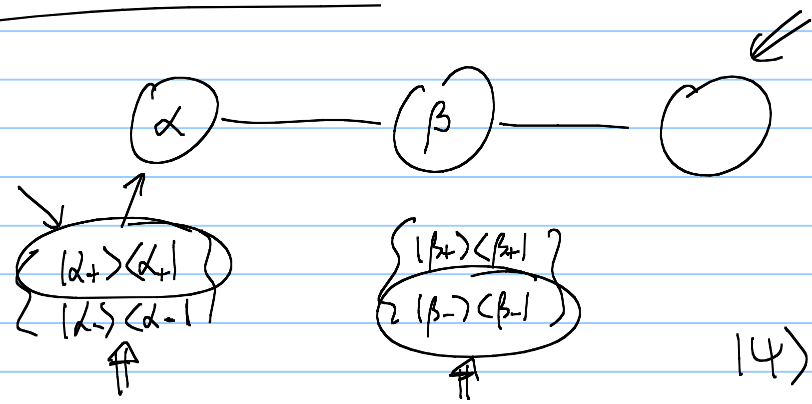
Lecture 2



$P_0 = |A|^2$

$P_1 = |B|^2$

$|\psi\rangle = \begin{pmatrix} A \\ B \end{pmatrix} \Leftarrow$ circuit

$|\psi'\rangle = \begin{pmatrix} B \\ A \end{pmatrix} = X\,|\psi\rangle$

$P_0 = |B|^2$

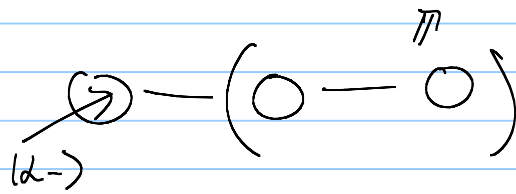
$P_1 = |A|^2$

mental readjustment

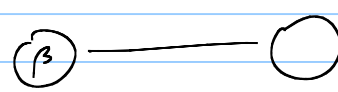
What if on the first measurement we got the $|\alpha_-\rangle\langle\alpha_-|$ outcome?

$$\langle \alpha_- | L_3 \rangle = -i \sin \alpha/2 \; |0+\rangle + \cos \alpha/2 \; |1-\rangle \quad \Leftarrow \text{BAD}$$

But we wanted: $-i \sin \alpha/2 \; |1-\rangle + \cos \alpha/2 \; |0+\rangle \quad \Leftarrow \text{GOOD} = \langle \alpha_+ | L_3 \rangle$



$$|\psi'\rangle = X_2 Z_3 \, |\psi_{good}\rangle$$



$|\beta_+\rangle\langle\beta_+|$

$|\beta_-\rangle\langle\beta_-|$

Remember $|\beta_\pm\rangle = e^{i\beta/2}|0\rangle \pm e^{-i\beta/2}|1\rangle$

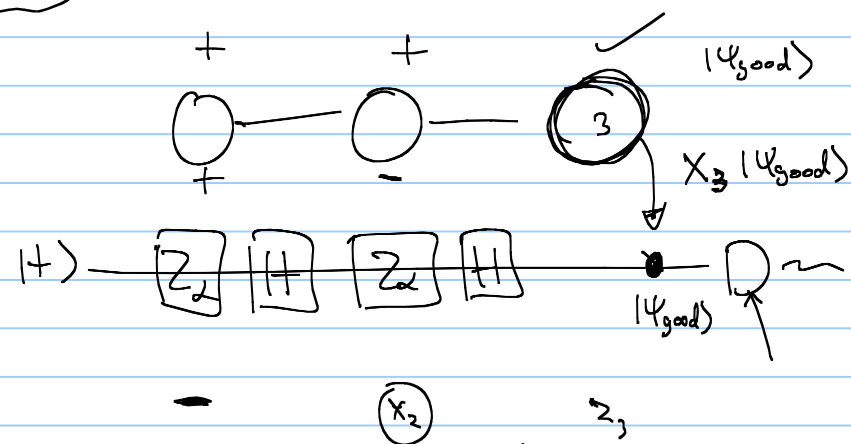
$|\beta_+\rangle_2\langle\beta_+|$ ⟵ $X_2|\psi\rangle$

$|\beta_-\rangle\langle\beta_-|$

Do the measurement

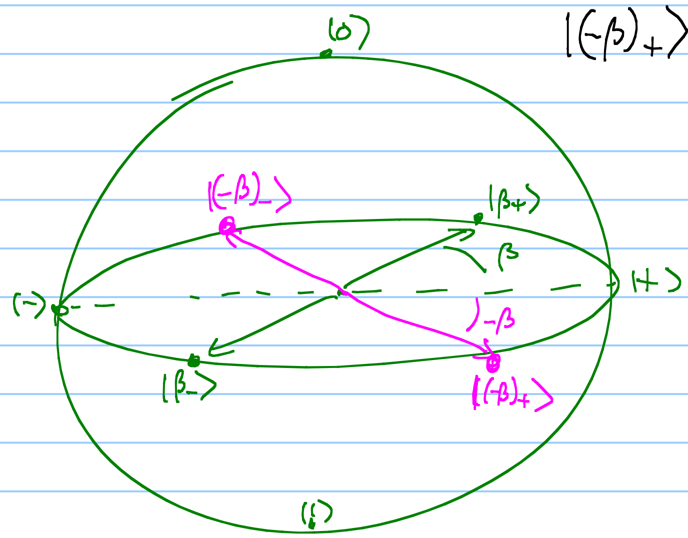
$$\boxed{\begin{array}{l} X|\beta_+\rangle\langle\beta_+|X \\ X|\beta_-\rangle\langle\beta_-|X \end{array}}$$ ⟵ instead.

$U|A\rangle\langle A|U^\dagger$



$$X|\beta_+\rangle = X\left(e^{i\beta/2}|0\rangle + e^{-i\beta/2}|1\rangle\right)$$
$$= \left(e^{i\beta/2}|1\rangle + e^{-i\beta/2}|0\rangle\right)$$
$$= e^{-i\beta/2}|0\rangle + e^{i\beta/2}|1\rangle$$

$|(-\beta)_+\rangle$



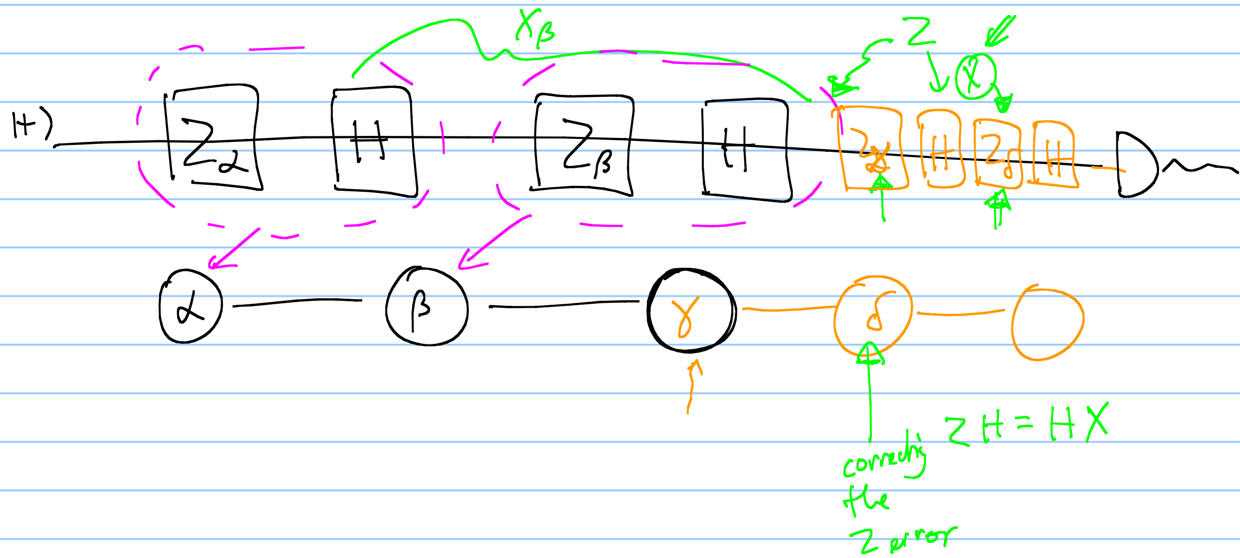
This is the key idea

Classically feed-forward & change the angle of the second measurement based on the outcome of the first.

Still have to correct the Z_3 error

$$|\psi''\rangle = \begin{pmatrix} A \\ -B \end{pmatrix}$$

Z error doesn't affect the probabilities P_0, P_1



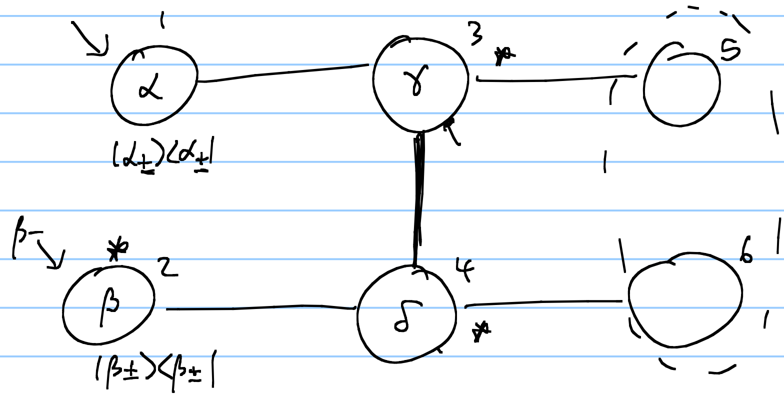
Summary

(i) $|\alpha_+\rangle\langle\alpha_+|$ $\qquad$ $|\alpha_-\rangle\langle\alpha_-|$

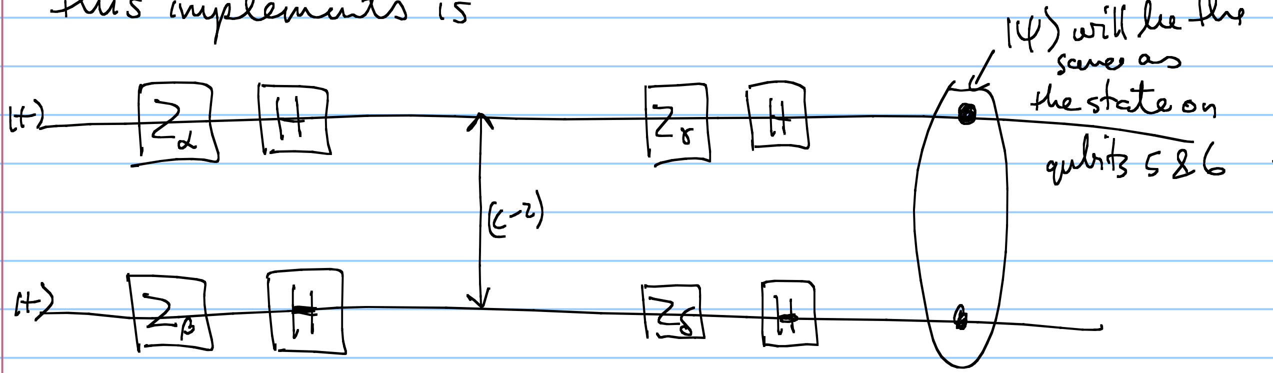
$\{|\beta_+\rangle\langle\beta_+|, |\beta_-\rangle\langle\beta_-|\}$ $\qquad$ $\{|-\beta_+\rangle\langle-\beta_+|, |-\beta_-\rangle\langle-\beta_-|\}$

$\begin{pmatrix} A \\ B \end{pmatrix}$ $\quad$ $\begin{pmatrix} B \\ A \end{pmatrix}$ $\quad$ $\begin{pmatrix} A \\ -B \end{pmatrix}$ $\quad$ $\begin{pmatrix} -B \\ A \end{pmatrix}$

2-qubit gates?



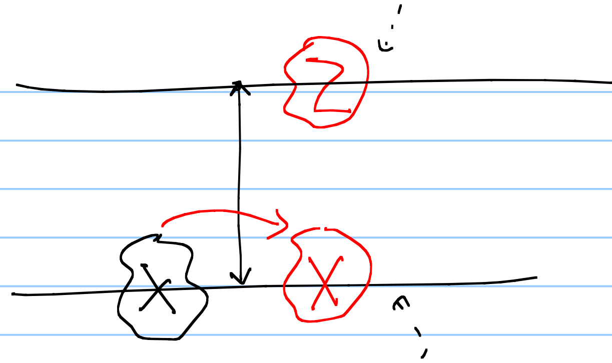
If we get all "+" (good) outcomes the circuit this implements is



Exercise 1 Prove these outputs are the same.

Exercise 2 Work out what measurements must be performed on qubits 3 & 4 if we get the $|\alpha_+\rangle$ outcome on qubit 1 but the $|\beta_-\rangle$ outcome on qubit 2

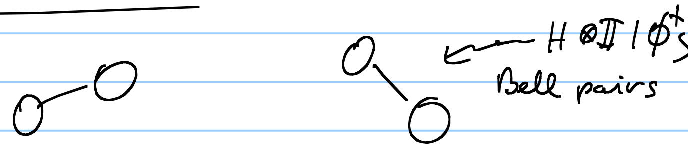
(Hint: Pauli errors going through a CZ gate can "fan out" e.g

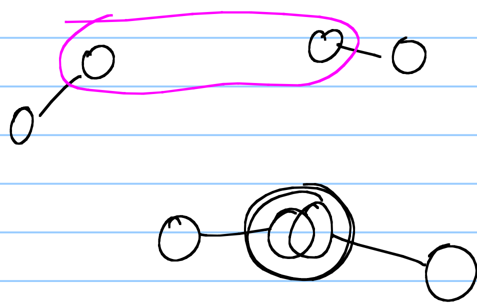


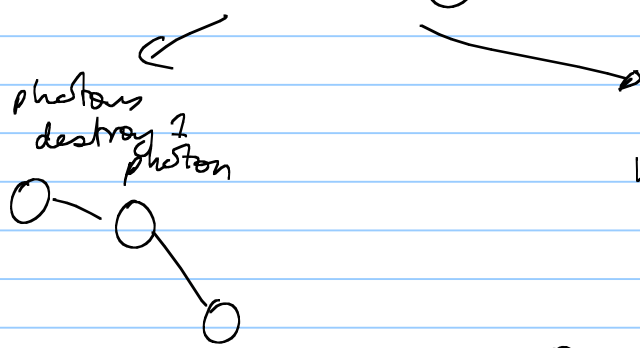
$CZ \; X\otimes I = X \otimes Z \; CZ$

OVERVIEW OF TECHNIQUES FOR BUILDING CLUSTER STATES

1/ Fusion



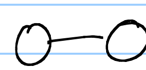




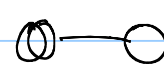
"Redundant Encoding"

$|0\rangle \Leftrightarrow |00\rangle \Leftrightarrow |000\cdots0\rangle$

$|1\rangle \Leftrightarrow |11\rangle \Leftrightarrow |1111\rangle$



$|0\rangle|+\rangle + |1\rangle|-\rangle$



$|00\rangle|+\rangle + |11\rangle|-\rangle$

more generally

$$\left(|A\rangle|0\rangle + |A^{\perp}\rangle|1\rangle\right)/\sqrt{2}$$

rest of the cluster

Bi-orthogonal or Schmidt decomposition

measure in $|\pm\rangle$ basis, removes qubits

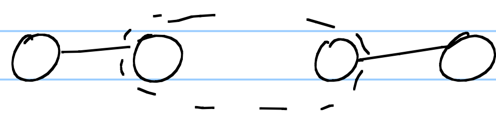
Redundant Encoding : $\left(|A\rangle|000\rangle + |A^{\perp}\rangle|111\rangle\right)/\sqrt{2}$

$$|\alpha_{\pm}\rangle = e^{i\alpha/2}|0\rangle \pm e^{-i\alpha/2}|1\rangle$$

$$\alpha^{i\alpha/2}|000\rangle \pm e^{-i\alpha/2}|111\rangle$$

not single qubit measurements

Abstractly



$|+\rangle|0\rangle + |-\rangle|1\rangle$ $\qquad$ $|0\rangle|+\rangle + |1\rangle|-\rangle$

$\Downarrow$

$|+\rangle|00\rangle|+\rangle + |-\rangle|11\rangle|-\rangle$

2

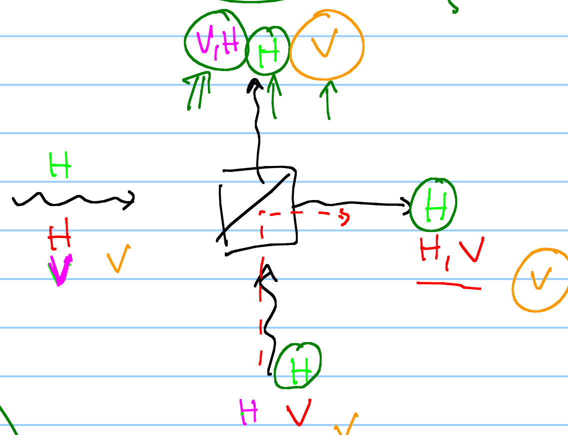
gate fails

$|V,H\rangle \Rightarrow |2_H\rangle + |2_V\rangle$

$|H\rangle \Rightarrow \boxed{|H\rangle + |V\rangle}$

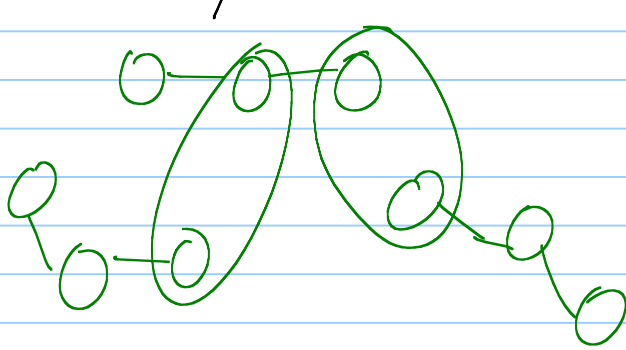
$|V\rangle \Rightarrow \boxed{|H\rangle - |V\rangle}$

45°

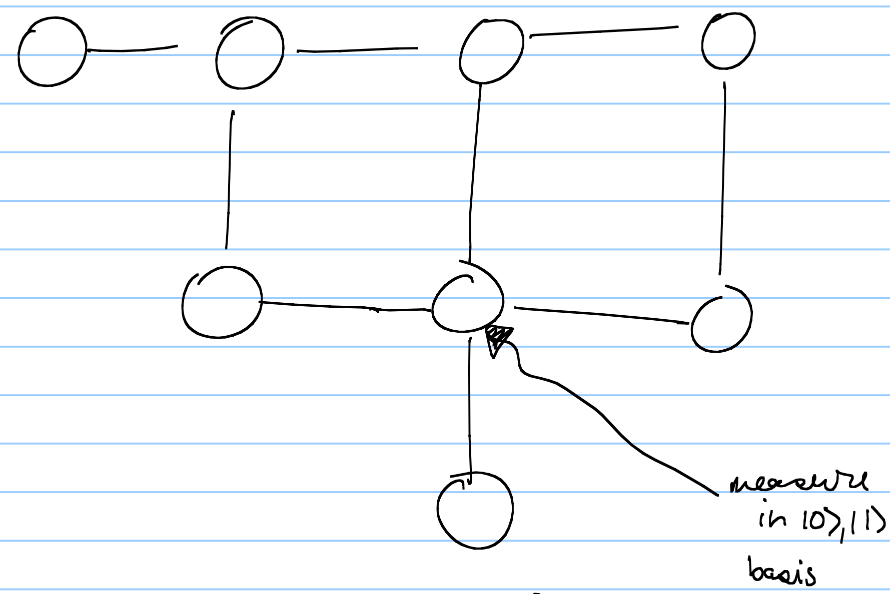


$|H\rangle = |0\rangle$

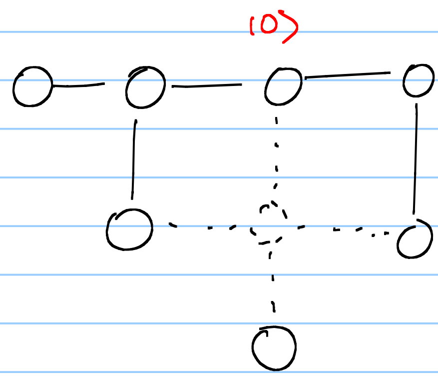
$|V\rangle = |1\rangle$

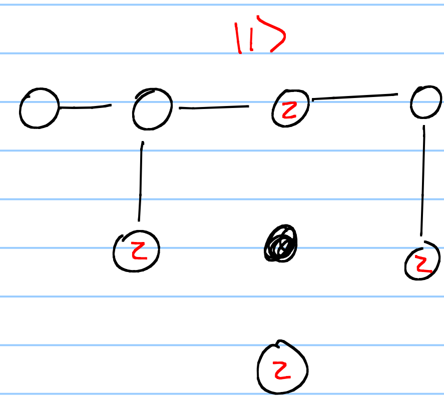


Exercise 3



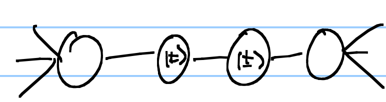
Show that this removes the qubit from the cluster state.



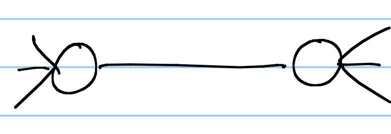


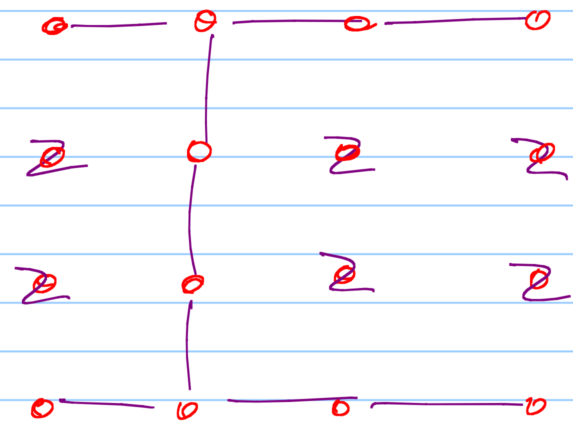
Exercise 4

Show that two X basis (is $|\pm\rangle$ basis, corresponding to $\alpha = 0$ in $|\alpha_\pm\rangle$) on a linear cluster has the following effect



becomes:





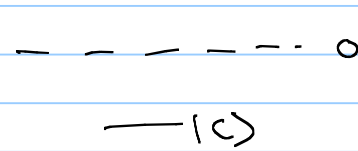
$0.9\,\mathbb{I} + 0.1\,|A\rangle\langle A|$

Technique 2

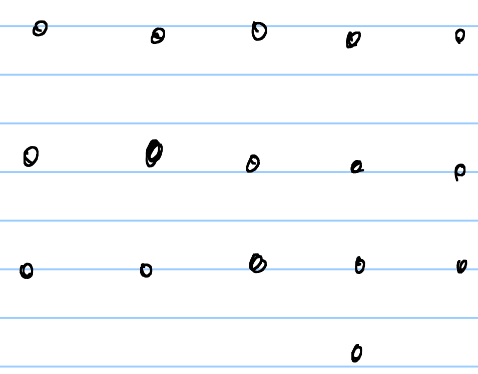
What if nature just gives us cluster states as the ground state of some molecule/crystal/nano tube etc for free?

What would a generic Hamiltonian giving this be?

(i) $H = -|C\rangle\langle C|$



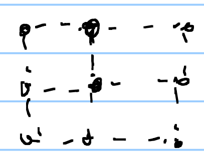
(ii) ? Local Hamiltonian



$\sum$ (products of Pauli operators which are only nontrivial for adjacent qubits)

$H_{nonlocal} = X \otimes X \otimes X \otimes \cdots \otimes X$

H_{local}

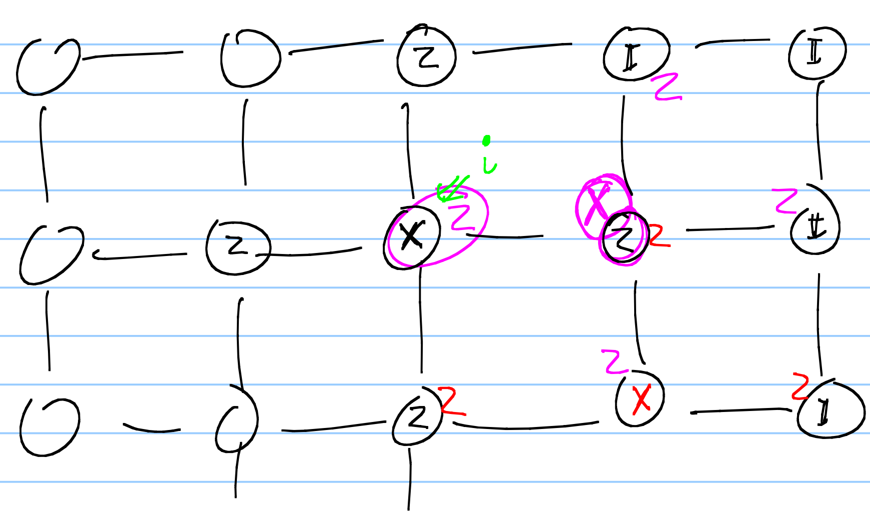


2-body Hamiltonian

$$\sum_{i \sim j} \underbrace{\left(X_i \otimes X_j + Y_i \otimes Y_j + Z_i \otimes Z_j \right)}_{\text{nontrivial Paulis}} \otimes \underbrace{I \otimes I}\cdots$$

connected pairs

Consider operators on 5 qubits



local $\otimes I$

$$\underbrace{Z \otimes Z \otimes X_i \otimes Z \otimes Z}_{K_i}$$

The following Hamiltonian has the cluster state as its ground state

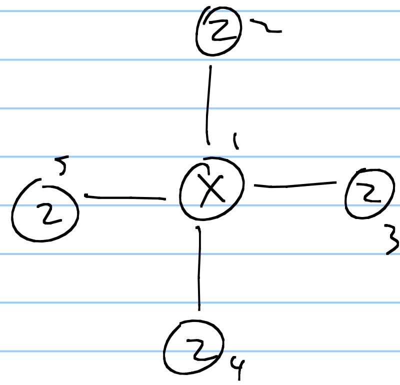
$$H = -\sum_i K_i$$

How to show this? Exercise 5

Think of the cluster a built from the CZ gates

$$|C\rangle = \prod_{i\sim j} CZ_{ij} |++\cdots+\rangle$$

What do the CZ gates do to the K_i operator



$$(CZ_{12}\, CZ_{13}\, CZ_{14}\, CZ_{15}) K_1 (\quad)^{\dagger}$$

Hints K_i all commute.

Hint The spectrum of the K_i are all ± 1