد امکان با حجمی کوچک) تا ساعت ۴ عصر به آدرس ای‌میل من (rezakhani@sharif.edu) بفرستید. ۷. موفق باشید.

A possible ideal-gas cycle operates as follows:
(i) from an initial state (p_1, V_1) the gas is cooled at constant pressure to (p_1, V_2);
(ii) the gas is heated at constant volume to (p_2, V_2);
(iii) the gas expands adiabatically back to (p_1, V_1).
Assuming constant heat capacities, show that the thermal efficiency is

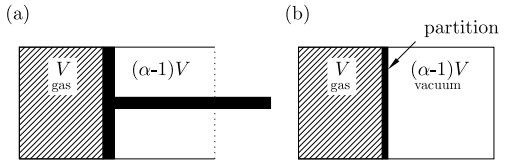
$$1 - \gamma \frac{(V_1/V_2) - 1}{(p_2/p_1) - 1}. \qquad (13.41)$$

(You may quote the fact that in an adiabatic change of an ideal gas, pV^γ stays constant, where $\gamma = c_p/c_V$.)



In a heat engine, heat can diffuse between the hot reservoir and the cold reservoir and in Chapter 10 we showed that this takes place on a timescale which scales with the square of the linear size of the system (see Example 10.4). The mechanical timescale of an engine typically scales simply with the linear size of the engine. Explain why this means that heat engines don't work on very small scales. [This is one why reason why the "engines" powering biological systems, which have to be extremely small, are not heat engines. Instead, useful energy is extracted directly from chemical bonds. Heat engines also often run on chemical fuel but use the fuel to heat one of the reservoirs and then extract work from the temperature difference thereby generated.]

Consider n moles of a gas, initially confined within a volume V and held at temperature T. The gas is expanded to a total volume αV, where α is a constant, by (a) a reversible isothermal expansion and (b) removing a partition and allowing a free expansion into the vacuum. Both cases are illustrated in Fig. 14.9. Assuming the gas is ideal, derive an expression for the change of entropy of the gas in each case.



Show that another expression for the entropy per mole of an ideal gas is

$$S = C_p \ln T - R \ln p + \text{constant}. \qquad (16.89)$$

Hence show that

$$\Delta G - \Delta H = T \left(\frac{\partial \Delta G}{\partial T} \right)_p , \qquad (18.10)$$

and explain what happens to these terms as the temperature $T \to 0$.

Show for the ideal gas that $(\partial U/\partial P)_T = 0$.

Give an argument why

$$\frac{\partial}{\partial V}(1/T) = \frac{\partial}{\partial U}(P/T)$$

should hold as a general thermodynamic identity. Hint: $\mathrm{d}S$ is exact.

Prove that between any two equilibrium states there exists only one adiabatic path. (Use a method different from the one we discussed in the class.)

Prove that the thermodynamic temperature is always nonnegative. What does it mean to have a negative temperature?

Find a differential equation for the probability distribution function $P(x,t)$ of a symmetric random walker who moves on a line back and forth with sufficiently small step sizes.

Calculate the range of possible final temperatures T_f in equilibrium for a system consisting of two partial systems A and B, if A and B have initial temperatures T_A, T_B and heat capacities C_V^A, C_V^B which are independent of temperature.

To this end, consider the limiting cases of a totally irreversible process ($\delta W = 0$) and a totally reversible process ($\delta W_{\max}$). Calculate the mechanical work that one can maximally extract from this system and the change of entropy of the partial systems in the irreversible case.
