A suggestion for spin-position gates in mobile electron-based quantum information processing

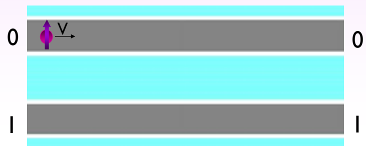
S. Alipour[1], A. T. Rezakhani[2], and M. Esmaelzadeh[3]

[1]Sharif University of Technology [2]University of Calgary [3]Iran University of Science and Technology

We investigate implementation of quantum computation by using spin and position degrees of freedom of electrons moving in the quantum wires. Such quantum wires can be realized in a two-dimensional electron gas, where electrons are assumed to move uniformly in either of two possible rails.
We show, how one can combine two previous methods for performing quantum computation.

The qubits in the system:

1. Spin qubits [1] 2. Dual-rail qubits [2]



A two-qubit state: $|\psi\rangle = |\sigma, k\rangle = |0,0\rangle$

Spin degree of freedom — Position degree of freedom

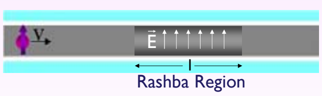
$\begin{cases} |\uparrow\rangle = |0\rangle \\ |\downarrow\rangle = |1\rangle \end{cases}$ $\begin{cases} k=0 & \text{electron in the top wire} \\ k=1 & \text{electron in the bottom wire} \end{cases}$

Single qubit gates:

1. Spin rotation gates [1]

We use Rashba Hamiltonian for spin degree of freedom.
Electron moves in an electric field: $\vec{E} = E\hat{n}$

Electron sees a magnetic field in its rest reference frame: $\vec{B} \sim \vec{V} \times \vec{E}$



Rashba Hamiltonian: $H_R = \alpha(\hat{n} \times \vec{k}).\vec{\sigma}$

$\hbar\vec{k} = m^*\vec{v}$ $\alpha = \dfrac{e\hbar}{2(m^*c)^2}E$ and $\vec{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$ Pauli matrices

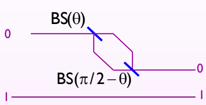
m^*: Electron's effective mass, l: the length of the Rashba region

Single-spin rotation gate: $e^{-iH_R t/\hbar} = e^{-i\alpha l(\hat{n}\times\hat{v}).\vec{\sigma}/\hbar^2}$

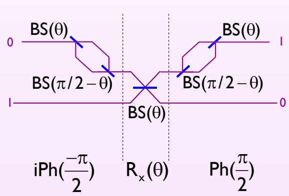
2. Position gates [3]

$U_{BS}: |k\rangle \to \cos\theta|k\rangle + i\sin\theta|1-k\rangle$

$U_{BS} = R_x(\theta) = \begin{pmatrix} \cos\theta & i\sin\theta \\ i\sin\theta & \cos\theta \end{pmatrix}$ Probability of transmission: $|\cos\theta|^2$ Probability of Reflection: $|\sin\theta|^2$



$Ph(\frac{\pi}{2}): |0\rangle \to i|0\rangle, |1\rangle \to |1\rangle$

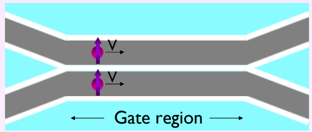


$Ph(\frac{\pi}{2})R_x(\theta)Ph(\frac{-\pi}{2}) = R_y(\theta)$

Two-qubit gates:

1. A two-qubit gate for spin qubits: SWAP [1]

Two quantum wires are more closer to each other in the gate region. In this region electrons can interact with each other and we can represent the effective Hamiltonian between two electrons by Heisenberg Hamiltonian.



$H_{exchange} = J(t)\vec{S}_1.\vec{S}_2 \Rightarrow U_{exchange} = e^{i\beta/4}e^{-i\beta/2U_{SWAP}}$

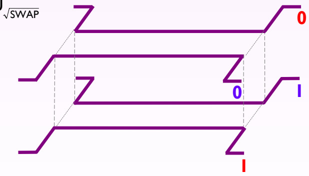
in which $\vec{S} = \frac{1}{2}\vec{\sigma}$, $\beta = \frac{1}{\hbar}\int_{t_1}^{t_2} J(t)dt$ and $U_{SWAP} = \frac{1}{2}(I + \vec{\sigma}_1.\vec{\sigma}_2)$

$U_{exchange}(\beta) = e^{i\beta/4}(\cos\frac{\beta}{2}I - i\sin\frac{\beta}{2}U_{SWAP}) \Rightarrow$ If $\beta = \frac{\pi}{2}$

$\Rightarrow U_{exchange}(\frac{\pi}{2}) = e^{-i\frac{\pi}{8}}U_{\sqrt{SWAP}}$

[3]

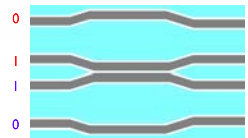
$U = I_{k_1k_2} \otimes \sqrt{SWAP}_{\sigma_1\sigma_2}$



Each electron is permitted to move in a dual-rail with differentiated color, without tunneling between two wires.

2. A two-qubit gate for dual-rail qubits: $S(\pi)$

$S(\pi) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & e^{i\pi} \end{pmatrix}$



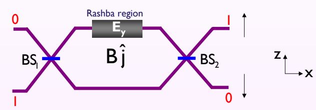
$H = (J(t)\vec{S}^{(1)}.\vec{S}^{(2)} - H_0)\delta_{k_1,1}\delta_{k_2,1} + H_0$ and $H_0 = H_0^{(1)} + H_0^{(2)}$ [3]

We can construct controlled-phase shifter, by using eight successive Coulomb couplers with $\beta = \pi/2$.
If both electrons move in the rail-1 ($k_1 = k_2 = 1$), then we have:
since $\sqrt{SWAP}^8 = I$ then the status of spin qubits will not change.

$U = (U_{exchange}(\frac{\pi}{2}))^8 |\sigma_1,1;\sigma_2,1\rangle\langle\sigma_1,1;\sigma_2,1| = e^{-i\pi}|\sigma_1,1;\sigma_2,1\rangle\langle\sigma_1,1;\sigma_2,1|$ [3]

3. A two-qubit gate for spin and dual-rail qubits: CNOT

$|\psi\rangle = \alpha|0,0\rangle_{\sigma,k} + \beta|1,0\rangle_{\sigma,k}$



- The local magnetic field between two wires causes an Aharonov-Bohm phase difference between electrons wave vectors in top and bottom branches.
- Because the electric field in Rashba region creates a magnetic field toward z-axis this only gives different phases to electrons with spin up and down without rotating them.

$\Rightarrow$ By controling magnetic field and the Rashba region we can tune interference between electron waves in top and bottom rails such that electrons with different spins separate in the output of the apparatus.

$U_{PBS}: |\uparrow,0\rangle \to |\uparrow,0\rangle, |\downarrow,0\rangle \to i|\downarrow,1\rangle, |\uparrow,1\rangle \to -|\uparrow,1\rangle, |\downarrow,1\rangle \to i|\downarrow,0\rangle$ [4]

$CNOT = \left(I_\sigma \otimes Ph(-\frac{\pi}{2})_k\right)U_{PBS}\left(I_\sigma \otimes Ph(-\frac{\pi}{2})\right)$ [3]

References :

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