Matrix Product States in Spin-1 Ladder

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Why MPS?

Which systems are we intrested in?
spin systems, (i.e. spin chains, spin ladders and ...)

why?
Spin systems are one of the candidates for quantum information processing devices (state transfer, ...)

What do we want to know?
- Construction of the ground state.
- Studying the properties of this state (Correlation functions, Correlation length, Entanglement).
- Quantum phase transitions and ...

How can we gain these?
One of the best candidate is *Matrix product states (MPS)*

Problem: Constructing the ground state is so hard to do (if possible at all).
Solution: MPS gives us a recipe for constructing the ground state and many other intresting properties of desired system.

How does MPS work?

System: An array of N, d-level objects (Fig. 1)
Boundary condition: Periodic

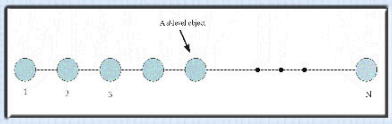

Fig 1) A chain of N, d-level spins with periodic boundary condition

Ground State:

$$|\Psi\rangle = \sum_{i_1,i_2,\cdots i_N} \psi_{i_1 i_2 \cdots i_N} |i_1, i_2, \cdots, i_N\rangle$$

Matrix product coefficients (symetrics of the system are refleted in A's):

$$\psi_{i_1 i_2 \cdots i_N} = \frac{1}{\sqrt{Z}} tr(A_{i_1} A_{i_2} \cdots A_{i_N})$$

Transfer matrix and normalization constant:

$$E = \sum_{i=0}^{d-1} A_i^* \otimes A_i \qquad Z = tr(E^N)$$

Average value of an observable:

$$\langle\Psi|O(k)|\Psi\rangle = \frac{tr(E^{k-1} E_O E^{N-k})}{tr(E^N)} \qquad E_O := \sum_{i,j=0}^{d-1} \langle i|O|j\rangle A_i^* \otimes A_j$$

and in thermodynamic limit:

$$\langle\Psi|O|\Psi\rangle = \frac{\langle\lambda_{max}|E_O|\lambda_{max}\rangle}{\lambda_{max}}$$

Two-point correlation functions:

$$\langle\Psi|O(k)O(l)|\Psi\rangle = \frac{tr(E_O(k)E_O(l)E^N)}{tr(E^N)} \qquad E_O(k) := E^{k-1}E_O E^{-k}$$

In thermodynamic limit and large distances:

$$\langle\Psi|O(1)O(r)|\Psi\rangle - \langle\Psi|O|\Psi\rangle^2 = \frac{\lambda_1^{r-2}}{\lambda_{max}^r}\langle\lambda_{max}|E_O|\lambda_1\rangle\langle\lambda_1|E_O|\lambda_{max}\rangle$$

Correlation length:

$$\xi = \frac{1}{\ln\frac{\lambda_{max}}{\lambda_1}}$$

Symmetries

- Our system is symmetric under this transformation

$$R|i\rangle = R_{ij}|j\rangle$$

that is

$$\Psi_{i_1 i_2 \cdots i_N} \rightarrow \Psi' := tr(A'_{i_1} A'_{i_2} \cdots A'_{i_N})$$

they are applied to A's in this way

$$(T_a)_{ij} A_j = [T_a, A_i] \qquad R_{ij} A_j = U(R) A_i U^{-1}(R)$$

Hamiltonian of the system:

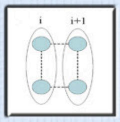

Fig. 7) Local hamiltonian h is written for two successive rungs. The total hamiltonian is the sum of local ones.

$$|\psi_{2,m}\rangle = \lambda_2\left(|2^{(2\oplus 0)}, m\rangle - |2^{(0\oplus 2)}, m\rangle\right)$$

$$|\psi_{1,m}\rangle = \lambda_1^{(1)}|1^{(2\oplus 2)}, m\rangle + \lambda_1^{(2)}|1^{(1\oplus 1)}, m\rangle + f(u,v,\lambda_1^{(k)})|1^{(0\oplus 1)}, m\rangle + \lambda_1^{(3)}|1^{(1\oplus 0)}, m\rangle + g(u,v,\lambda_1^{(1)})|1^{(1\oplus 2)}, m\rangle + g(u,v,\lambda_1^{(1)})|1^{(2\oplus 1)}, m\rangle$$

$$|\psi_{0,0}\rangle = \lambda_0\left(|0^{(1\oplus 1)}, 0\rangle + h(u,v)|0^{(0\oplus 0)}, 0\rangle\right)$$

$$h_{i,i+1} = \sum_{j=0}^{2}\sum_m |\psi_{j,m}\rangle\langle\psi_{j,m}|$$

$$H = \sum_i h_{i,i+1}$$

Outlook

- **Calculating concurrence of the system in terms of u and v.**
- **Studying behavior of the entanglement in terms of parameters of the system.**
- **Obtaining the hamiltonian in terms of spin operators to understand the interactions of the system.**
- **Studying the first excited state.**
- **Studying non-homogeneous spin systems.**
- **Studying problems with open boundary condition.**

References:

[1] A. Klumper, A. Schadschneider and J. Zittarz, J. Phys. A (1991) L293; Z. Phys. B. 87 (1992) 281; Europhys. Lett., 24 (1993) 293.
[2] M. Fannes, B. Nachtergaele and R. F. Werner, Commun. Math. Phys. 144, 443 (1992).
[3] D. Perez-Garcia, F. Verstraete, M. M. Wolf and J. I. Cirac; quant-ph/0608197v1.
[4] M. Asoudeh, V. Karimipour, and A. Sadrolashrafi; Phys. Rev. B 75, 224427 (2007).
[5] Michael M. Wolf, Gerardo Ortiz, F. Verstraete and J. Ignacio Cirac; cond-mat/0512180 v1.

Our Problem

Spin-1 Ladder (Fig. 2) with full rotational symmetry:

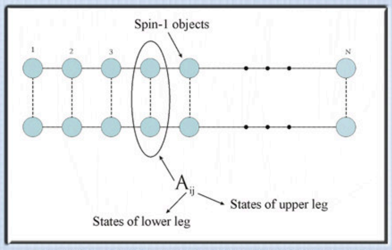

Fig. 2) Spin-1 Ladder. To each posible state of a rung a matrix is associated.

Results

Two point correlation function between first and rth site, for S in the x, y or z direction:

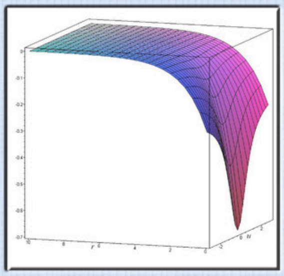

Fig. 6) Two point correlation function of S versus u and r for v=1.5.

Two point correlation function between first and rth site, for s in the x, y or z direction:

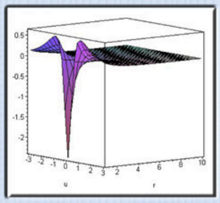

Fig. 6) Two point correlation function of s versus u and r for v=1.

Average value of S^2 in x, y or z direction:

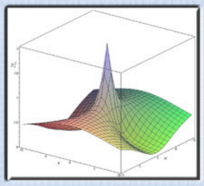

Fig. 5) Average value of S^2 versus u and v.

Inverse of correlation length:

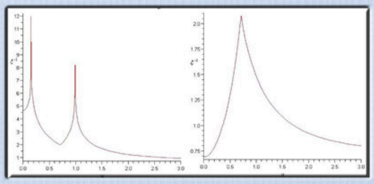

Fig. 4) Correlation length versus u for v=0 (left), v=0.9 (right).

Applying the full rotational symmetry to A's causes all parameters to vanish except two: u, v.
Eigenvalues of transfer matrix (Fig. 3):

$$\lambda_1 = \frac{5}{3} + v^2 + 2u^2 \qquad \lambda_2 = v^2 + u^2 - \frac{5}{6} \qquad \lambda_3 = v^2 - u^2 + \frac{1}{6}$$

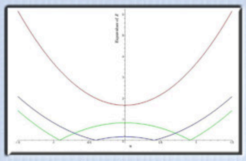

Fig. 3) Eigenvalues of transfer matrix, E versus u for v=0.

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