PERCOLATION THEORY, OPTICAL QUANTUM COMPUTING & COMPUTATIONAL "PHASES" OF MATTER

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Collaborators: Kieling & Eisert

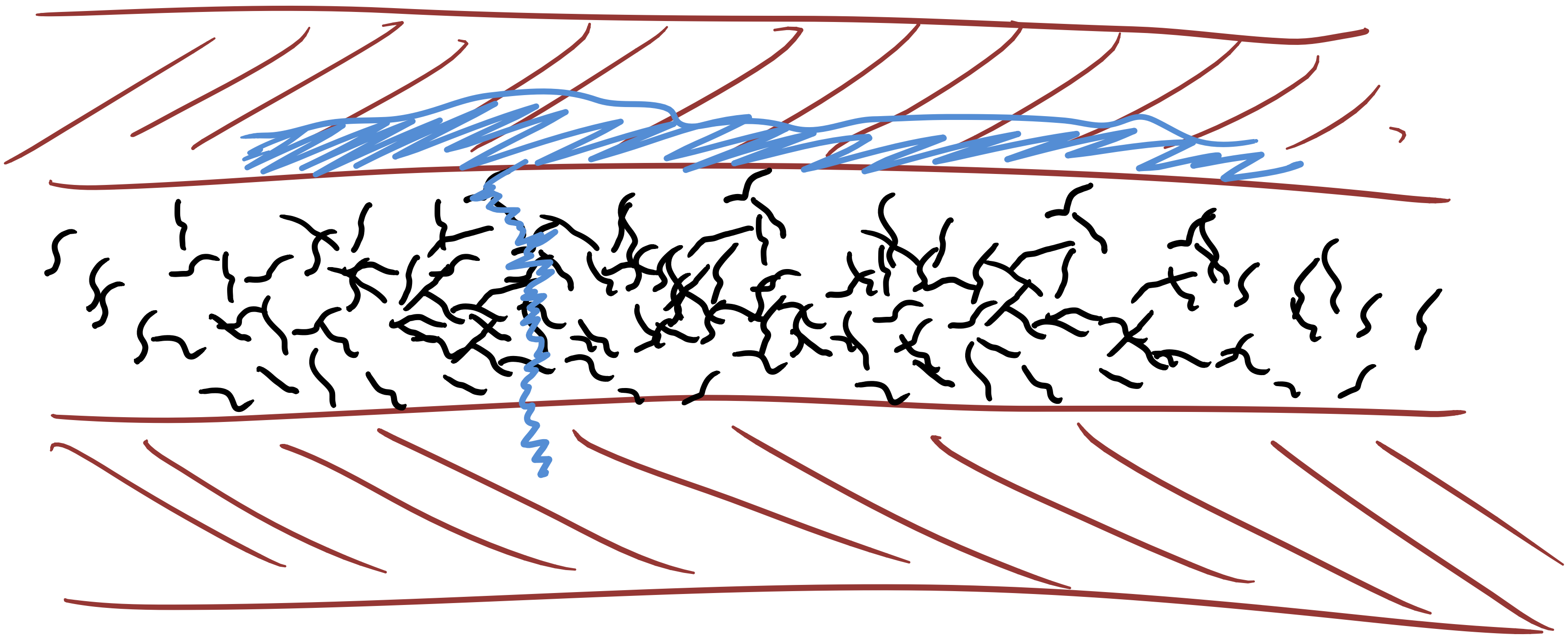
Barrett, Bartlett, Doherty, Jennings

OUTLINE

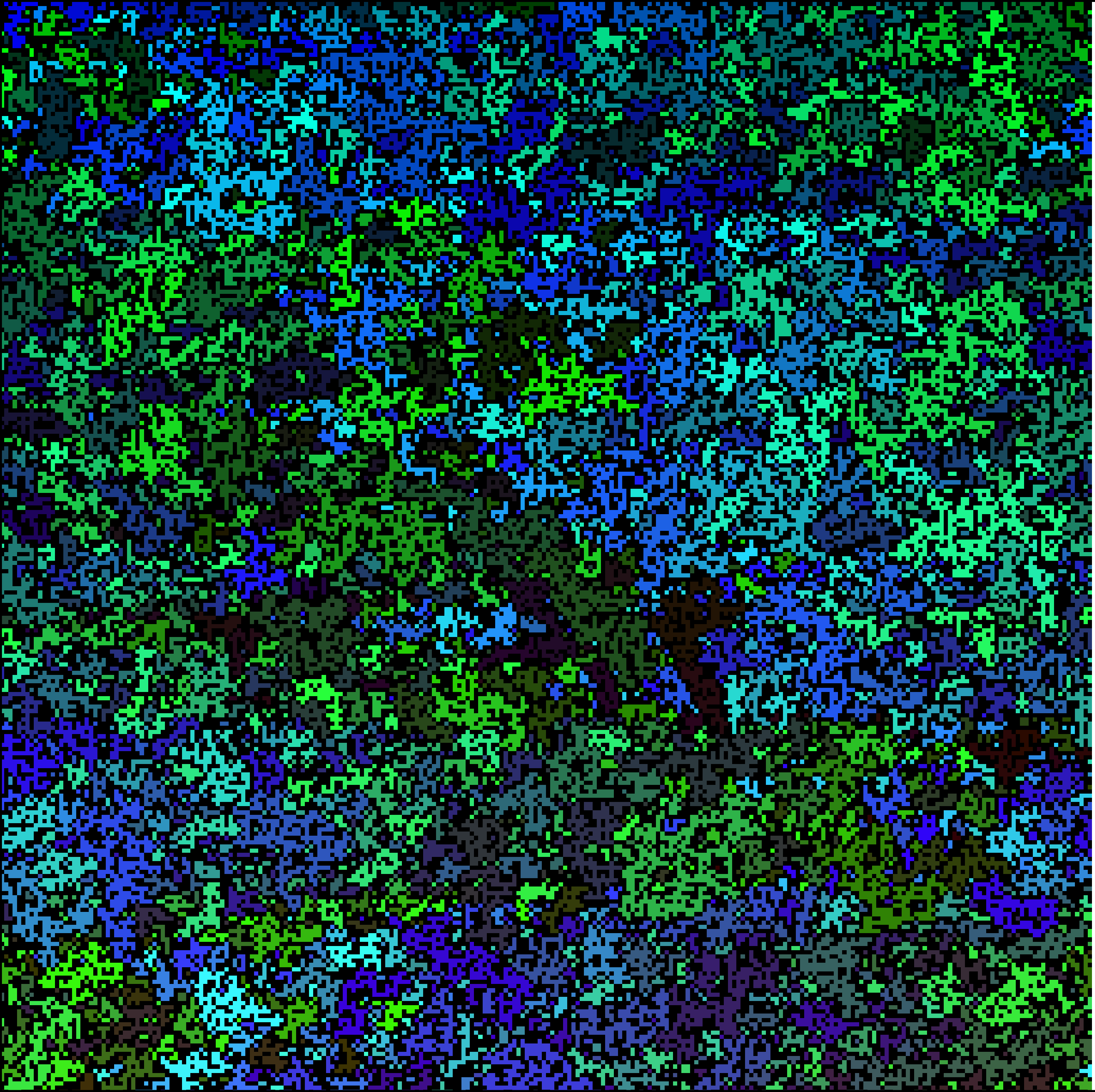
- Percolation phenomena
- Applications to LOQC
- Applications to new phases of matter in a many-body system

WHAT IS PERCOLATION?

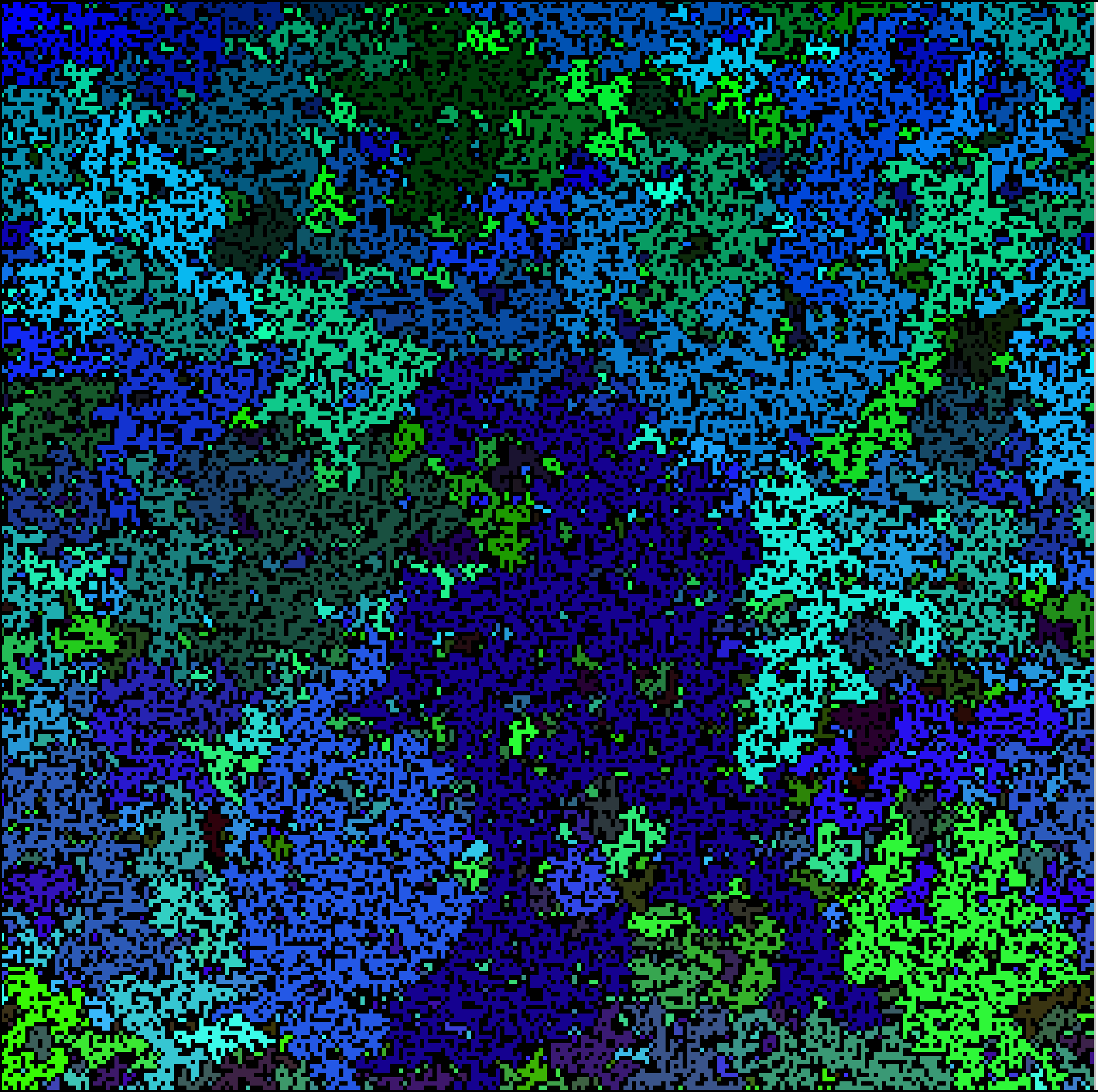
- Toy model for porous rock



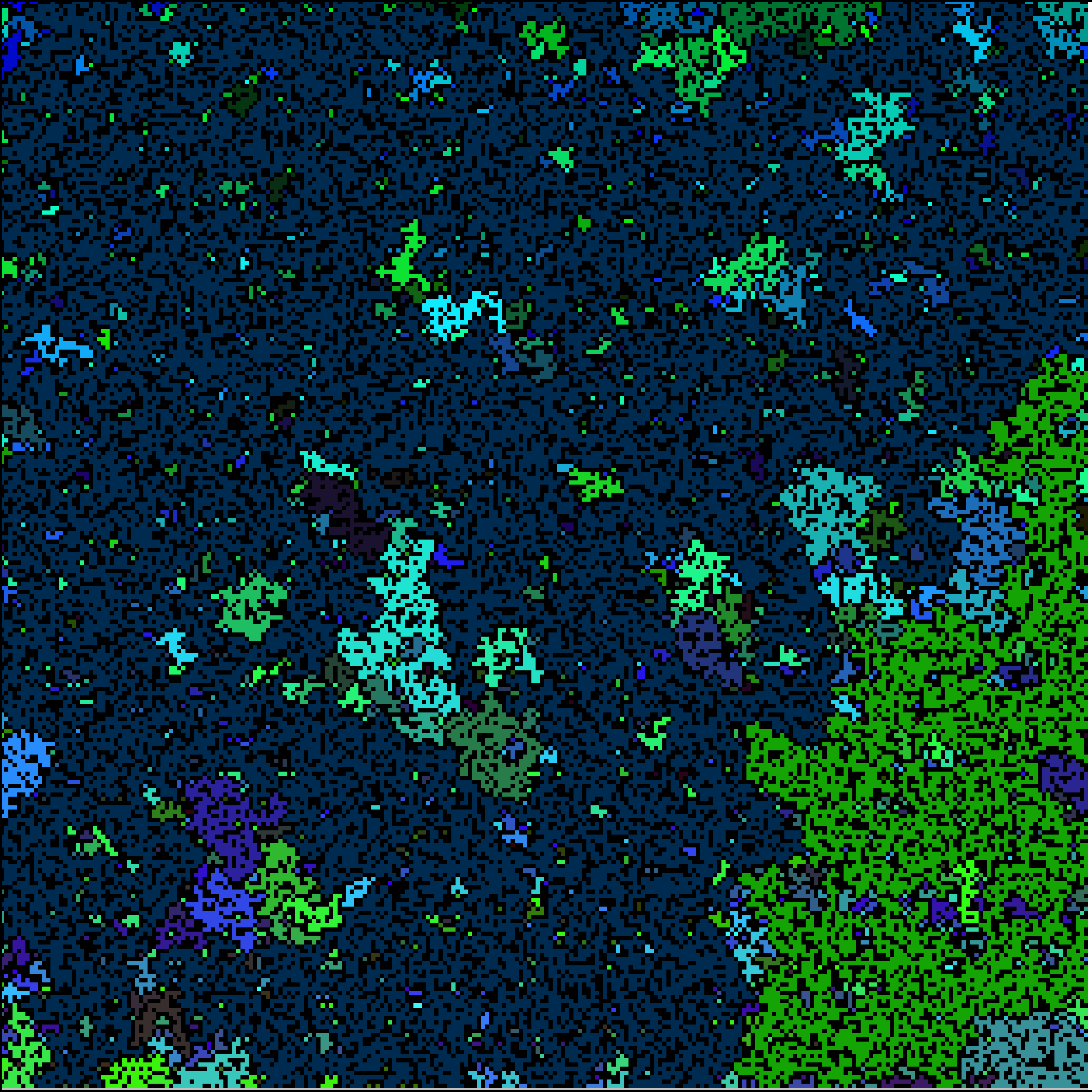
SITE PERCOLATION:



$p = 0.5$

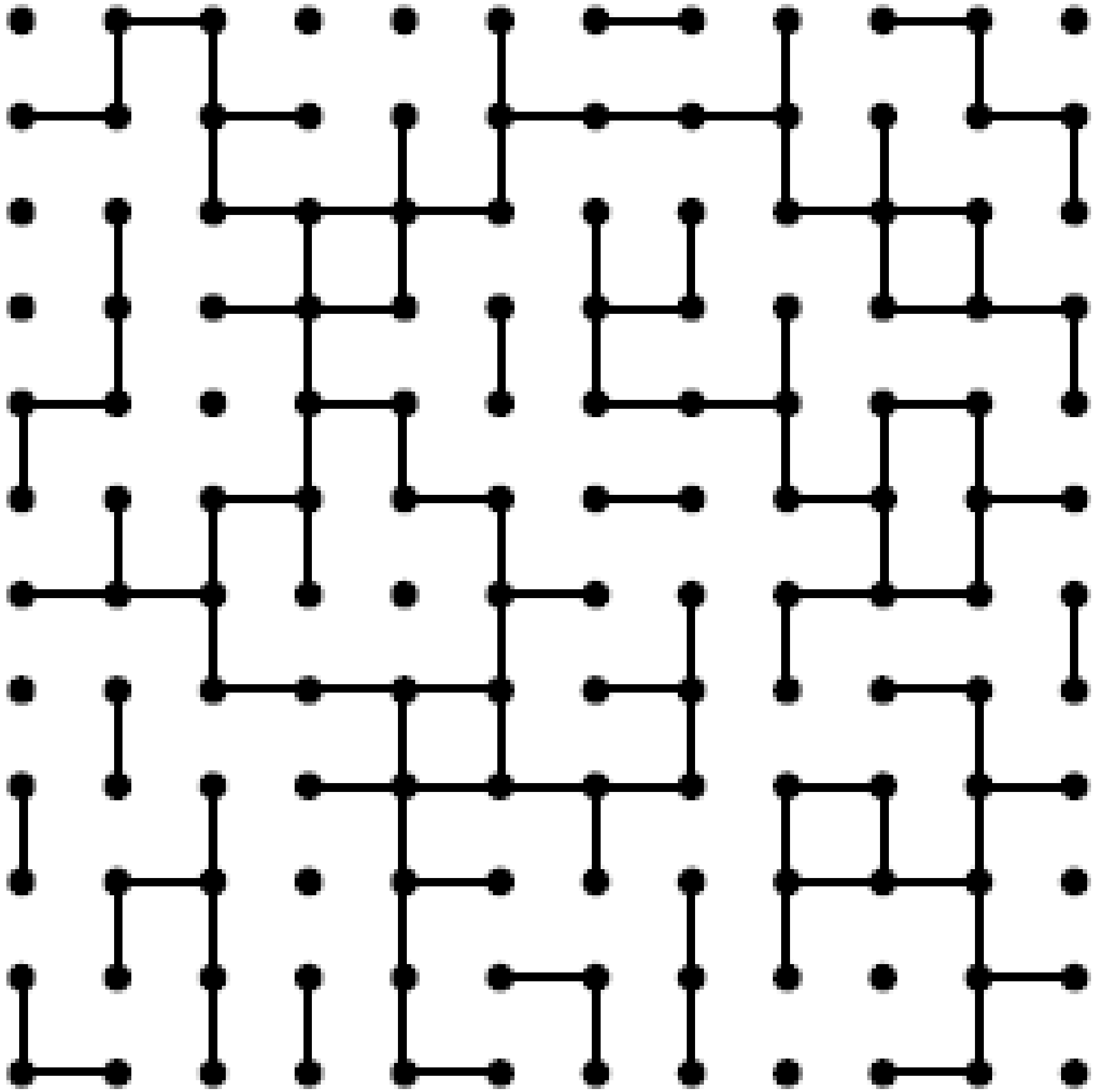


$p = 0.57$



$p = 0.61$

BOND PERCOLATION



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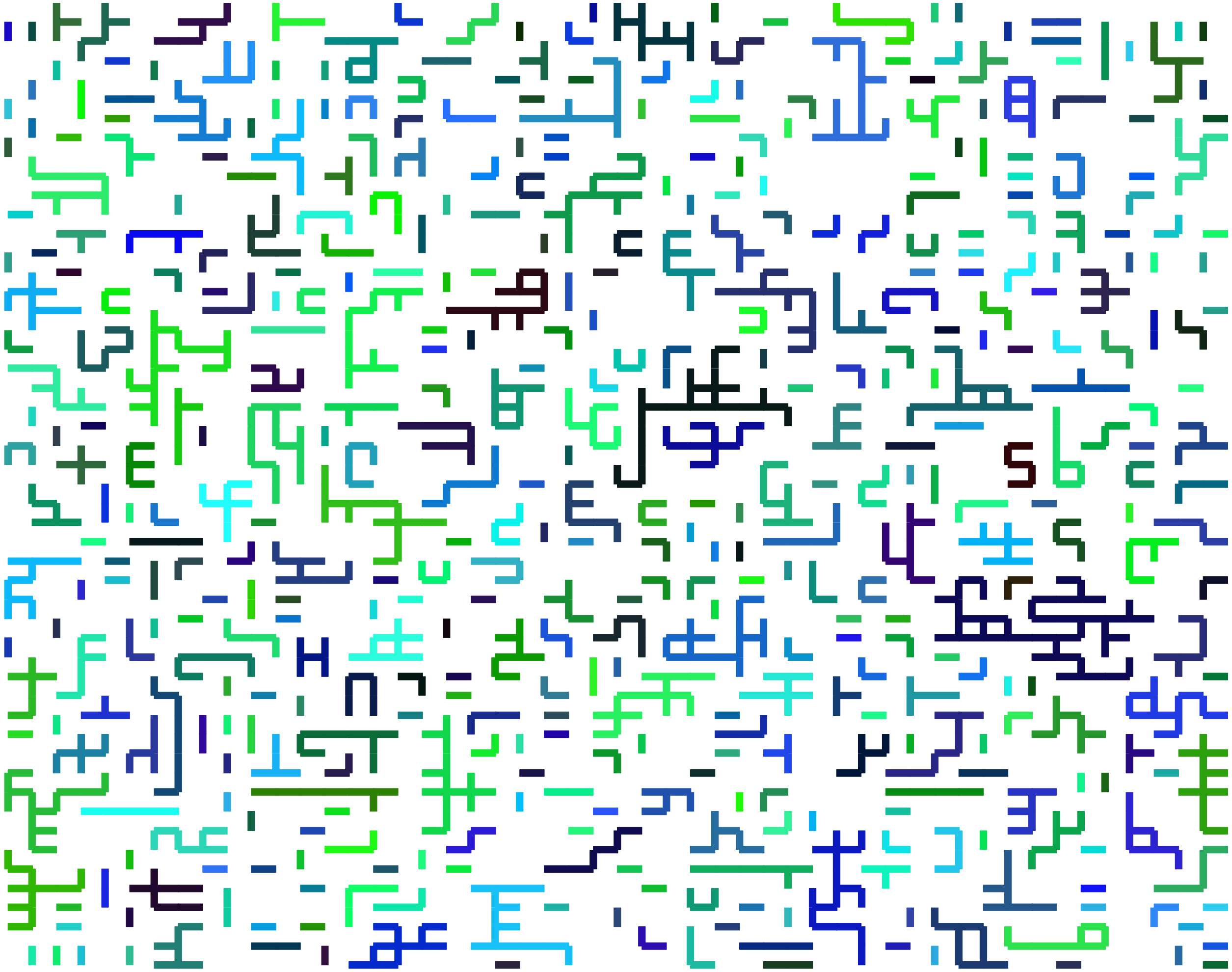
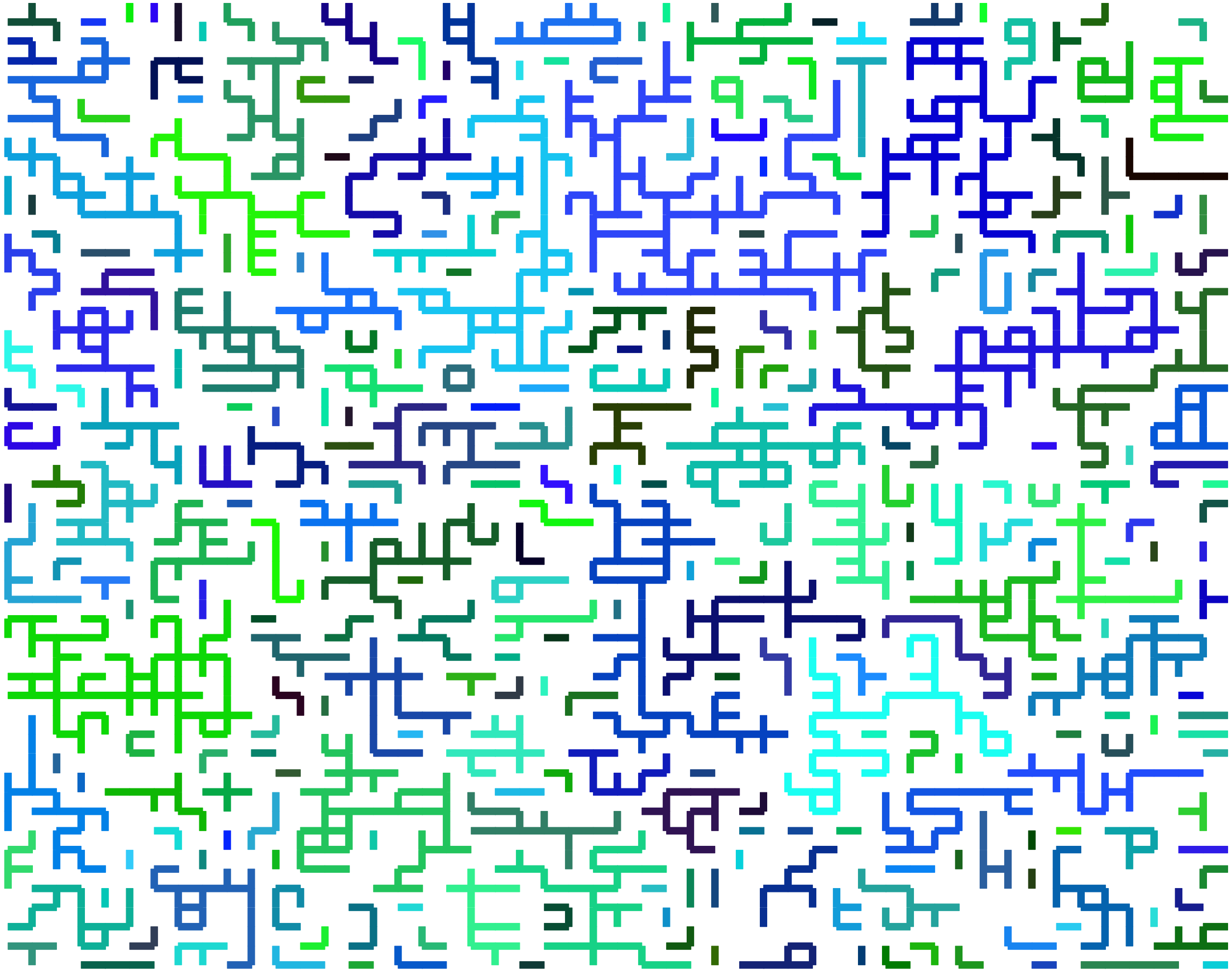
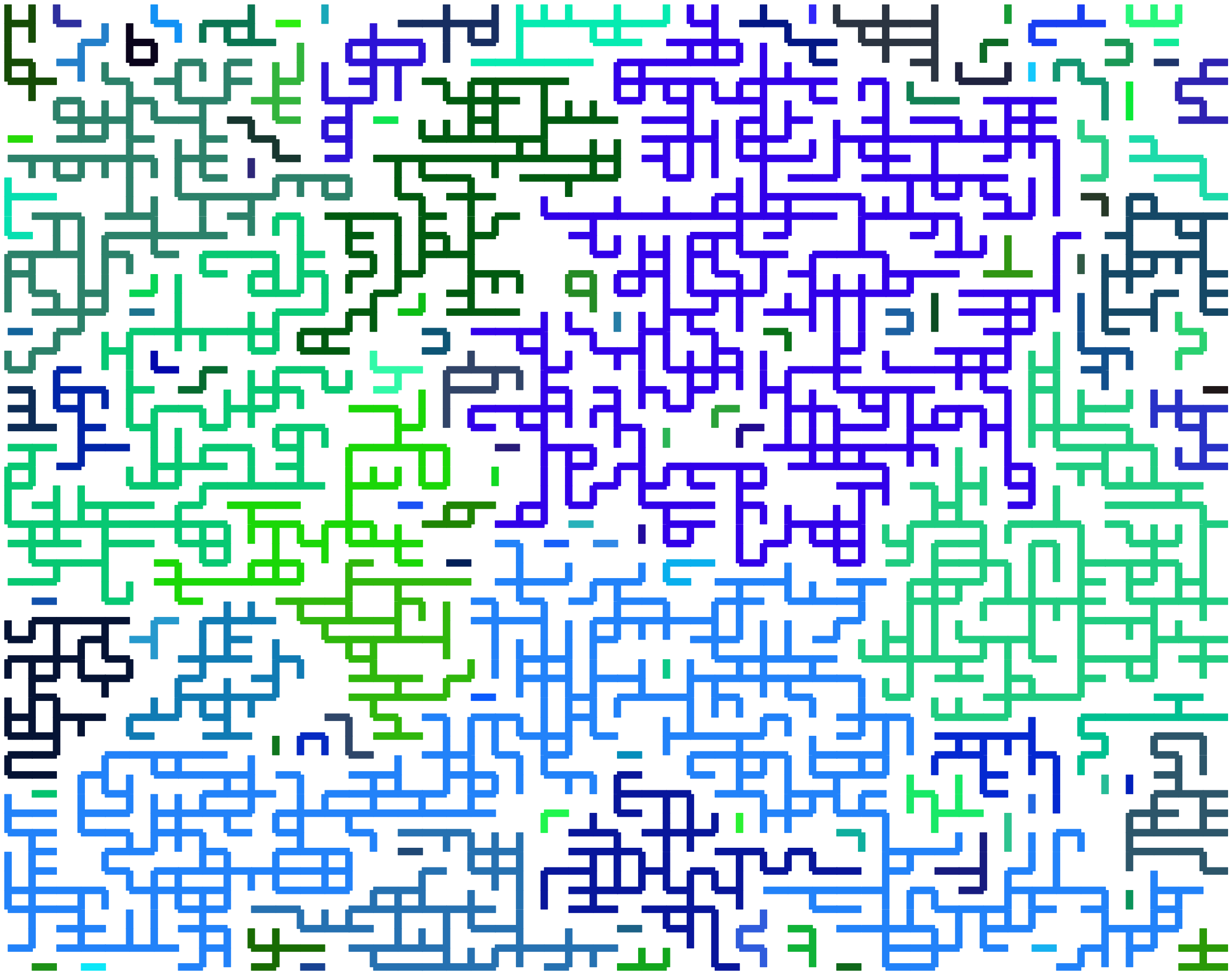


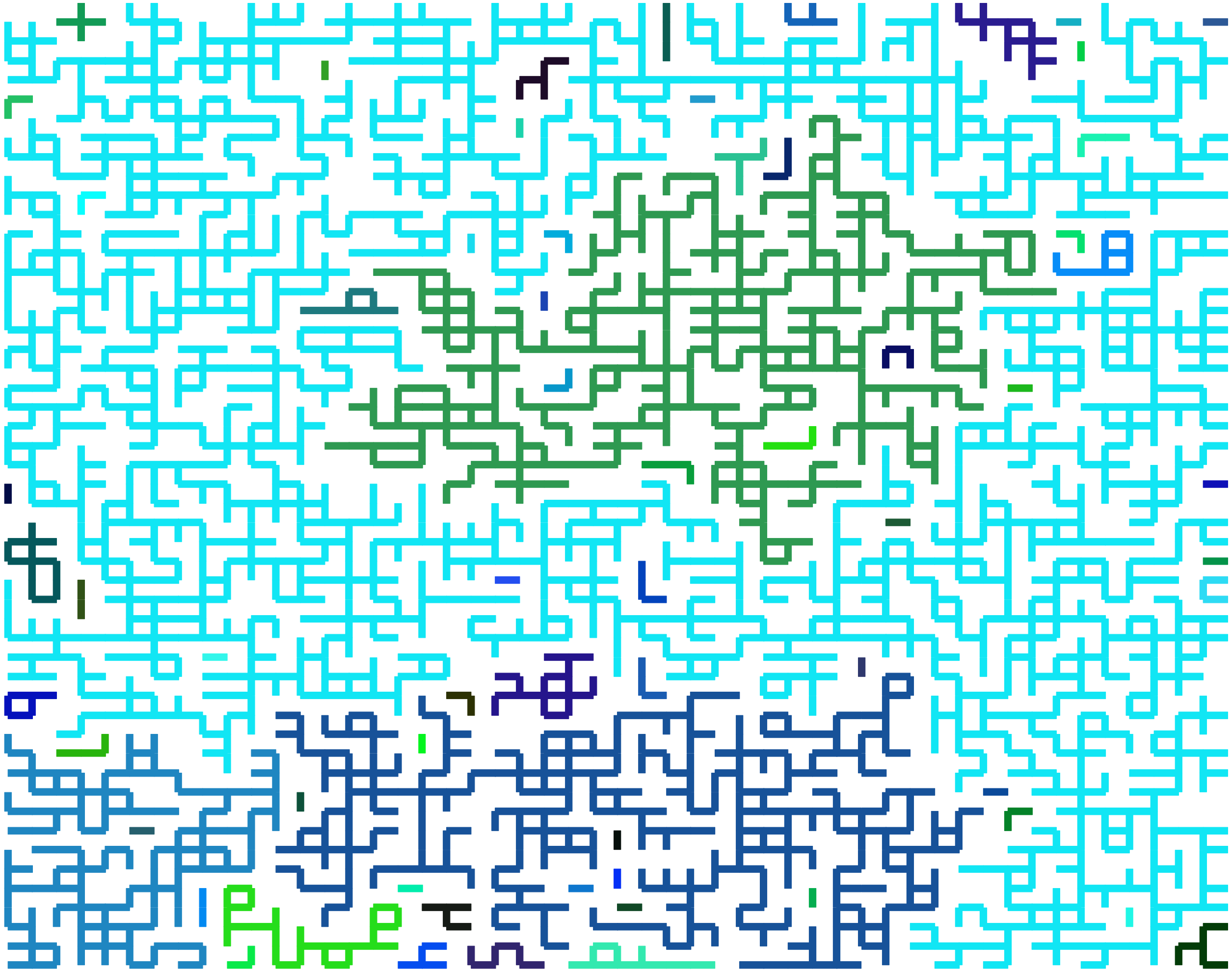
$p=0.3$



$p = 0.4$



$p = 0.48$



p = 0.51

SOME QUANTITATIVE FEATURES OF PERCOLATION:

- There is a lattice-dependent critical probability p_c such that for $p > p_c$ & $N \to \infty$ the existence of a spanning cluster becomes certain

SOME QUANTITATIVE FEATURES OF PERCOLATION:

- There is a lattice-dependent critical probability p_c such that for $p < p_c$ & $N \to \infty$ the NON-existence of a spanning cluster becomes certain

SOME QUANTITATIVE FEATURES OF PERCOLATION:

lattice	p_c (site percolation)	p_c (bond percolation)
cubic (body-centered)	0.246	0.1803
cubic (face-centered)	0.198	0.119
cubic (simple)	0.3116	0.2488
diamond	0.43	0.388
honeycomb	0.6962	0.65271*
4-hypercubic	0.197	0.1601
5-hypercubic	0.141	0.1182
6-hypercubic	0.107	0.0942
7-hypercubic	0.089	0.0787
square	0.592746	0.50000*
triangular	0.50000*	0.34729*

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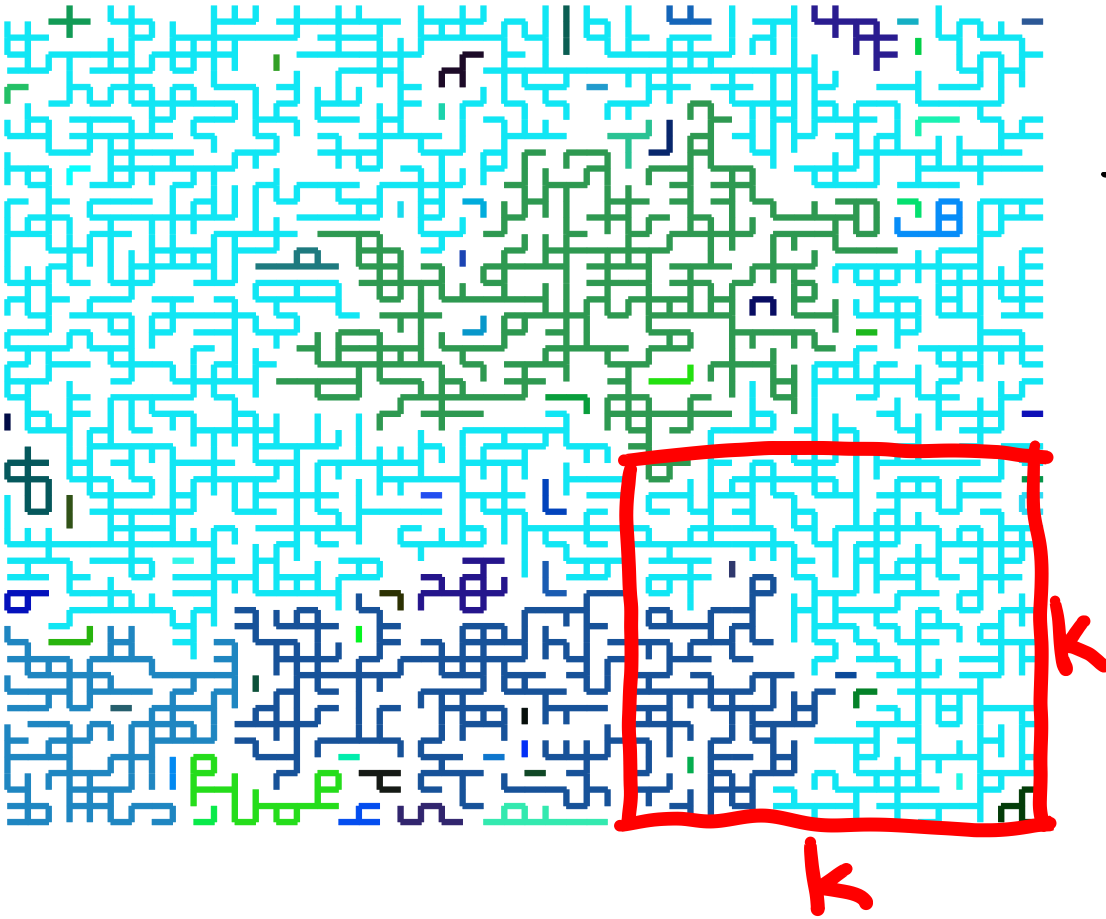

If $p > p_c$ the probability a $k \times k$ block "spans" goes like

$$\boxed{1 - \mathrm{poly}(k)\, e^{-c_1 k}}$$

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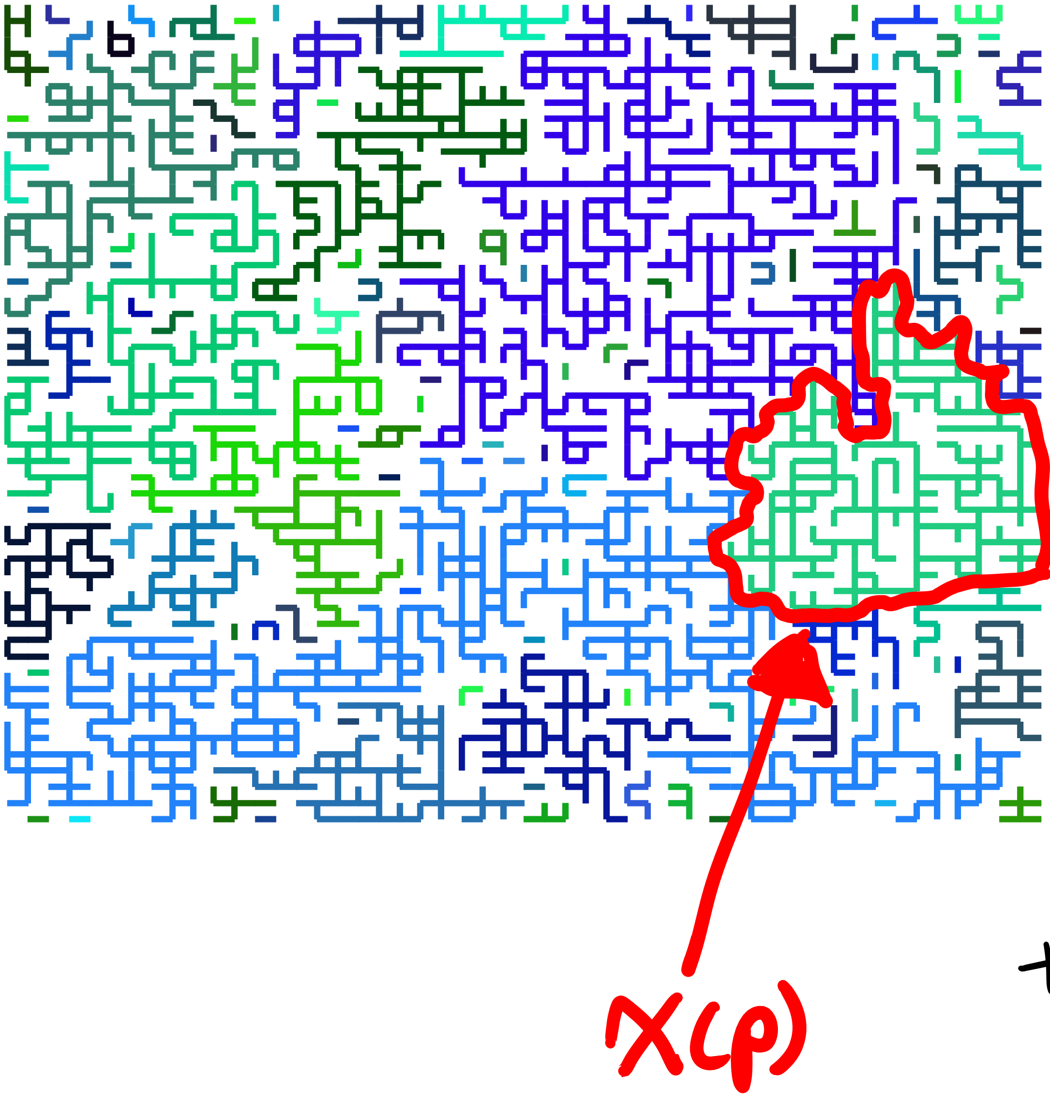


If $p < p_c$ the probability a cluster has n sites/bonds goes like

$$e^{-\frac{1}{2}n/\chi^2}$$

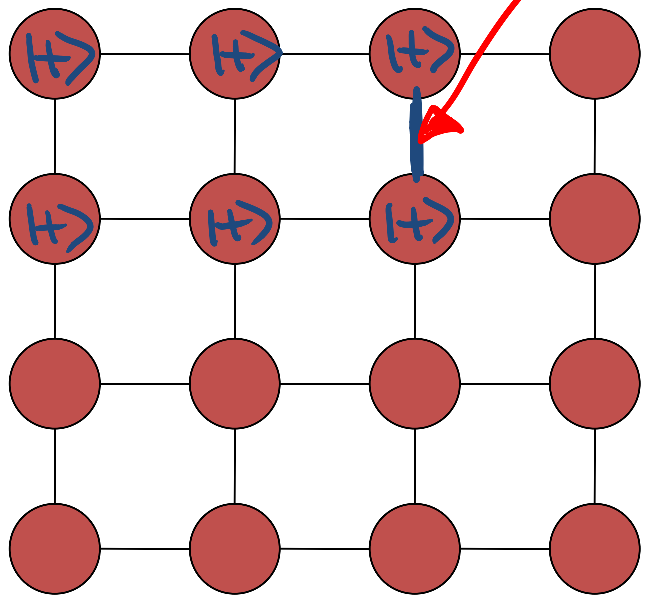
where χ is the mean cluster size

Brief Review of Cluster state LOQC

• Goal is to create multi-photon entangled "cluster" state:

$$CZ \equiv \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & -1 \end{pmatrix}$$

Graphical Representation:



$$|+\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right)$$

$$|C\rangle \equiv \prod_{i\sim j} CZ_{ij} \, |+\rangle^{\otimes n}$$

• Given such a state, universal QC is possible via single photon measurements and classical feedforward

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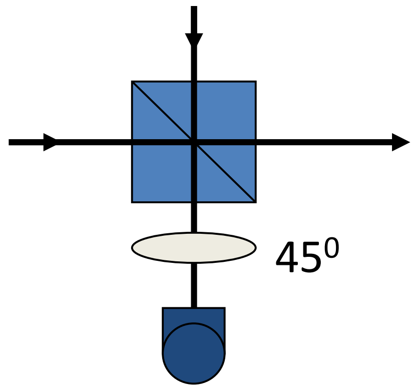
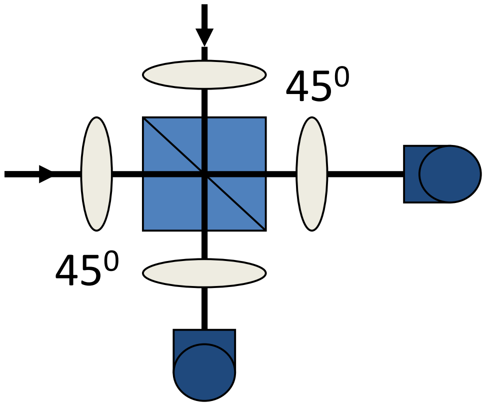
- Large Clusters can be built up from smaller pieces using very simple gates:



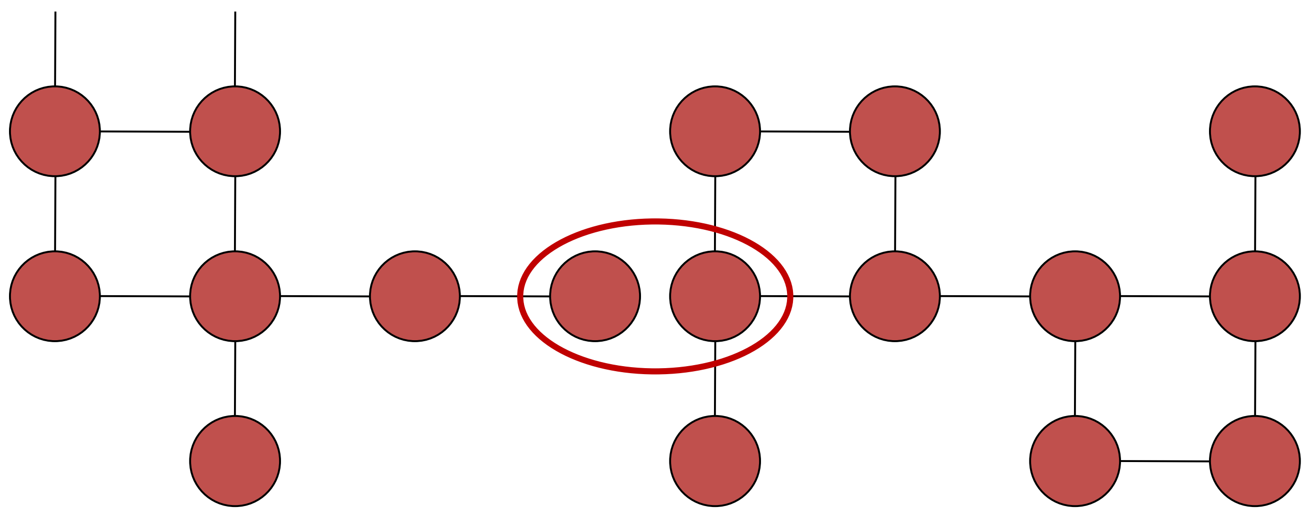
Type-I

Gates succeed with probability 1/2



Type-II

Fusing pieces of cluster states:



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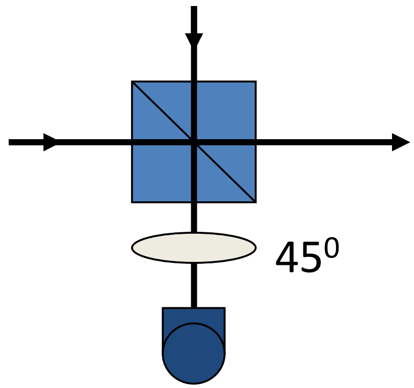

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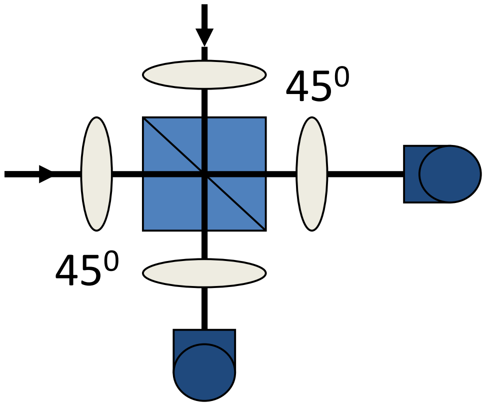

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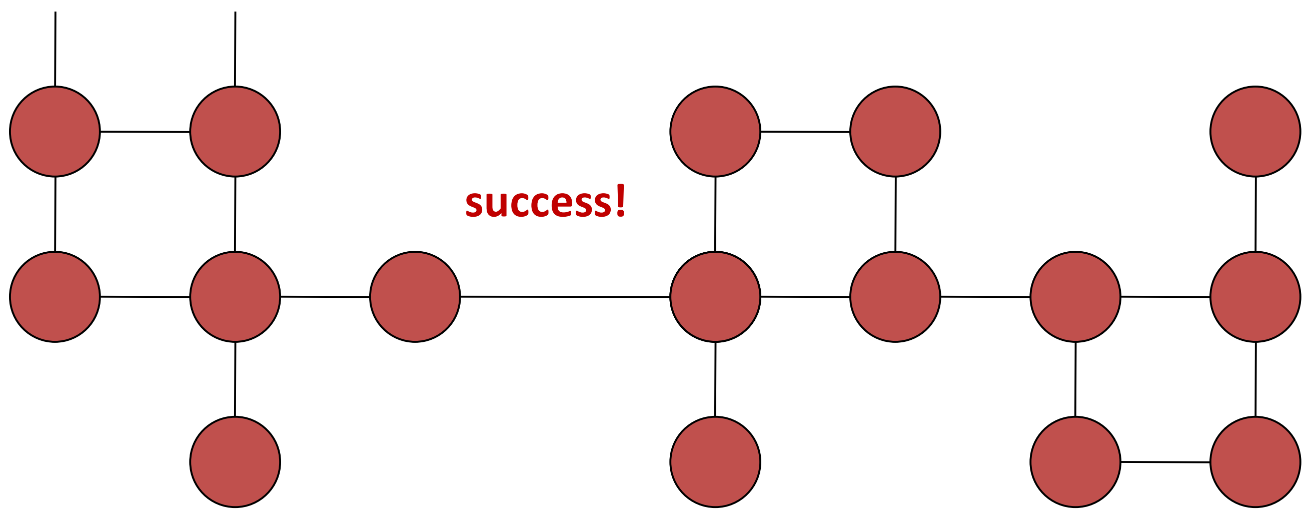

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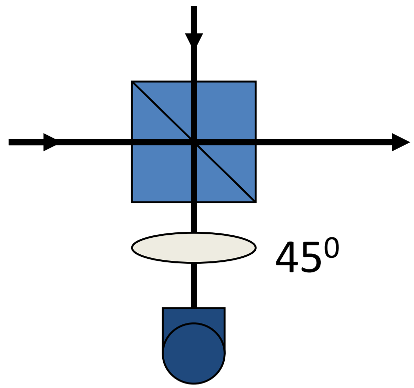

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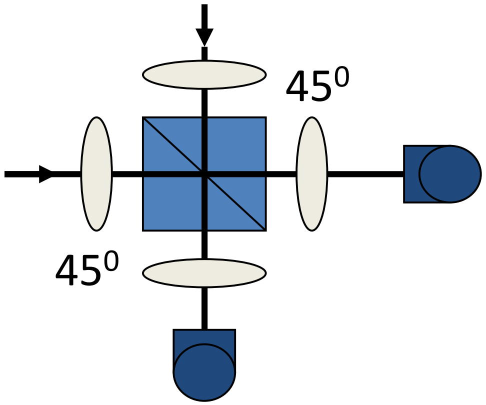

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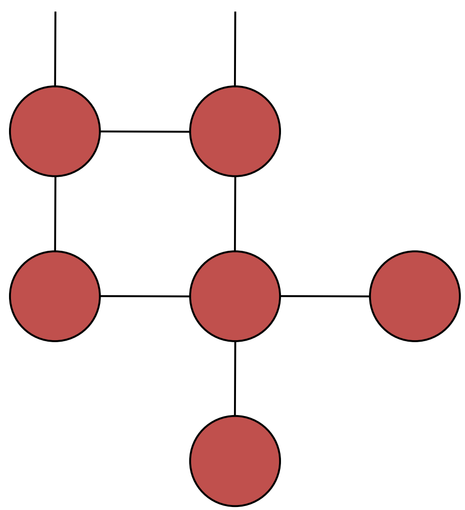
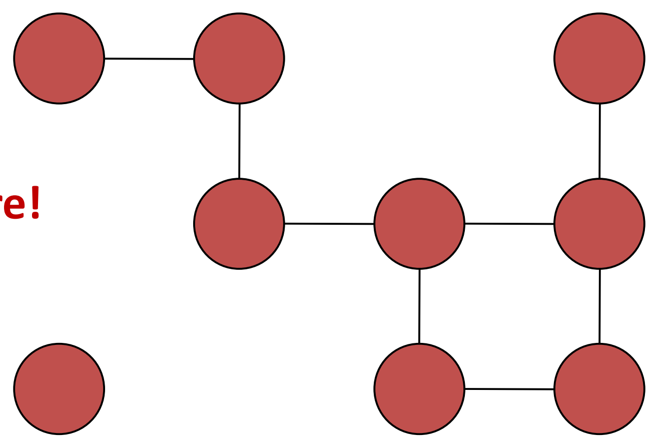


failure!



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LINEAR OPTICAL QUANTUM COMPUTATION

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  - Memory
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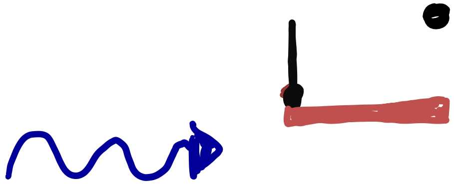

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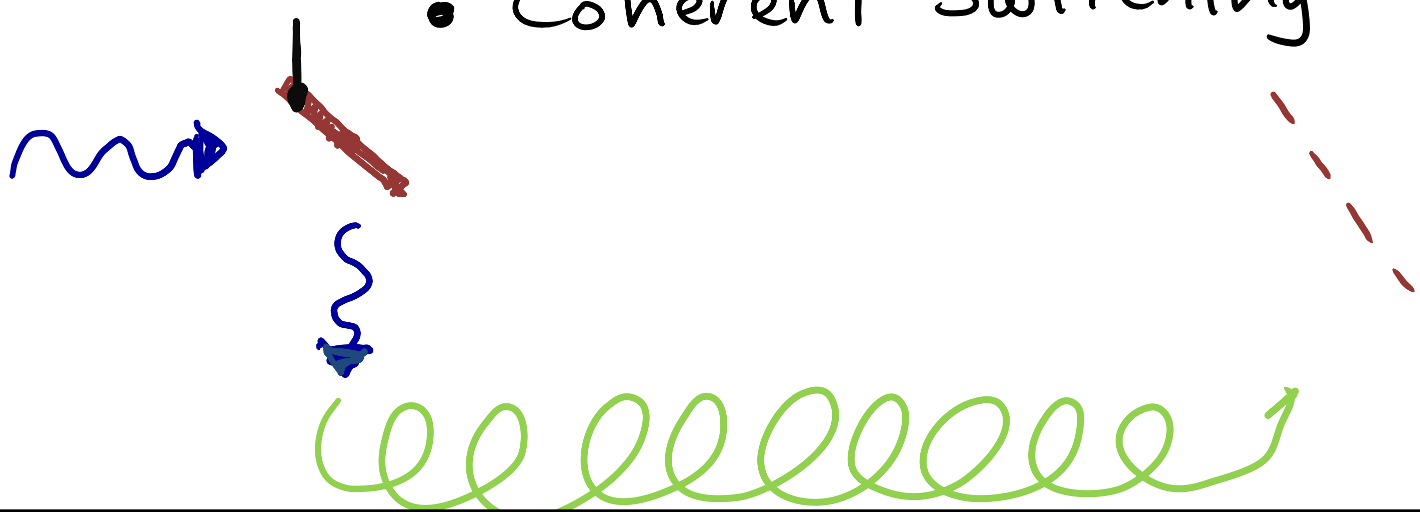

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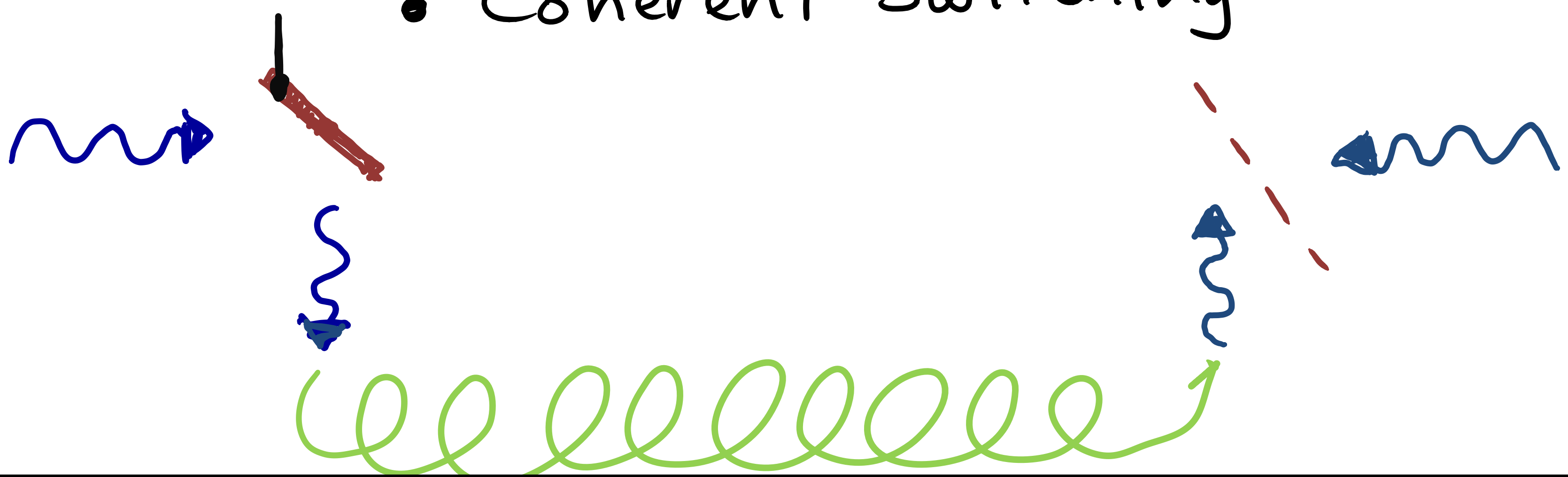
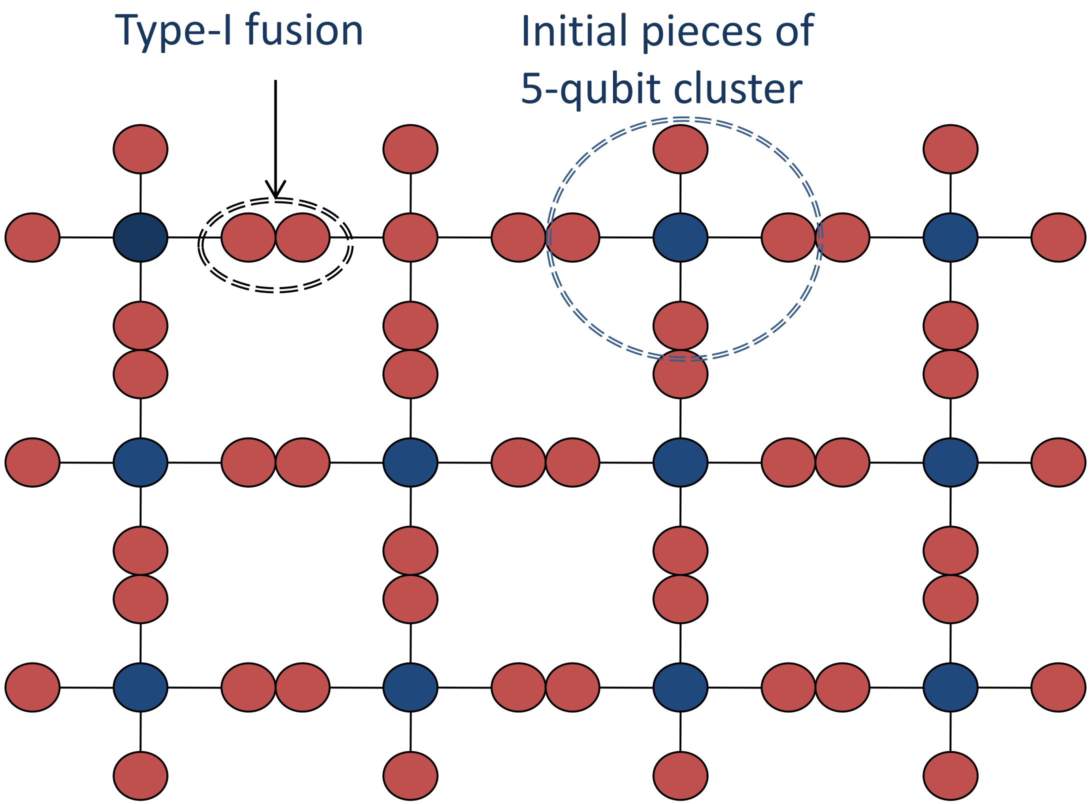
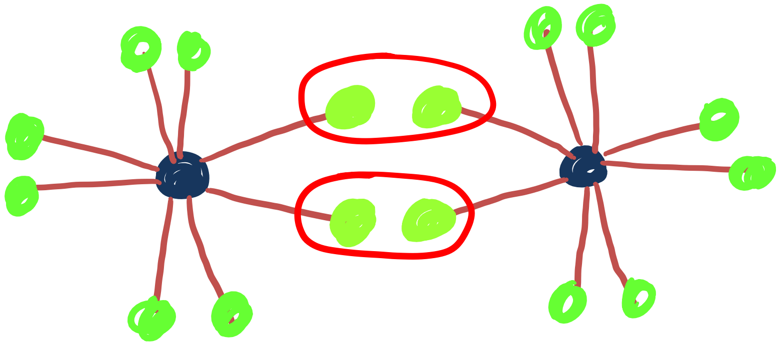


PERCOLATION CAN HELP:



SOLUTION:

Prob of success = 3/4 >> p_{crit}



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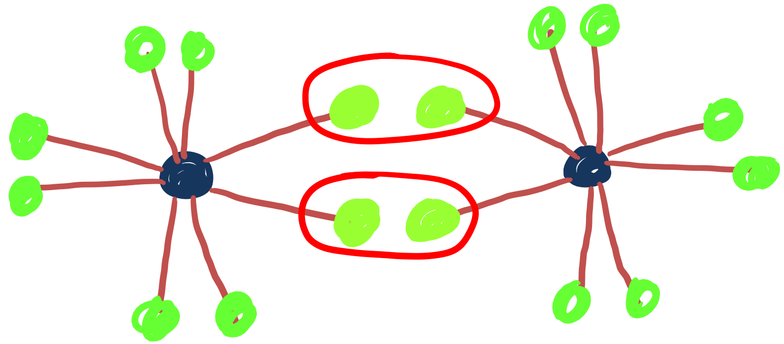
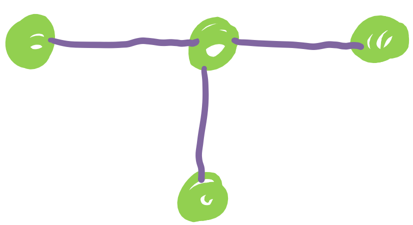
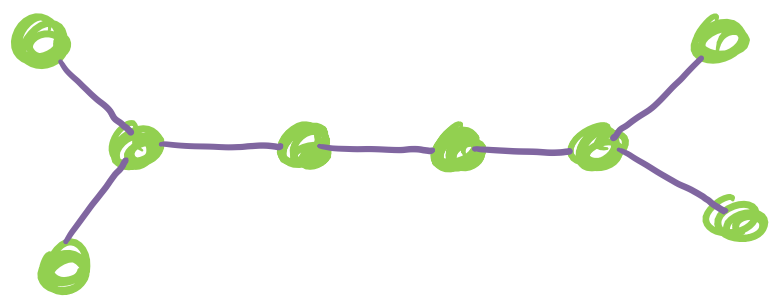
Prob of success = $3/4 \gg P_{crit}$



BUT: We need a square lattice cluster state <u>not</u> some random cluster!





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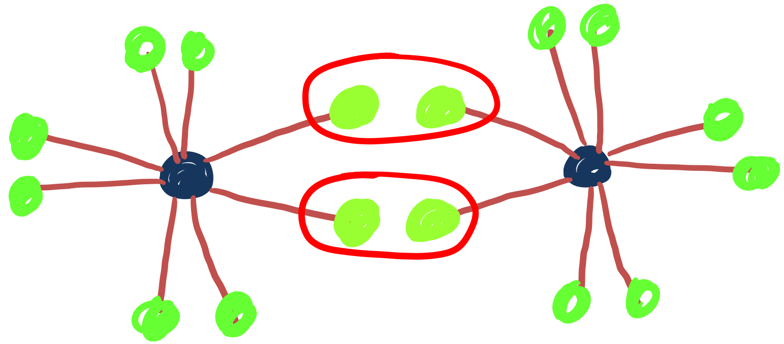

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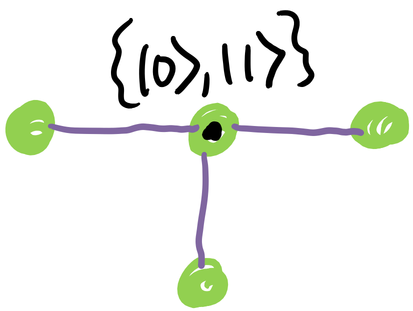

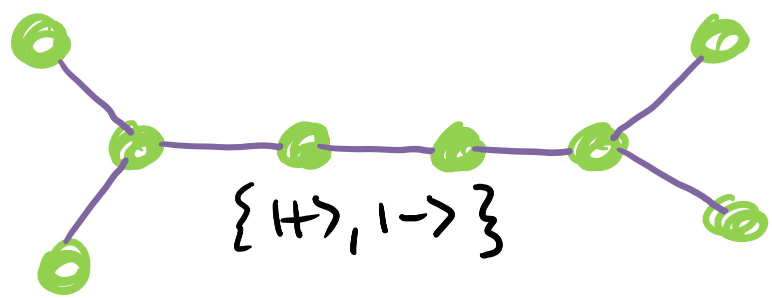

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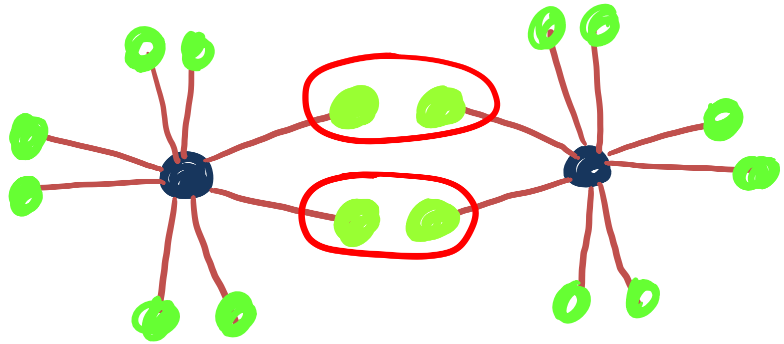

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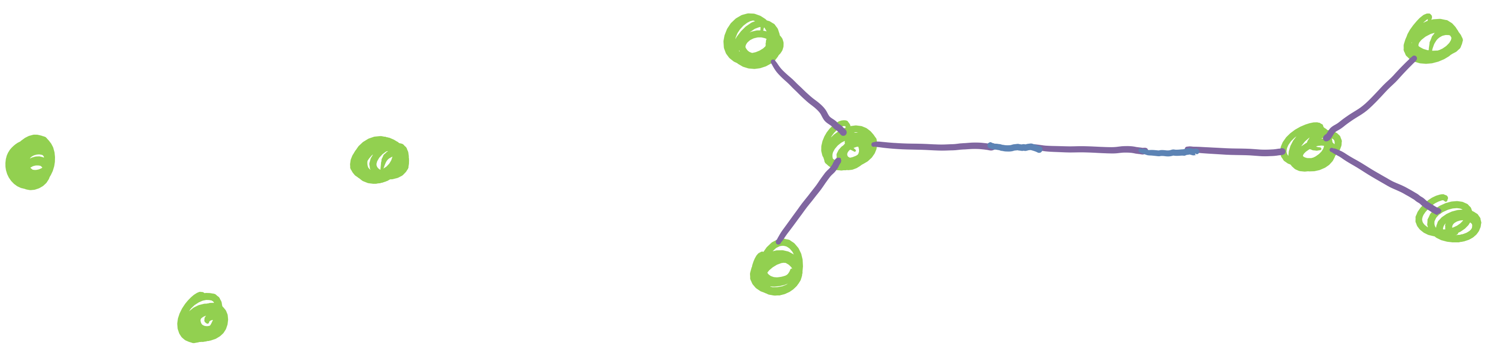
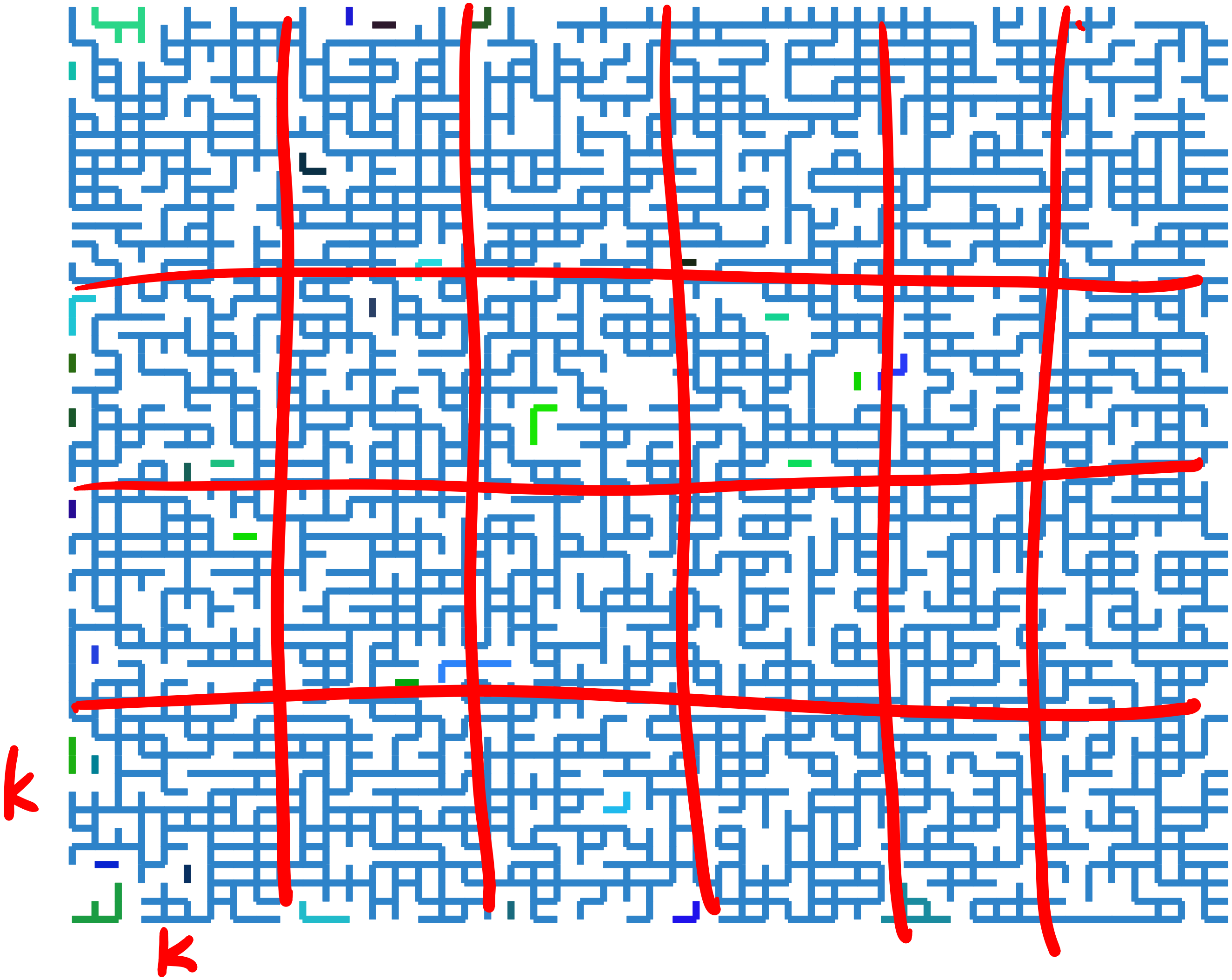
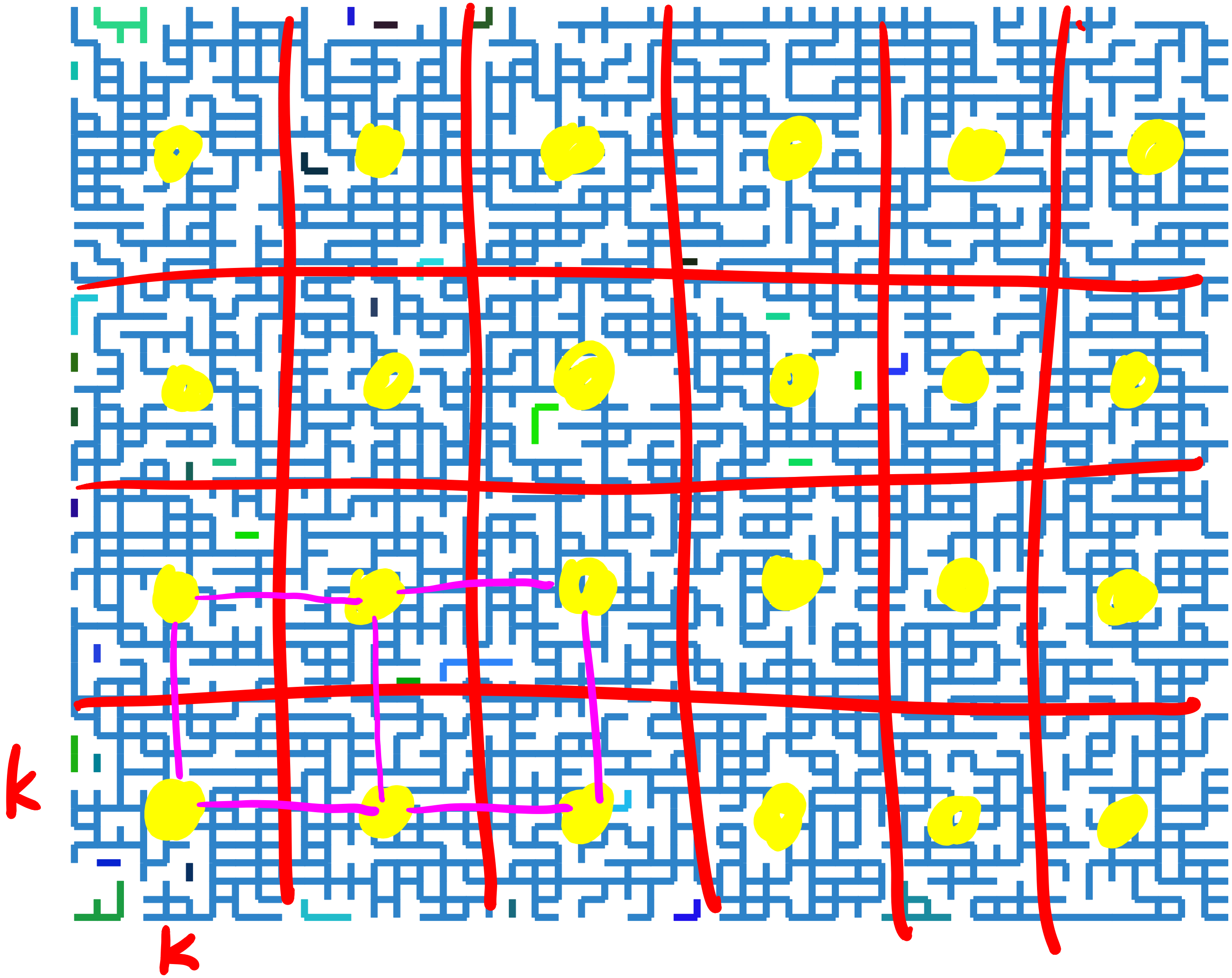






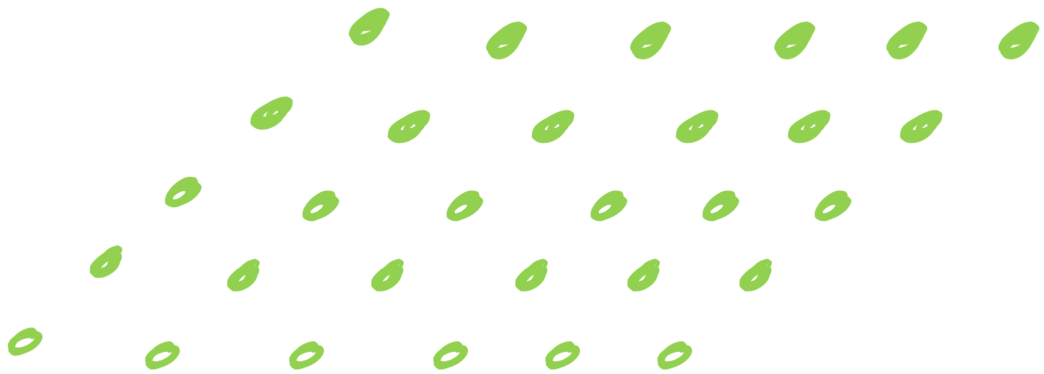
Prob success $= \left(1 - \text{poly}(k)\, e^{-ck}\right)^{N/k} \to 1$ for $k = N^{\epsilon}$

ie Require $N^{2+\epsilon}$ resources

As good as deterministic gates!

Percolation & Q-computational phases of matter

System:



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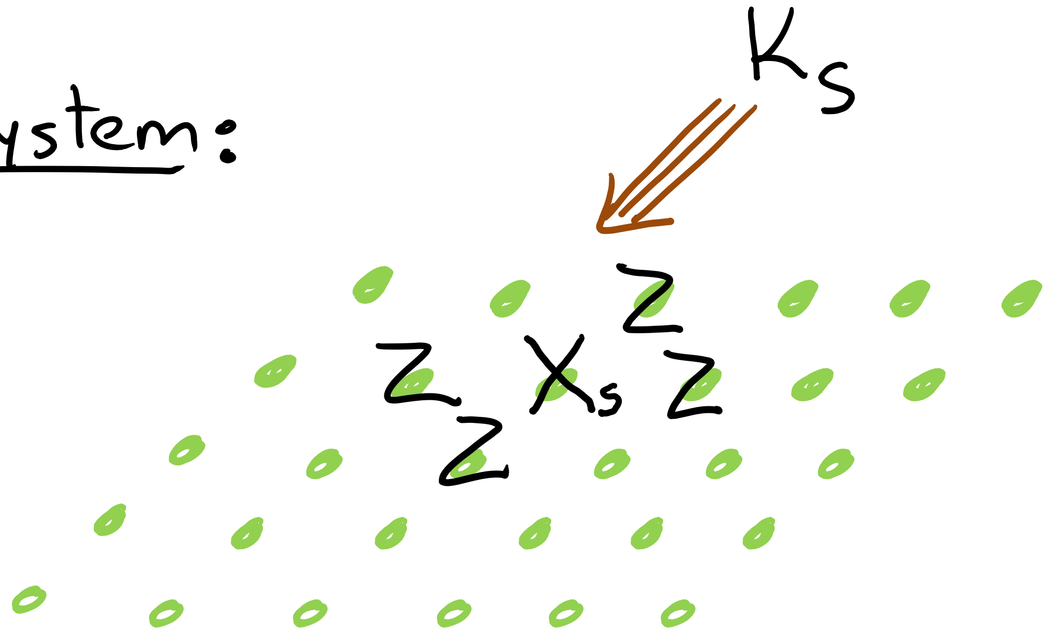

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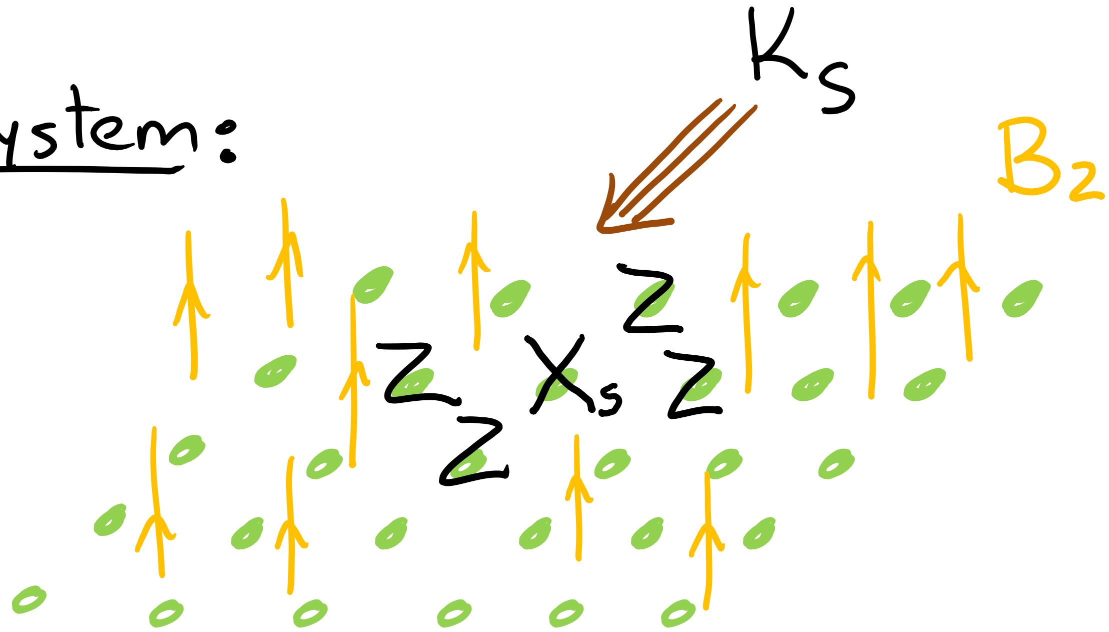


$$H = -\sum_s K_s - B_z \sum_s Z_s$$

Ground state is exactly solvable:

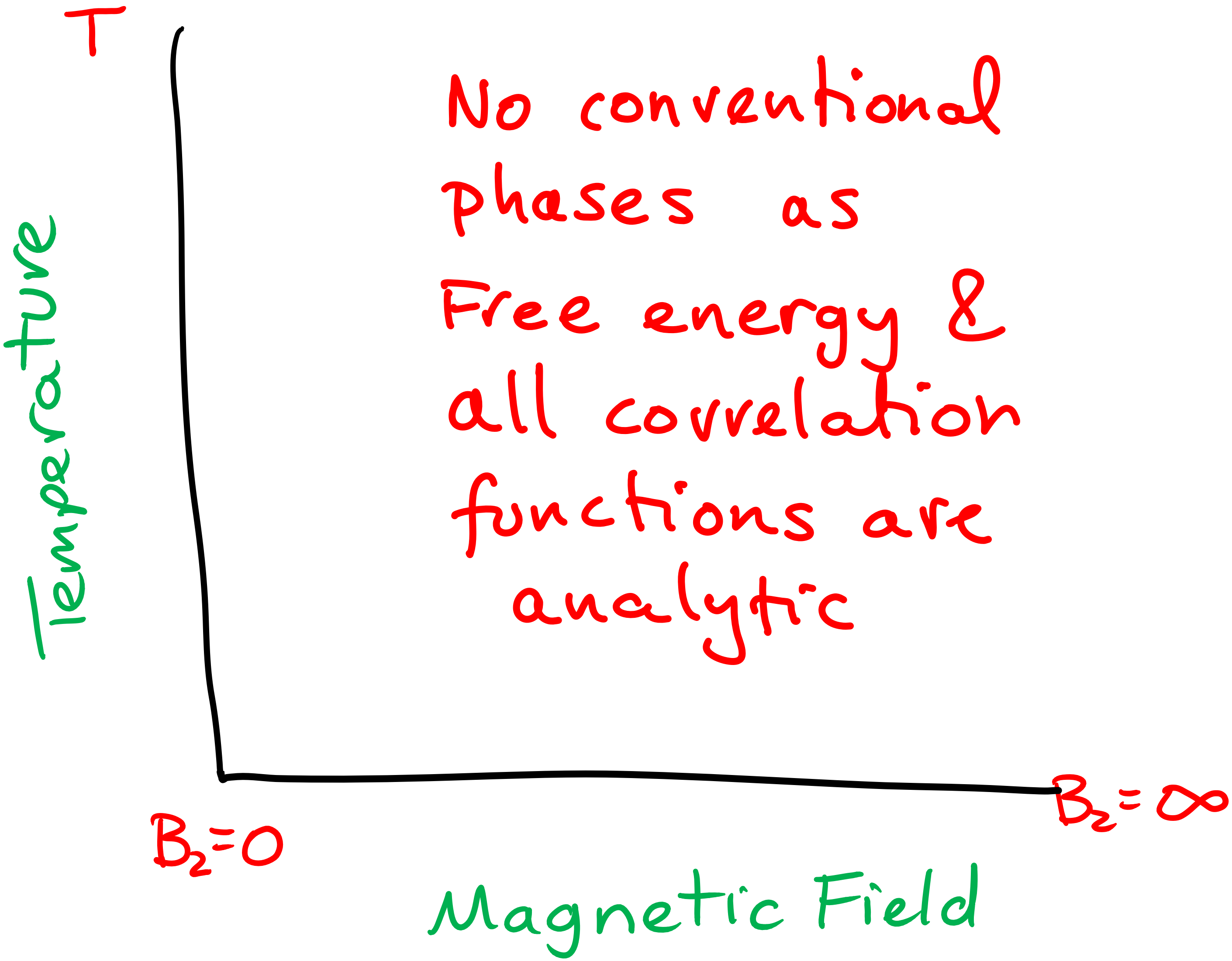
$$|G\rangle = \prod_{i\sim j} CZ_{ij} \left(a(B_2)|0\rangle + b(B_2)|1\rangle \right)^{\otimes N}$$

$B_2 = 0 \rightarrow a = b = \frac{1}{\sqrt{2}} \leftrightarrow$ Regular cluster state

$B_2 = \infty \rightarrow a = 0, b = 1 \leftrightarrow |00\cdots\cdots 0\rangle$

Q: Is there a "phase transition" as we vary B_2 in the "quomputationess" of $|G\rangle$?

Phase Diagram:



Recall entanglement distillation:

$$\left[\begin{pmatrix} b/a & 0 \\ 0 & 1 \end{pmatrix}_A \otimes \mathbb{I}_B\right] \quad a|00\rangle + b|11\rangle$$

$$\Rightarrow \frac{b}{a} a|00\rangle + b|11\rangle$$

$$= b\underbrace{(|00\rangle + |11\rangle)}_{\text{Maximally entangled}}$$

Similar trick works here

- Measure each qubit with $\left\{ \begin{pmatrix} b/a & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} \sqrt{1-\frac{b^2}{a^2}} & 0 \\ 0 & 0 \end{pmatrix} \right\}$

$$\begin{pmatrix} b/a & 0 \\ 0 & 1 \end{pmatrix}_1 \prod_{i\sim j} CZ_{ij} \left(a|0\rangle + b|1\rangle\right)^{\otimes n}$$

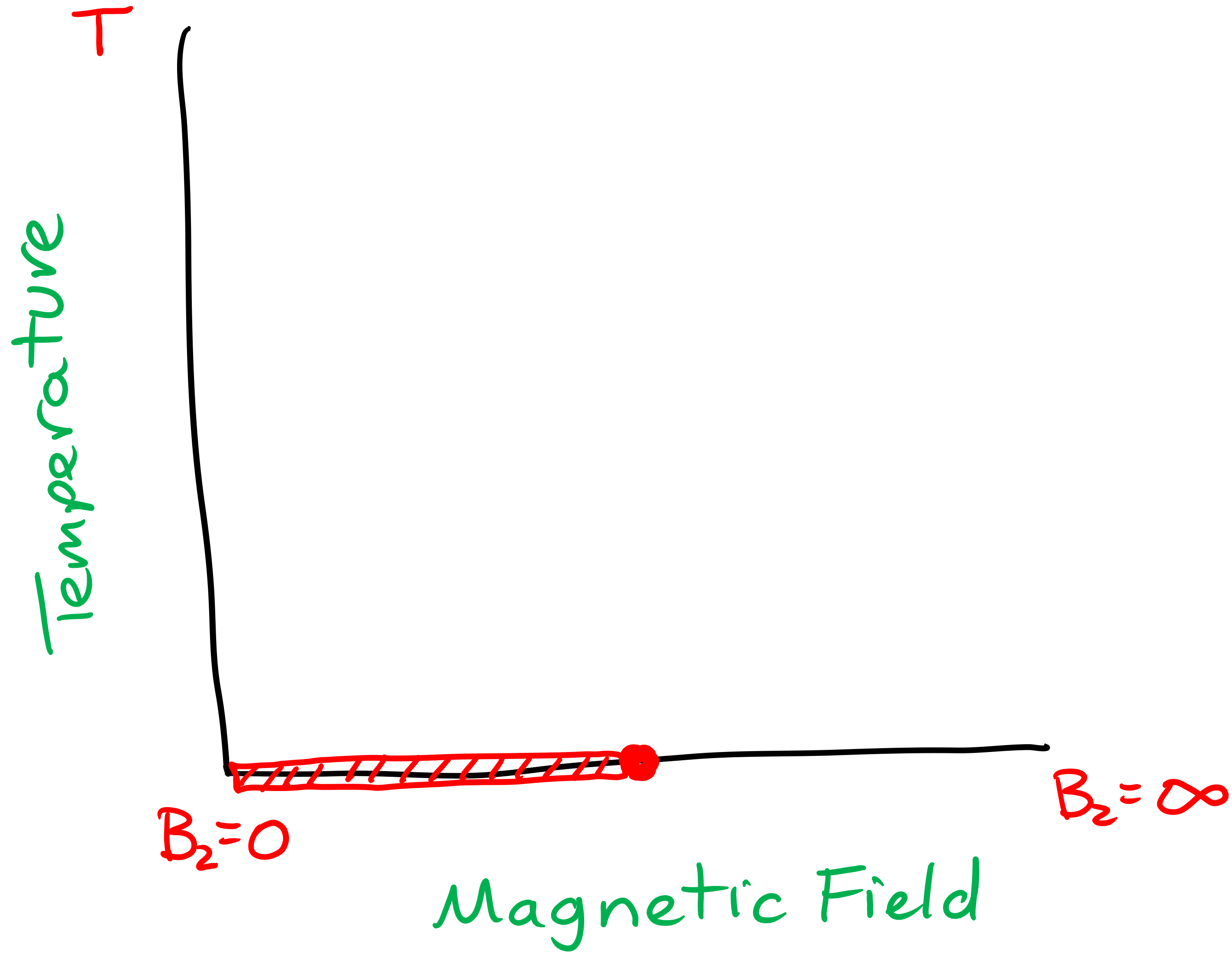
$$\Rightarrow \prod_{i\sim j} CZ_{ij} \left(a|0\rangle + b|1\rangle\right)^{\otimes n-1} \cdot |+\rangle_1$$

$\Uparrow$ ideal cluster

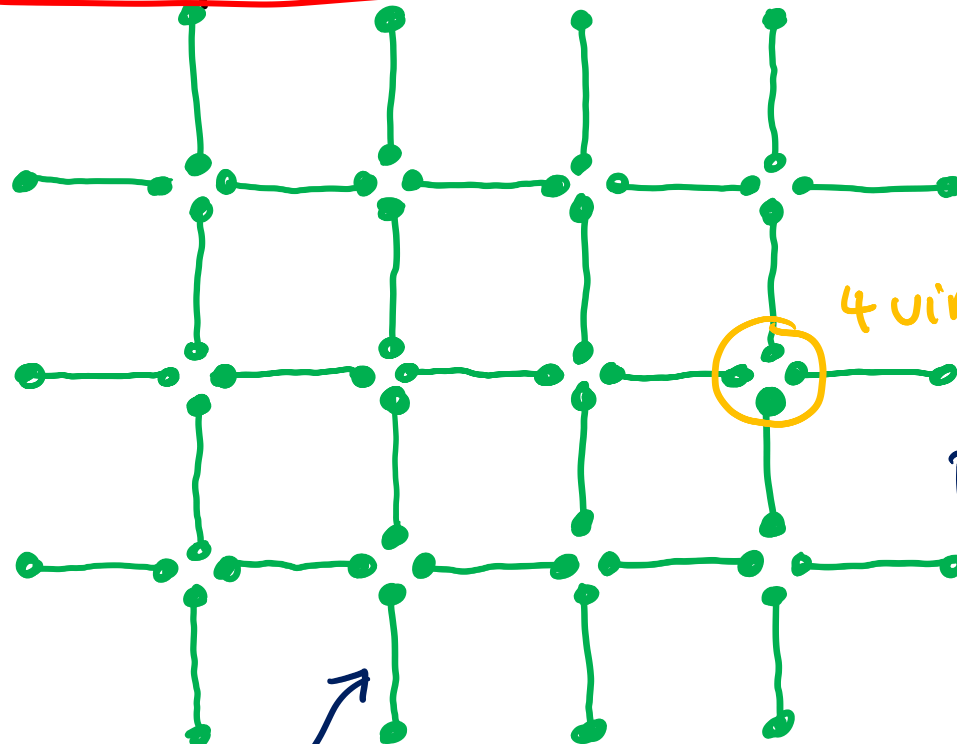
$\Rightarrow$ Becomes a SITE PERCOLATION

If $|b(B_2)|^2 > p_c$ then Q-comp. possible

Phase Diagram:



Finite Temperature?



4 virtual qubits

Fuse via

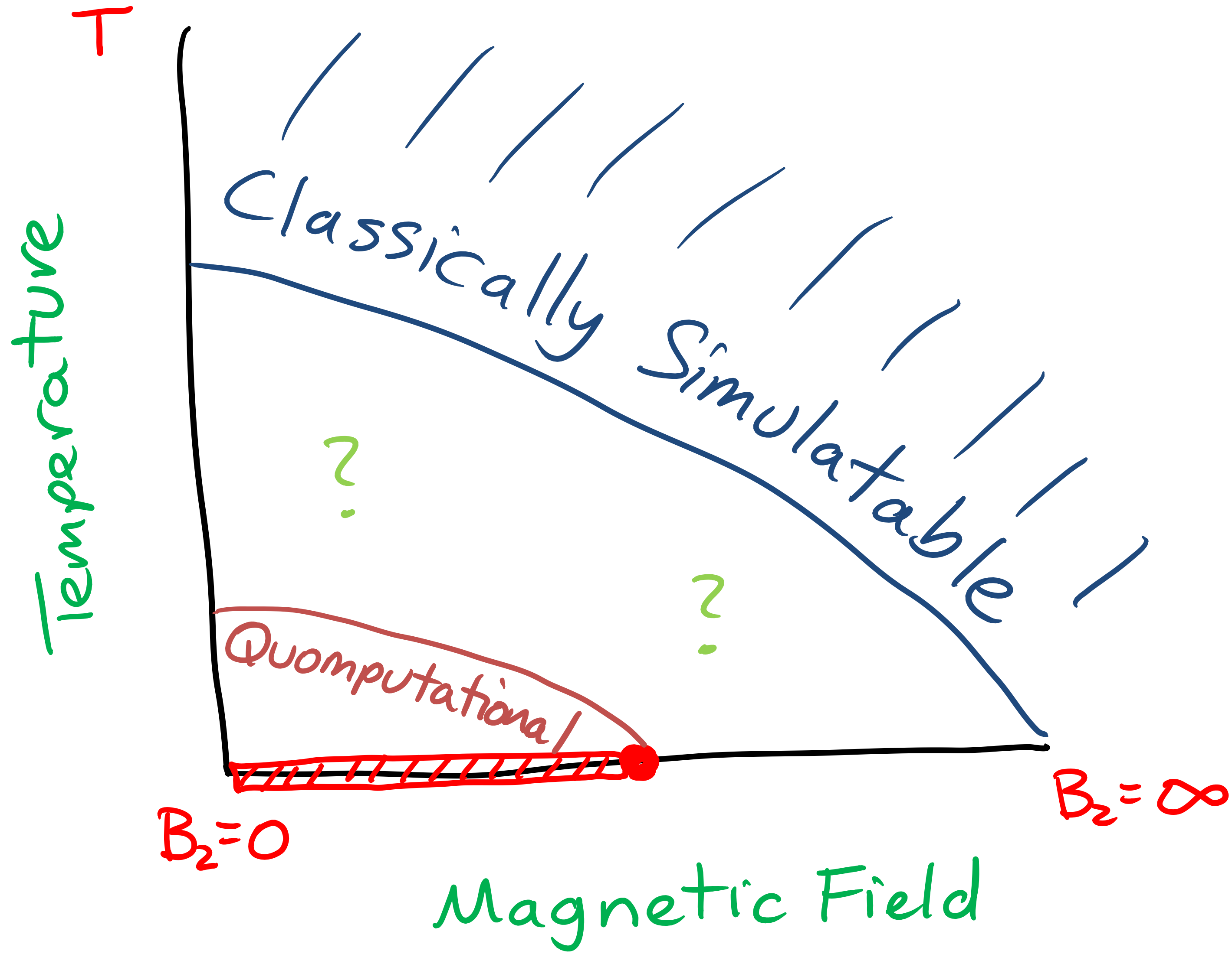
$$P = |0\rangle\langle 0000| + |1\rangle\langle 1111|$$

Two qubit mixed state $\rho(T, B)$

Idea: $\rho = p_e |\Psi_e\rangle\langle\Psi_e| + (1-p_e)\,\sigma_{separable}$

If $p_e < p_{crit}^{bond}$ then finite "clusters" of entanglement

Phase Diagram:



CONCLUSION

- PERCOLATION IS COOL
- AND SOMETIMES USEFUL

  FOR PHYSICISTS WHO DON'T WORK

  FOR ~~EV~~OIL COMPANIES