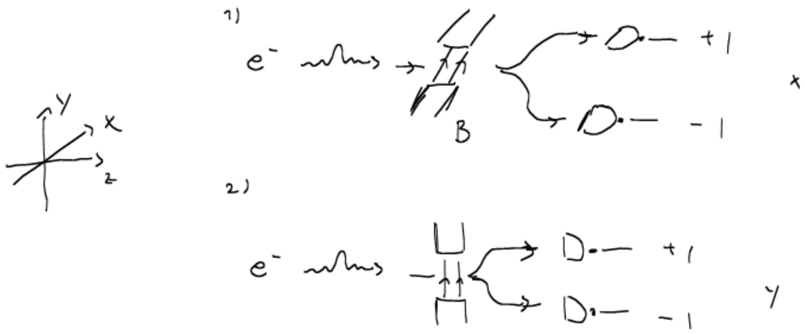

FIG. 1: Recall the physical interpretation of Pauli matrices. 1) An electron described by the pure state vector $|\psi\rangle$ passes through a magnetic field aligned along the x-axis. Depending on the state, it is deflected along one of the two indicated paths and will cause the front or the rear detector to click. With the former event, we associate the number $x = +1$, with the latter the number $x = -1$. The expectation value $\langle x \rangle$ is given by $|\langle \psi | X | \psi \rangle|^2$. 2) Analogous situation for Y-measurements.

GHZ argument. Consider the "GHZ"-state

$$|GHZ\rangle = \frac{1}{\sqrt{2}}(|0,0,0\rangle + |1,1,1\rangle).$$

The GHZ-state is a stabilizer state.

1. Find a generating set for the stabilizer group S. Write down the entire stabilizer group.

   Hints: a) $X \otimes X \otimes X \in S$, b) there are three elements in S which carry a negative sign (e.g. $-Y \otimes Y \otimes X \in S$).

2. Recall the meaning of the statement "$|GHZ\rangle$ is a $+1$-eigenvector of $X \otimes X \otimes X \in S$": Assume three electrons are in the state described by $|GHZ\rangle$. Conducting an X-measurement on each electron (see Fig. 1), we obtain three numbers $x_1, x_2, x_3 \in \pm 1$. The fact that $X \otimes X \otimes X \in S$ means that the product $x_1 x_2 x_3$ is guaranteed to equal $+1$.

   One could try to construct a "classical model" of the GHZ-state. By this we mean that all the observables have definite values: $\langle X_i \rangle = x_i, \langle Y_i \rangle = y_i, \langle Z_i \rangle = z_i$, for $x_i, y_i, z_i \in \pm 1$. Of course, all the elements in the stabilizer group should evaluate to $+1$ for these assignments. So for example, $X \otimes X \otimes X \in S \Rightarrow x_1 x_2 x_3 = 1$.

   Prove that *it is impossible* to find such a classical assignment.

   Hints: Concentrate on the elements of S mentioned under 1a) and 1b). Argue that one can distinguish two cases: a) $x_1 = x_2 = x_3 = +1$ and b) two of the x_i equal -1 and the third equals $+1$. For both cases, show that one cannot find an assignment of the y_i's which reproduces the quantum observations.

3. An amateur philosopher might ponder: "Does a falling tree make a sound if nobody's there to hear it?".

   In no more than four sentences, comment on the connection between this statement and point 2.

Example $n = 2$

$$|\underline{\Psi}\rangle = \frac{1}{\sqrt{2}}\left(|0\underline{0}\rangle + |1\underline{1}\rangle\right)$$

$$Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Claim:

- $\underbrace{X \otimes X}_{\to g_1} |\underline{\Psi}\rangle = |\underline{\Psi}\rangle$
- $\underbrace{Z \otimes Z}_{\to g_2} |\underline{\Psi}\rangle = |\underline{\Psi}\rangle$

$\Rightarrow$ $\underbrace{g_1 g_2 g_2 g_1 \cdots g_2}_{\text{group generated by } g_1, g_2} |\underline{\Psi}\rangle = |\underline{\Psi}\rangle$

$$\langle g_1, g_2 \rangle = S$$

$$g_1 \cdot g_2 = (X \otimes X)(Z \otimes Z)$$

$$= (XZ) \otimes (XZ)$$

$$= (-iY) \otimes (-iY)$$

$$= -Y \otimes Y$$

$$g_2 \cdot g_1 = \ldots = -Y \otimes Y$$

$S =$

$\mathbb{1} \otimes \mathbb{1} \quad \rightsquigarrow \quad g_1^0 \cdot g_2^0$ $\qquad g_1 = X \otimes X$

$X \otimes X \quad \rightsquigarrow \quad g_1^1 \cdot g_2^0$ $\qquad g_1^0 = \mathbb{1} \otimes \mathbb{1}$

$Z \otimes Z \quad \rightsquigarrow \quad g_1^0 \cdot g_2^1$

$-Y \otimes Y \quad \rightsquigarrow \quad g_1^1 \cdot g_2^1$

$$P_S = \frac{1}{4} \left(\mathbb{1} \otimes \mathbb{1} + X \otimes X + Z \otimes Z - Y \otimes Y \right)$$

$$|\Phi\rangle = \frac{1}{\sqrt{2}} \left(|00\rangle + |11\rangle \right)$$

$$\operatorname{tr}\left[|\Phi\rangle\langle\Phi| \; Z \otimes Z \right] = 1 \checkmark$$

$$\operatorname{tr}\left[|\Phi\rangle\langle\Phi| \; X \otimes \mathbb{1} \right] = 0$$

$Y \otimes \mathbb{1} \quad \rightsquigarrow 0$

$Z \otimes \mathbb{1} \quad \rightsquigarrow 0$

$$\rightsquigarrow \operatorname{tr}_2 \left[|\Phi\rangle\langle\Phi| \right] = \rho_1 = \tfrac{1}{2} \mathbb{1}$$

Tomorrow

- graph states
- error correction