Problem 1. Solve the Legendre equation

\[(1 - x^2)y'' - 2xy' + n(n + 1)y = 0\]

by direct series substitution.

a) Verify that the indicial equation is

\[k(k - 1) = 0\]

b) Using \(k = 0\) \((k = 1)\), obtain series of even (odd) powers of \(x\).

c) Show that the both solution diverge for \(x = \pm 1\) if the series continue to infinity.

d) Finally, show that by an appropriate choice of \(n\), one of two series may be converted into a polynomial.

Problem 2. By direct differentiation and substitution show that

\[y_2(x) = y_1(x) \int x \exp\left[-\int P(t)dt\right] \left[y_1(s)\right]^2 ds\]

satisfy the ODE

\[y'' + Py' + Qy = 0\]

If \(y_1(x)\) satisfy it.

Problem 3. The quantum mechanical angular momentum operator is given by \(L = -i(r \times \nabla)\). show that the eigenvalue problem of the form

\[L \cdot L \psi = l(l + 1)\psi\]

lead to associated Legendre equation.

Problem 4. In this problem we study the solution of Electrostatic field between two metallic cyldinder by the Transformation of the coordinates techniques.

We define Cylindrical bipolar coordinate system as:

\[x = a \frac{\sinh u}{\cosh u - \cos t}\] \(1\)

\[y = a \frac{\sin t}{\cosh u - \cos t}\] \(2\)

\[z = z\] \(3\)

a) Find \(t = cte\) and \(u = cte\) surface equation and plot them.

b) Consider two cylinder with paralell axeses where their distance \(d > r_1 + r_2\) from each other. If they contain charge of \(+Q\) and \(-Q\) (per-unit length) respectively, find Electric field in the outside of two cylindre.