Temperature effects on quantum cloning of states and entanglement [1]

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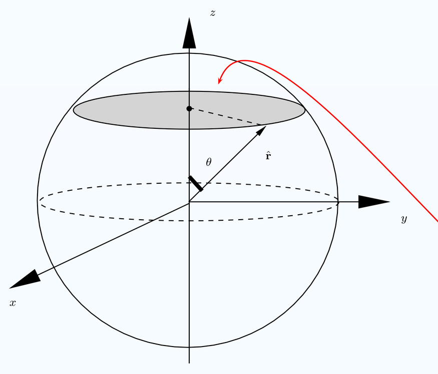
Introduction

- No-cloning theorem of { quantum states [2] / entanglement [3] }
- Quantum computation and cloning
- Approximate quantum cloners:

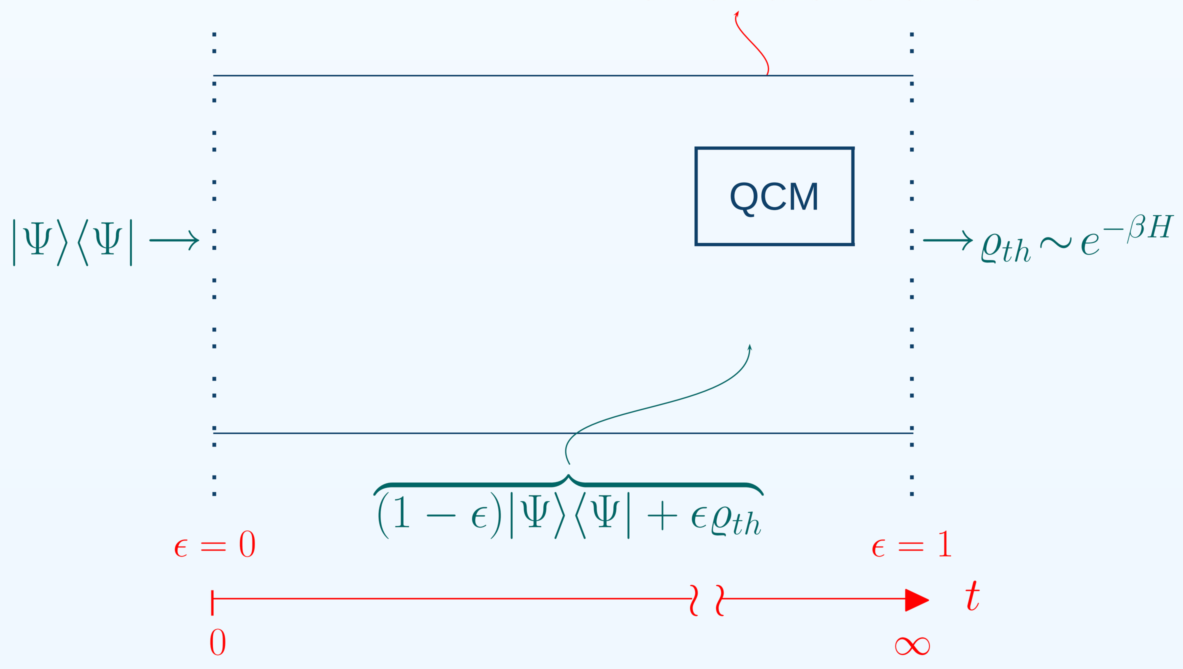


:: universal cloning (UC) machines [2]
:: phase-covariant cloning (PCC) machines of
– equatorial qubits [2]
– orbital states [4]
:: entanglement cloners [3]

Our model

Our aim is to investigate temperature effects on the performance of quantum cloning machines.

Decoherence [5]: Interaction with thermal environment $\Rightarrow$ ***thermalization***: pure states $\rightsquigarrow$ mixed states:



["QCM" stands for "*Quantum Cloning Machine*"].

We assume that: $\begin{cases} t_{env.} \ll \tau_c, T_{diss.} = \min\{T_1, T_2, T_O\} \\ \tau_c \lesssim T_{diss.} \end{cases}$

- T_1 and T_2: time-scales with respect to energy and phase relaxation processes, respectively.
- T_O: time-scale dictated by all other relaxation sources.
- τ_c: time-scale of the cloning process

Summary of the results

In the following sections, we show that:

- when only the blank copy and the ancilla state are affected, a redefinition of cloning transformations removes thermal effects.
- this thermalization may reduce performance of a quantum cloner even below classical cloners.
- there exist some instances in which the quality of cloning for phase-covariant cloners is less than that of universal cloners.
- an optimal entanglement cloner preserves its higher performance (than the other schemes of entanglement broadcasting) even when thermal noise comes into play.

Dissipative hardware

(1) Dissipative (mixed) ancilla

The possibility of optimal cloning with any pure state [2] $\Longrightarrow$ optimal fidelity with mixed ancilla is achievable.

(2) Dissipative (mixed) ancilla + blank

Attaching some new auxiliary system $M \longrightarrow$ redefinition of the cloning transformation [6] $\Longrightarrow$ optimal fidelity again

$\Rightarrow$ optimal cloning with thermally diluted machinery

Duplicating a thermally diluted qubit

The following matrix transformation can represent optimal universal and phase-covariant clonings:

$$\begin{pmatrix} a & b \\ b^* & 1-a \end{pmatrix}_a \rightarrow \begin{pmatrix} \mu^2 a + \nu^2 & 2\mu\nu b \\ 2\mu\nu b^* & \mu^2(1-a)+\nu^2 \end{pmatrix}_{a(a')}.$$

Here $\begin{cases} \text{UC} \rightarrow \nu^2 = \frac{1}{6} \\ \text{PCC} \rightarrow \nu^2 = \frac{1}{4}\left(1 - \frac{1}{\sqrt{1+2\tan^4\theta}}\right) \end{cases}$

Our Hamiltonian model is $H = \omega_0\sigma_z/2$. We also define $\eta = \omega_0\beta/2$:

$$\therefore \begin{cases} \omega_0 > 0\,,\ T\uparrow \ \Rightarrow\ \eta\downarrow\text{: opposed behavior} \\ \omega_0 < 0\,,\ T\uparrow \ \Rightarrow\ \eta\uparrow\text{: same behavior} \end{cases}$$

Bures fidelity: $F(\rho,\sigma) = \left(Tr\sqrt{\rho^{1/2}\sigma\rho^{1/2}}\right)^2 \ \Rightarrow F(|\Psi\rangle, \varrho^{out}):$

$$F(\theta,\epsilon,\eta) = \mu^2[1-\epsilon+\epsilon(e^{-\eta}\cos^2\tfrac{\theta}{2} + e^{\eta}\sin^2\tfrac{\theta}{2})/Z] + (\mu\nu - \mu^2/2)(1-\epsilon)\sin^2\theta + \nu^2,$$

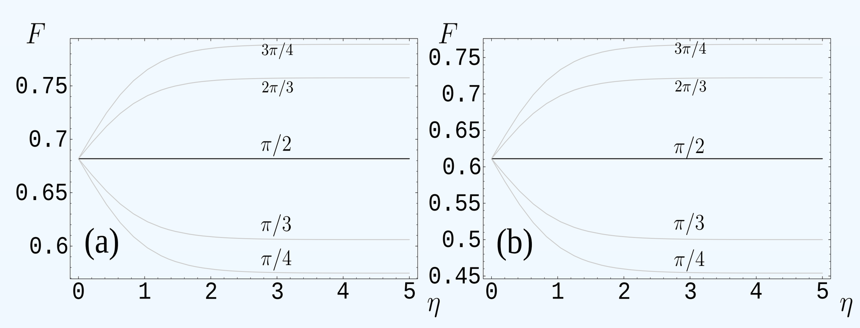
$$\therefore\quad \partial_\eta F = \frac{-\mu^2\epsilon}{2\cosh^2\eta}\cos\theta \Rightarrow \forall\epsilon\ :\ \begin{cases} \theta < \frac{\pi}{2} \ \Rightarrow\ F(\eta)\downarrow \\ \theta = \frac{\pi}{2} \ \Rightarrow\ F(\eta) = constant \\ \theta > \frac{\pi}{2} \ \Rightarrow\ F(\eta)\uparrow \end{cases}$$

And in high temperature limit ($\eta \rightarrow 0$)

$$\partial_\epsilon F = -\mu(\nu\sin^2\theta + \frac{\mu}{2}\cos^2\theta) \ \Rightarrow\ \forall\,\theta\ :\ F(\epsilon)\downarrow$$

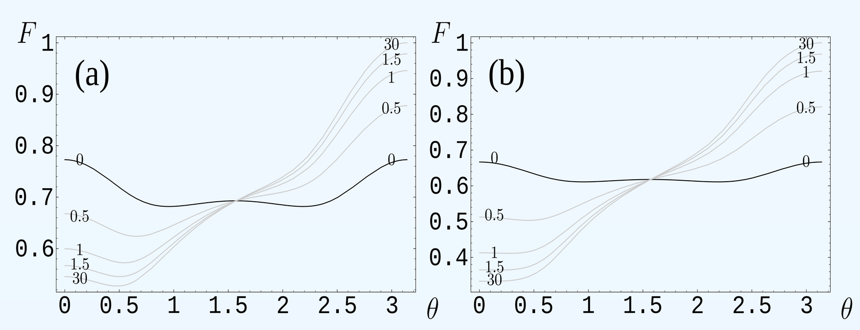
positive

Universal cloning

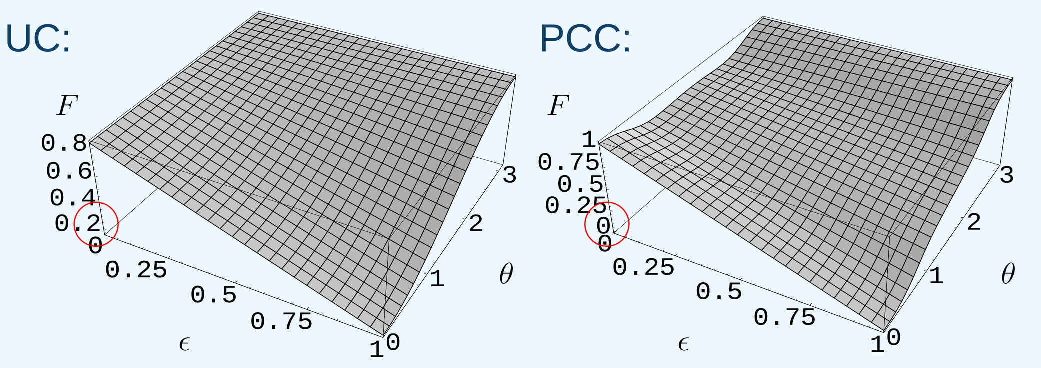


$\epsilon = 5/11$ $\epsilon = 2/3$

Phase-covariant cloning



At low temperature and $\omega_0 > 0$ ($\eta \rightarrow \infty$):



universal: $\forall\theta \in [0,\pi)\ \Rightarrow\ F(\epsilon)\downarrow$

phase-covariant: for θs $\gtrsim 2.52$ and less than π rad $\ \Rightarrow\ F(\epsilon)\uparrow$

Important point:

for some (ϵ,θ,η) we see that $F^{UC} > F^{PCC}$

In the case of universal cloning:

$$\epsilon < \frac{\cosh\eta}{e^{-\eta}\sin^2\frac{\theta}{2} + e^{\eta}\cos^2\frac{\theta}{2}} \ \Rightarrow\ F_{qua.} > F_{class.} = \frac{1}{2}$$

otherwise a classical cloner is better than quantum cloner.

$$\Rightarrow\ \left.\begin{matrix} \eta > 0\ \&\ \theta \geqslant \pi/2 \\ \eta < 0\ \&\ \theta \leqslant \pi/2 \end{matrix}\right\} \ \Rightarrow\ F_{qua.} > F_{class.}$$

Cloning of thermal entanglement

Clean state: $|\Psi_\alpha^-\rangle_{ab} = \alpha|01\rangle_{ab} - \sqrt{1-\alpha^2}|10\rangle_{ab} : \alpha\in\mathbb{R}$ & $|\alpha| \leqslant 1$

Hamiltonian model: $H = J(\sigma_x^a\sigma_x^b + \sigma_y^a\sigma_y^b)$

Via: local cloning: (i) with two optimal UC machines [7]

Non-local cloning: { (ii) with UC machine of 4-level quantum states [2] / (iii) by an optimal entanglement cloner [3] }

► When $\epsilon = 0$ (no external noise), we could write the state of each clone in cases (ii) & (iii), and nonlocal copies of case (i) in the following compact form

$$\varrho^{out} = \tfrac{1-M}{4}(|00\rangle\langle00| + |11\rangle\langle11|) + [\tfrac{1+M}{4} + L(2\alpha^2-1)]|01\rangle\langle01| + [\tfrac{1+M}{4} - L(2\alpha^2-1)]|10\rangle\langle10| - M\alpha\sqrt{1-\alpha^2}(|01\rangle\langle10| + |10\rangle\langle01|),$$

machine parameter

where

$$M_i = (2/3)^2\ ,\quad M_{ii} = 3/5\ ,\quad M_{iii} = 6A^2 + 4AC$$

$$A = \frac{1}{3}\sqrt{\frac{1}{2} + \frac{1}{\sqrt{13}}}\ ,\quad C = \frac{A}{2}(\sqrt{13}-3)\ ,\quad L = \frac{3}{26}(1 + 2M + \sqrt{1+4M-9M^2}).$$

► When temperature comes into play, the above equation takes the following general form

$$\varrho^{out} = (\tfrac{M\epsilon}{Z} + \tfrac{1-M}{4})(|00\rangle\langle00| + |11\rangle\langle11|) + [M(\tfrac{1-\epsilon}{2} + \tfrac{\epsilon\cosh\gamma}{Z}) + \tfrac{1-M}{4} + L(1-\epsilon)(2\alpha^2-1)]|01\rangle\langle01| + [M(\tfrac{1-\epsilon}{2} + \tfrac{\epsilon\cosh\gamma}{Z}) + \tfrac{1-M}{4} - L(1-\epsilon)(2\alpha^2-1)]|10\rangle\langle10| - M[(1-\epsilon)\alpha\sqrt{1-\alpha^2} + \tfrac{\epsilon}{Z}\sinh\gamma](|01\rangle\langle10| + |10\rangle\langle01|),$$

in which $Z = 2(1+\cosh\gamma)$ and $\gamma = 2\beta J$.

$$\therefore\quad \left.\begin{matrix} \epsilon = 1\ \&\ \forall\gamma \\ \alpha = \pm\frac{1}{\sqrt{2}}\ \&\ \forall\epsilon\ \&\ \forall\gamma \end{matrix}\right\} \ \Rightarrow \varrho^{out} = M\varrho^{in} + (\frac{1-M}{4})I$$

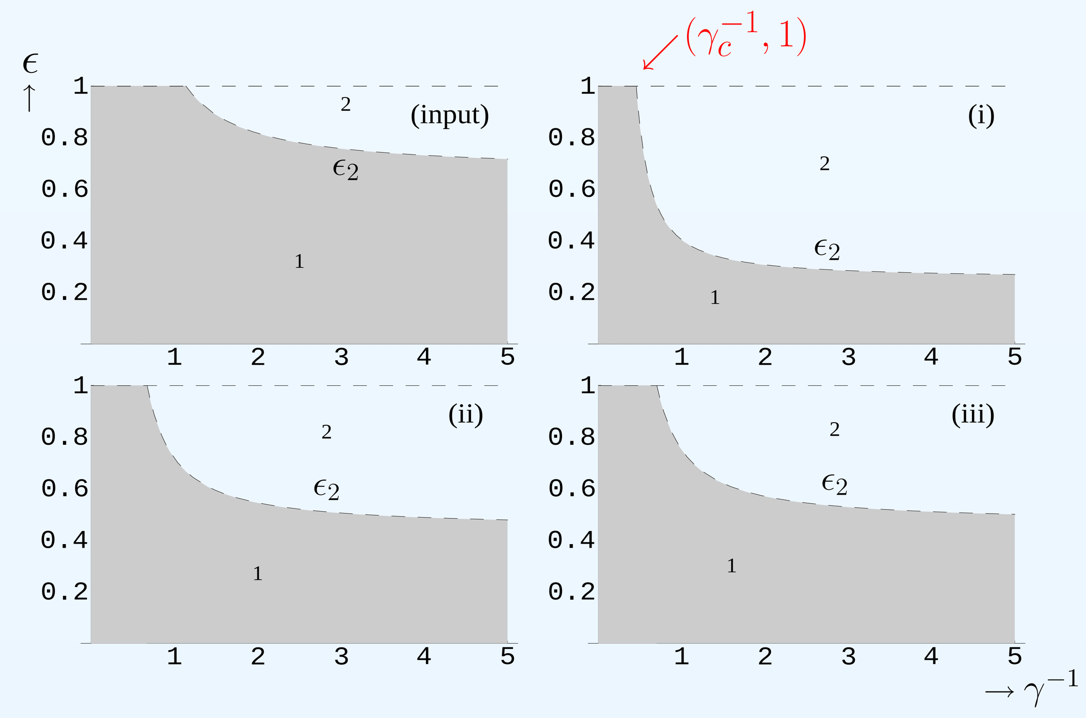
Applying positive partial transposition criterion [8] results in some temperature and state-dependent regions over which the output cloned pairs are inseparable. For example, at room temperature, our clones are entangled when (for more details see [1])

$$0 \leqslant \epsilon < (1 - \frac{1}{3M})\ \&\ |\alpha^2 - \frac{1}{2}| < \frac{\sqrt{(3M(1-\epsilon)-1)(M(1-\epsilon)+1)}}{4M(1-\epsilon)}.$$

Remark1.– For some "T" $\in$ intermediate (high & low) temperatures, $\exists$ intervals of $\alpha^2(\epsilon)$ in which the cloned pairs are separable: $M\uparrow \ \Rightarrow$ the length of these intervals $\downarrow$. Recall: $M_{iii} > M_{ii} > M_i$.

Remark2.– For a given $\alpha^2(\epsilon) \in$ moderate (two limits of) temperatures, the range of $\epsilon(\alpha^2)$ in which the clones are entangled $\uparrow$ when $M\uparrow$.

Entanglement phase diagrams of input and output states (achieved from three different schemes of entanglement cloning/broadcasting), when $\alpha = 1/\sqrt{2}$ are



The regions labeled by 1 (2) indicate (no-) entanglement regions. Here, we also have $\gamma_c = \ln[(M+1+2\sqrt{M^2+M})/(3M-1)]$ & $\epsilon_2 = [(M-1+4M\delta)(1+\cosh\gamma)]/\{2M[1-\sinh\gamma + 2\delta(1+\cosh\gamma)]\}$.

$\Rightarrow$ The advantage of optimal entanglement cloner M_{iii} over other studied scenarios in the sense of robustness against thermal perturbations.

Acknowledgment

Center of Excellence in Complex Systems and Condensed Matter (CeCsCm)

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References

[1] S. Baghbanzadeh and A. T. Rezakhani, arXiv:quant-ph/0704.1177.
[2] V. Scarani *et al.*, Rev. Mod. Phys. **77**, 1225 (2005).
[3] L. P. Lamoureux *et al.*, Phys. Rev. A **69**, 040301 (2004).
[4] V. Karimipour and A. T. Rezakhani, Phys. Rev. A **66**, 052111 (2002).
[5] M. Merkli *et al.*, Phys. Rev. Lett. **98**, 130401 (2007).
[6] A. Roy *et al.*, Phys. Lett. A **286**, 1 (2001).
[7] V. Bužek *et al.*, Phys. Rev. A **55**, 3327 (1997).
[8] A. Peres, Phys. Rev. Lett. **77**, 1413 (1996).