Beam Splitters, Interferometers and Hong-Ou-Mandel Effect for Interacting Bosonic and Fermionic Walkers in a Lattice

Leonardo Banchi

University College London, UK

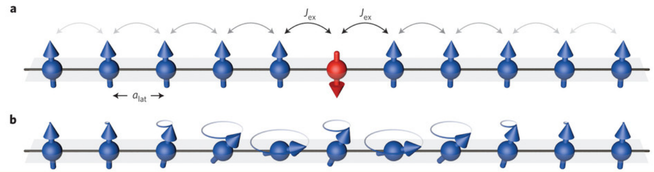
Aim of the work

Implement linear optics operations in an optical lattice

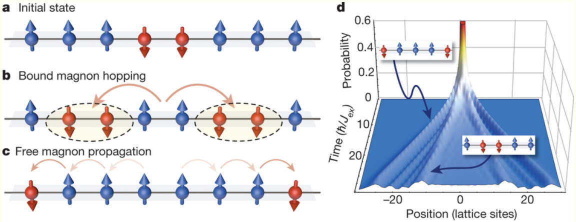
- Particles in tight binding lattices may be either static or mobile depending on the potential barriers
- Static particles to store information
- Mobile particles to process information
- Focus on distant particles for scalability

Recent experimental advances

T. Fukuhara *et al.*, Nature Physics (2013)



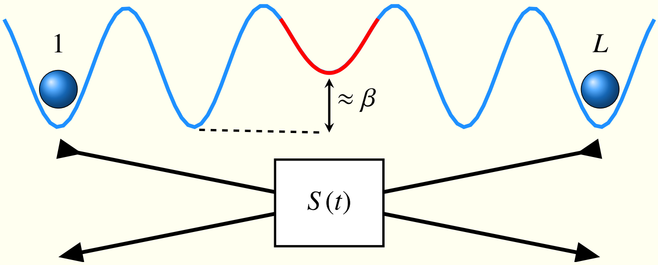
T. Fukuhara *et al.*, Nature (2013)



Content of this talk

- Transmission via hopping
- Fundamental operations (beam splitter, phase shifter) via suitably inserted impurities
- Hong-Ou-Mandel effect
- Mach-Zehnder interferometry
- Crossover from bosonic to fermionic behavior close to the superfluid-Mott transition
- Discussion towards KLM quantum computation in a lattice

Fundamental operation: beam splitter



$$H = -\frac{J}{2}\sum_{j=1}^{L-1}\left[a_j^\dagger a_{j+1} + \text{h.c.}\right] + \sum_{j=1}^{L}\left[\frac{U}{2}n_j(n_j-1) - \mu_j n_j\right]$$

$$\mu_j = \mu + J\beta\delta_{j,L/2}$$

Fundamental operation: beam splitter

- Transmission and reflection coefficients

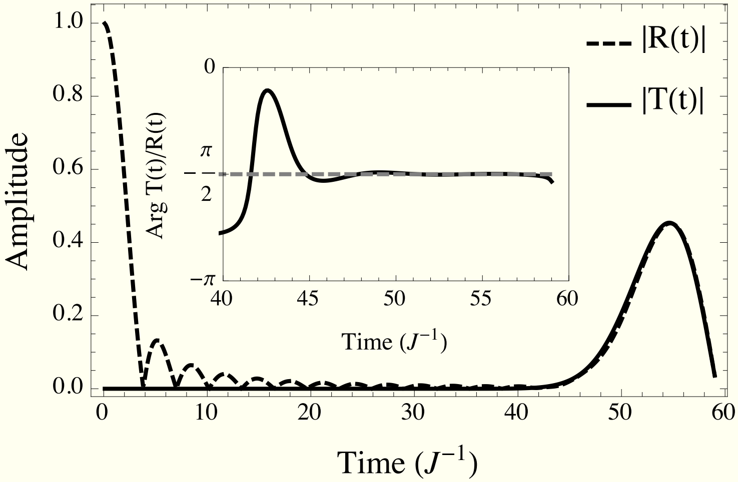
$$T(t) = \langle 0|\, a_L\, e^{-itH}\, a_1^\dagger\, |0\rangle \qquad R(t) = \langle 0|\, a_1\, e^{-itH}\, a_1^\dagger\, |0\rangle$$

- Effective Beam Splitter operator (scattering matrix) at the transmission time $t^* \simeq L$

$$S(t^*) = \begin{pmatrix} R(t^*) & T(t^*) \\ T(t^*) & R(t^*) \end{pmatrix} \approx D \begin{pmatrix} \frac{\beta}{i+\beta} & \frac{-i}{i+\beta} \\ \frac{-i}{i+\beta} & \frac{\beta}{i+\beta} \end{pmatrix} + \mathcal{O}(L^{-1}) \,,$$

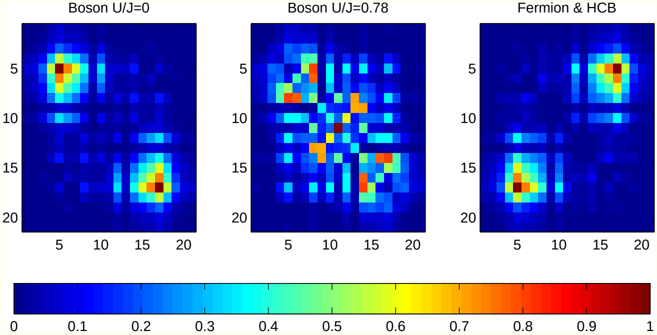
- $\tilde{S}{=}S(t^*)/D$ is the unitary beam splitter operation.
- $D{=}\mathcal{O}(L^{-1/3})$ is a damping factor

Fundamental operation: beam splitter



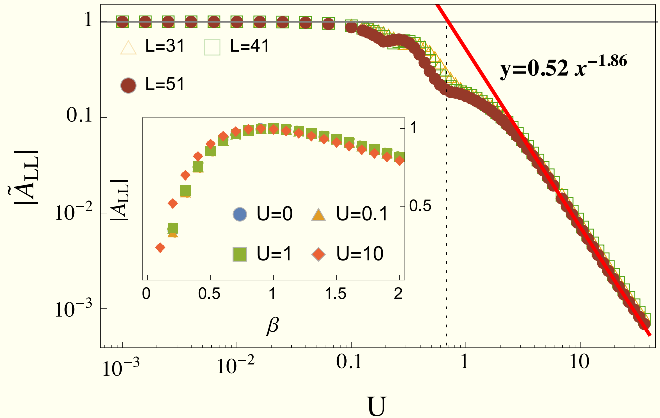
$L = 51$, $\beta = 0.95$

Two particles: Hong-Ou-Mandel effect



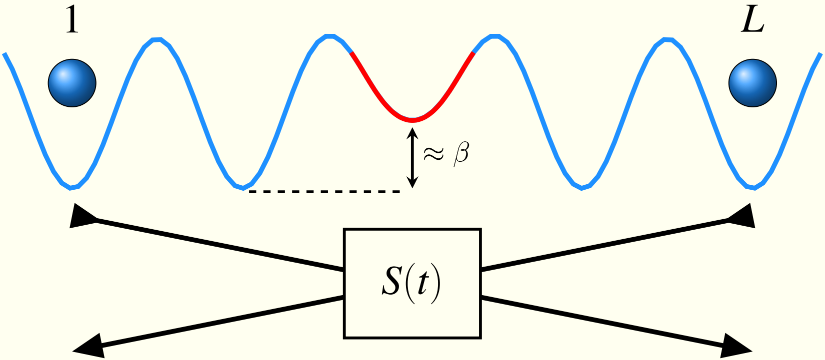
- Probability P_{ij} to measure one particle in position i and the other in position j.
- Fermion/Hard-core-boson behavior when $U \to \infty$ $(U/J \gtrsim 10)$

Bunching Probability

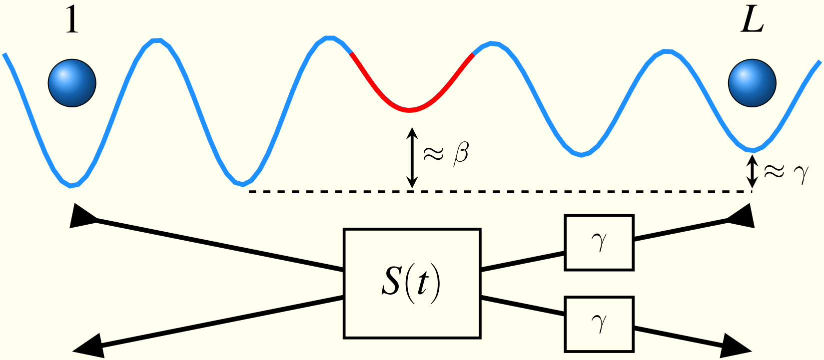


- **Transition from bunching to anti-bunching** close to the Mott phase transition point
- Stable bunching effect for $U < 0.2J$

Beam splitter and Phase shifter

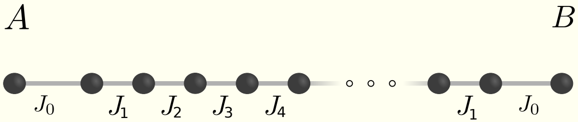


Beam splitter and Phase shifter



Routes to perfection: engineered lattice

Christandl et al., PRL, 2004 Albanese et al., PRL, 2004

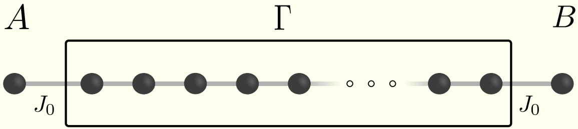


$$J_\ell = \sqrt{(\ell+1)(N-\ell)}$$

Mirror inversion after a certain time t^*

$$|\psi(x, t+t^*)\rangle = |\psi(N-x, t)\rangle$$

Routes to perfection: minimal engineering



Banchi, Apollaro, Cuccoli, Vaia, Verrucchi, NJP 2011

Banchi, Bayat, Verrucchi, Bose, PRL 2011

With optimally tuned J_0

- Dynamics generated by excitations with almost linear dispersion relation
- **NOT** an effective two-site Hamiltonian, J_0 is comparable with J
- Ballistic coherent dynamics ($t^* \approx L$)

Optimal dynamics with the XX model

- Dispersion relation ($J \equiv 1$)

$$\omega_k = \cos k$$

- Density of excitations

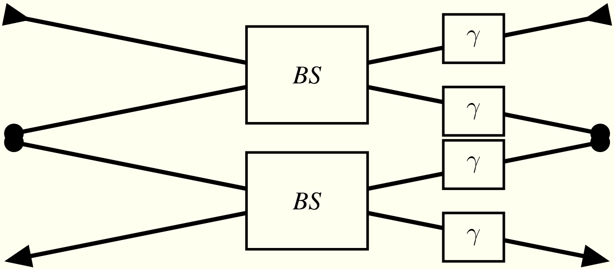
$$\rho(k) \approx \frac{1}{\Delta^2 + \cot^2 k} \qquad \Delta = \frac{J_0^2}{2 - J_0^2}$$

  peaked around the inflection point $k = \pi/2$ with width Δ

- Group velocity of the "wave-packet" close to the inflection point

$$v_k = \partial_k \omega_k \simeq \text{const.} \times \left[1 + \left(\frac{2}{N J_0^6} - \frac{1}{2}\right) k^2 + O(k^4)\right]$$

Application: Mach-Zender interference

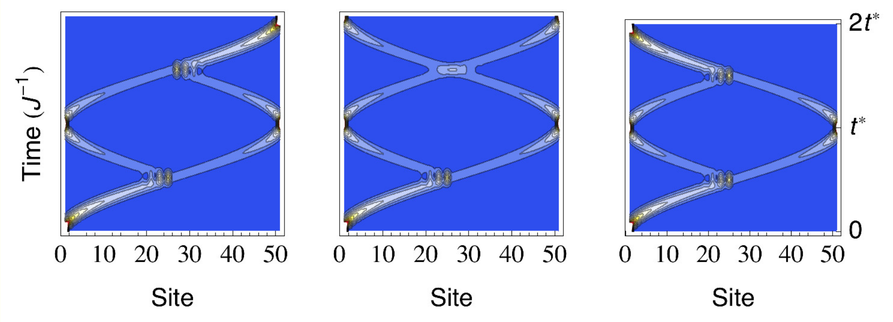


S matrix for the process (50/50 BS)

$$S(t^*) \simeq D\tilde{S}(t^*) \qquad \tilde{S}_\phi \simeq \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & -ie^{i\phi} \\ -ie^{i\phi} & e^{2i\phi} \end{pmatrix}$$

With fully engineered chain $D = 0.989$ when $L = 51$

Application: Mach-Zender interference



Different output ports depending on the phase shift

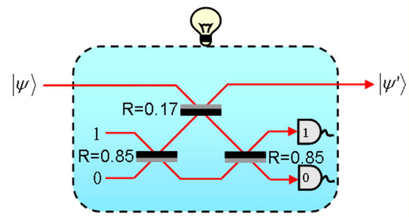
Imperfections

- Non-local impurity: $\beta_j = \beta^{50/50} \exp[-(N+1-j)^2/\sigma^2]$
  - unaffected when FWHM $\lesssim 0.66a$
  - small deviations ($\lesssim$5% with "renormalized" $\beta^{50/50}$) when FWHM $\lesssim 8a$
- Open ends via a strong impurities β_{wall} at the boundaries
  - deviations $< 5\%$ when $\beta_{\text{walls}} \gtrsim 3$
- Curvature $\mu_j^{\text{eff}} = \mu - J\omega^2 j^2/2$
  - No deviations from the ideal case when $\omega \lesssim 0.03$

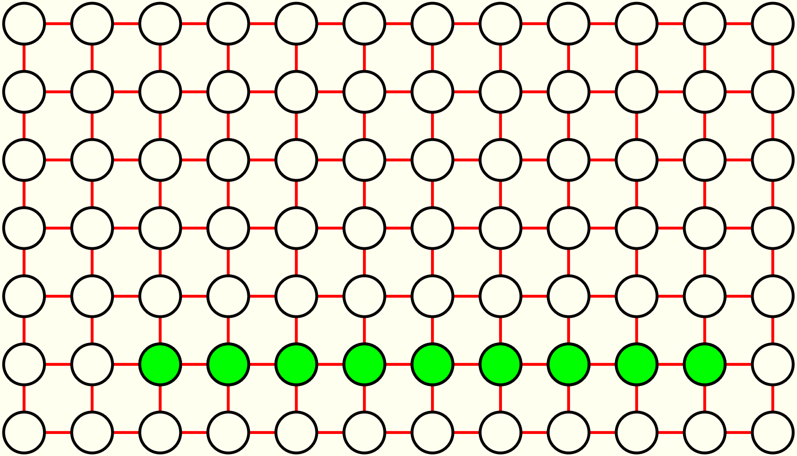
Discussion towards KLM on a lattice

Knill-Laflamme-Milburn quantum computer

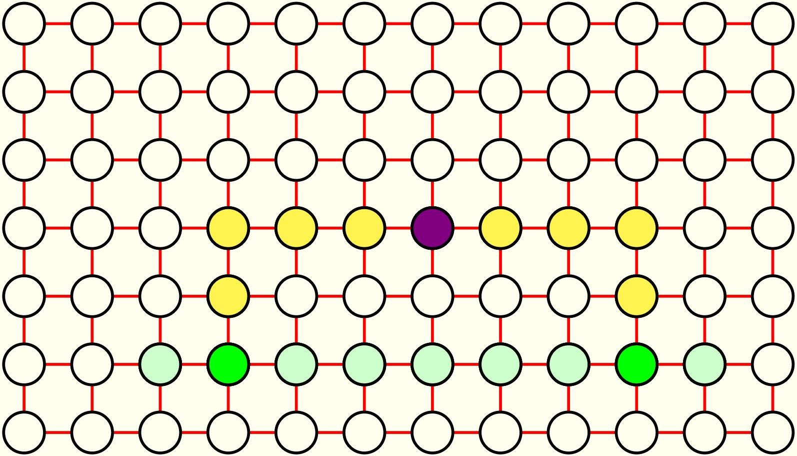
- Linear optics quantum computer
- The fundamental non-linear interaction (**non-linear phase shift**) implemented with linear optics, auxiliary photons and measurements



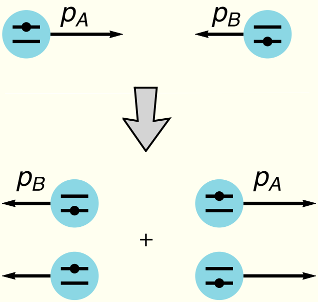
Discussion towards KLM on a lattice



Discussion towards KLM on a lattice



Quantum gates from particle scattering



Can we exploit natural interactions to implement quantum gates?

Flying qubits in one dimension

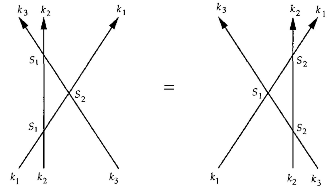
- Lieb-Liniger Model

$$H = -\frac{\partial^2}{\partial x_1^2} - \frac{\partial^2}{\partial x_2^2} + 2U\delta(x_1 - x_2),$$

- Scattering matrix for bosons/fermions ($p_2 > p_1$)

$$S(p_2, p_1) = \frac{(p_2 - p_1) \pm iU\,\mathrm{SWAP}_{12}}{p_2 - p_1 + iU},$$

- Integrability (Bethe-Ansatz) from S



Quantum gate between flying qubits

$$S^{B/F}|\uparrow\rangle_A|\uparrow\rangle_B = e^{i\phi_{B/F}}|\uparrow\rangle_A|\uparrow\rangle_B$$

$$S^{B/F}|\downarrow\rangle_A|\downarrow\rangle_B = e^{i\phi_{B/F}}|\downarrow\rangle_A|\downarrow\rangle_B$$

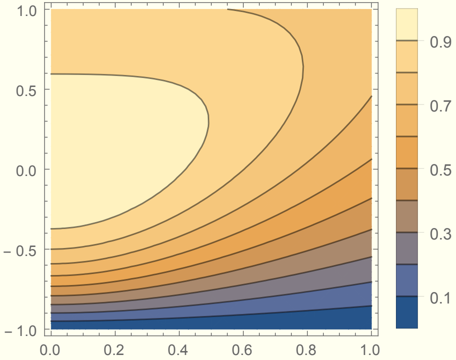
$$S^{B/F}|\uparrow\rangle_A|\downarrow\rangle_B = \frac{p_{A+B}|\uparrow\rangle_A|\downarrow\rangle_B \mp iU|\downarrow\rangle_A|\uparrow\rangle_B}{p_{A+B} + iU}$$

$$S^{B/F}|\downarrow\rangle_A|\uparrow\rangle_B = \frac{p_{A+B}|\downarrow\rangle_A|\uparrow\rangle_B \mp iU|\uparrow\rangle_A|\downarrow\rangle_B}{p_{A+B} + iU}$$

Maximal entanglement when

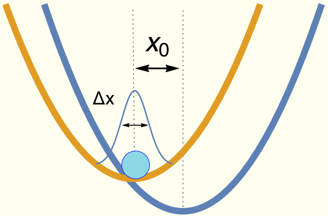
$$p_{A+B} = U$$

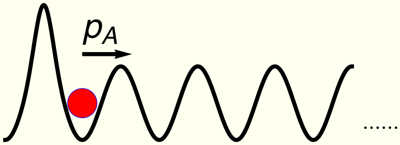
Gaussian wave-packet



- Mean $U(1+\delta)$
- Variance $U\eta$

Implementations





$$H = \sum_{j,\alpha} \frac{J_j}{2} \left[a^{\dagger}_{j,\alpha} a_{j+1,\alpha} + \text{h.c.} \right] + \sum_{j,\alpha,\beta} \frac{U_j^{\alpha\beta}}{2} n_{j,\alpha} n_{j,\beta} \; ,$$

Concluding remarks

Linear optics on a lattice

- Operations via local optical potentials
- Considered applications: beam splitters, phase shifters, Hong-Ou-Mandel effect, Mach-Zender interferometry
- Transition from bosonic to fermionic behavior. Critical point?
- Main possible application: KLM quantum computer on a lattice, generation of non-classical states

Quantum gate via scattering

- Natural interactions can generate maximal entanglement for physical parameters

- Linear optics on a lattice

  E. Compagno, LB, S. Bose, **arXiv: 1407.8501**

- Quantum gate between flying qubits

  LB, E. Compagno, V. Korepin, S. Bose, soon in arXiv