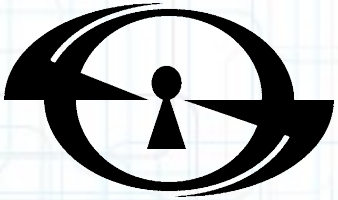


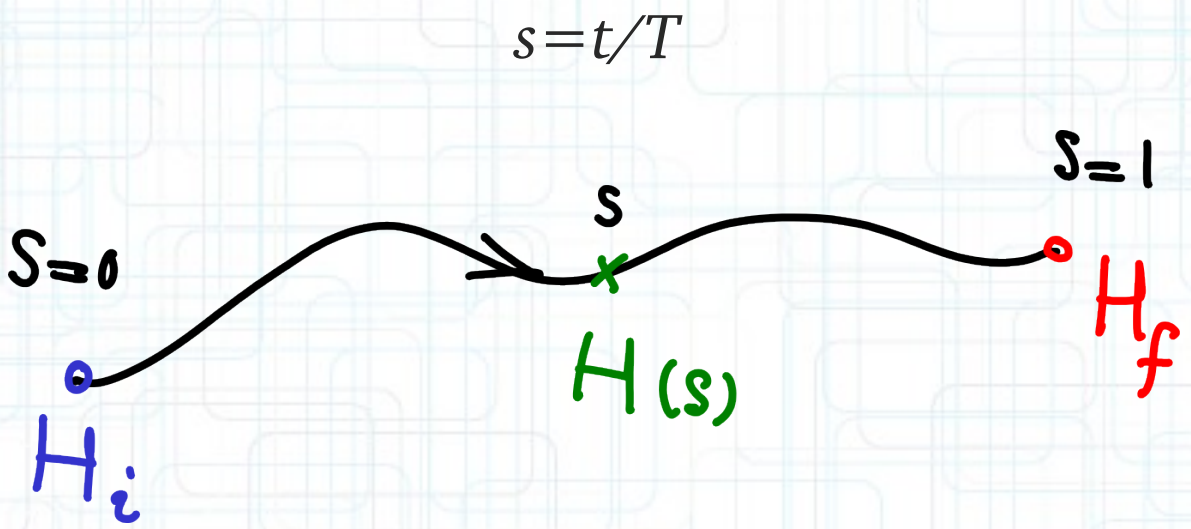
Iran International Conference on Quantum Information, Sharif University of Technology, Tehran, Iran, 8-12 Sep 2012

A Lieb-Robinson Bound for Adiabatic Evolution

Mear M. R. Koochakie,
Joint work with: Sahar Alipour, Ali Rezakhani

Sharif University of Technology

Adiabatic Theorem



$$|G(0)\rangle \xrightarrow{\text{slow enough } \dot{s}} \approx |G(1)\rangle$$

Born M., Fock V., 1928

Adiabatic Quantum Computation, Adiabatic error

Farhi, E., *et. al.*, 2000

$$\delta = \| U|G(0)\rangle - |G(1)\rangle \|$$

$$U_{\mathrm{ad}}(s)|G(0)\rangle \equiv |G(s)\rangle$$

Kato, 1950

$$\delta \leq \| U - U_{\mathrm{ad}} \|$$

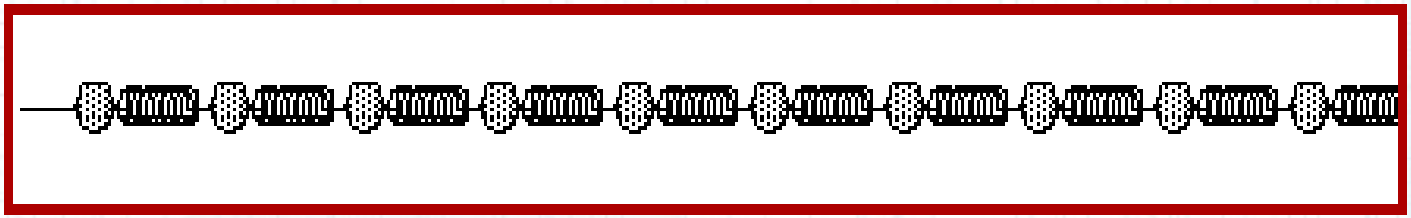
Adiabatic Criterion

Avron, J., Seiler, R., Yaffe, L., Comm. Math. Phys., 1987

$$\delta \leqslant C \frac{\max_t \|\dot{H}(t)\|}{\Delta_{\min}^2} \frac{1}{T} + \mathcal{O}(\frac{1}{T^2})$$

Jansen, S., Ruskai, M., Seiler R., J. Math. Phys., 2007

Locality and wave speed

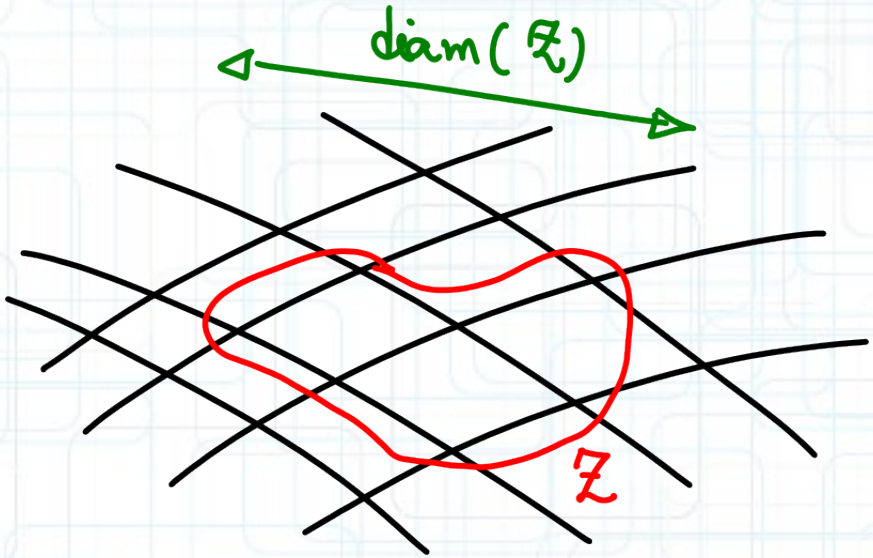


http://electron9.phys.utk.edu/phys135d/modules/m10/waves.htm

Lieb-Robinson Theorem

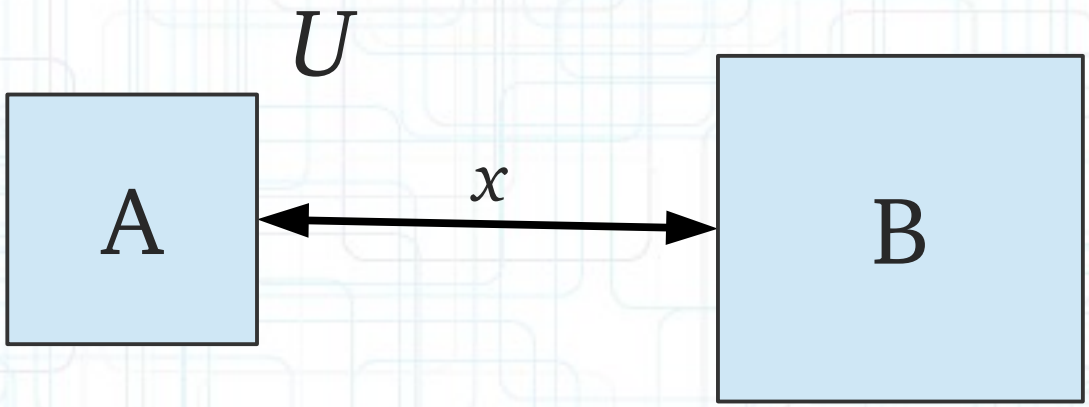
Lieb, E.H. and Robinson, D.W., Comm. Math. Phys. 28, 1972

$$H = \sum_{Z} H_Z$$



$$\|H_Z\| = \mathcal{O}\left(\exp(-\mu \operatorname{diam} Z)\right)$$

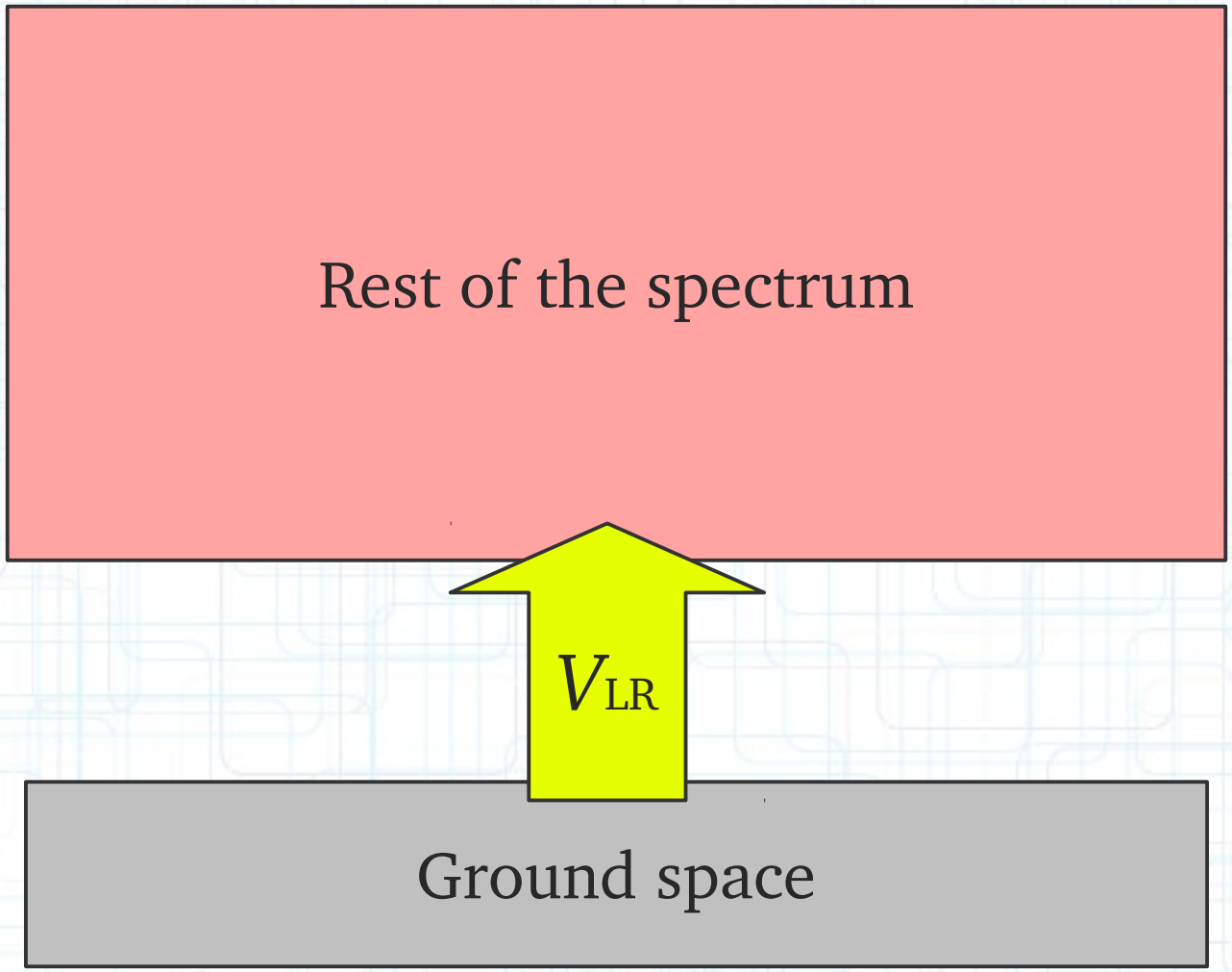
Hastings, Nachtergaele, Osborne,...



$$\|[A^t(x), B]\| < \alpha \exp(at - \mu x)$$

$$V_{\mathrm{LR}} = \frac{a}{\mu}$$

Basic Idea

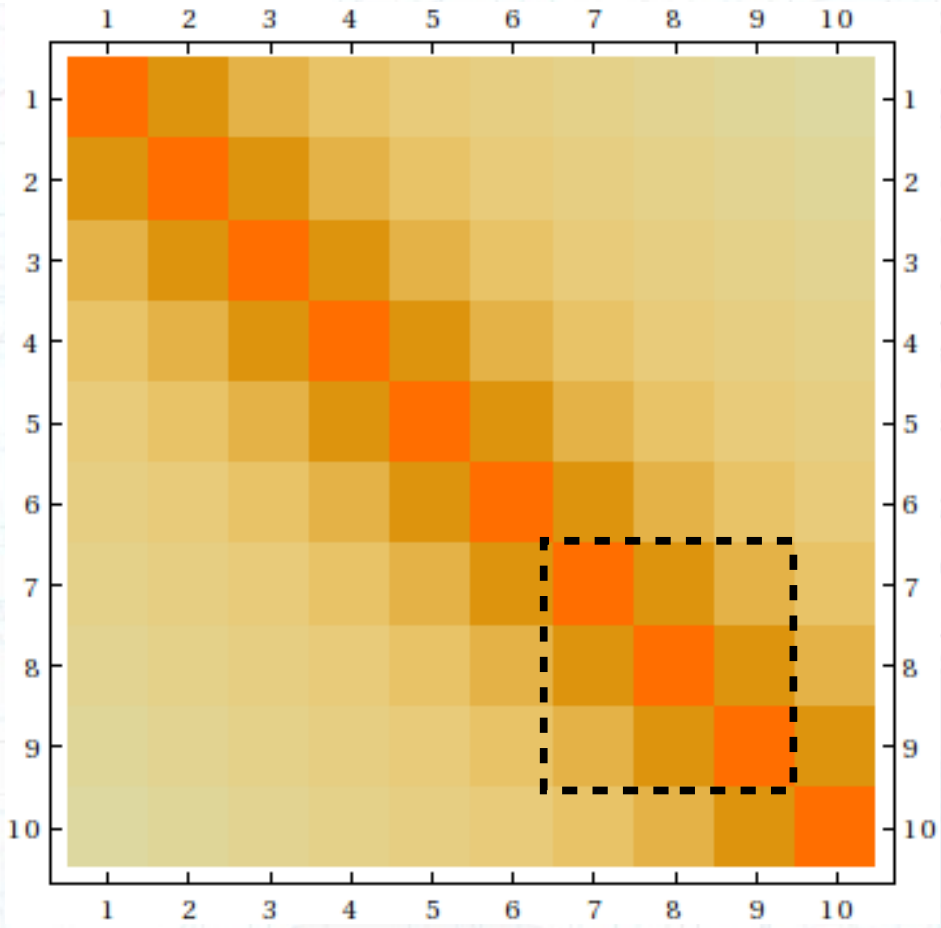


Locality in Energy Space

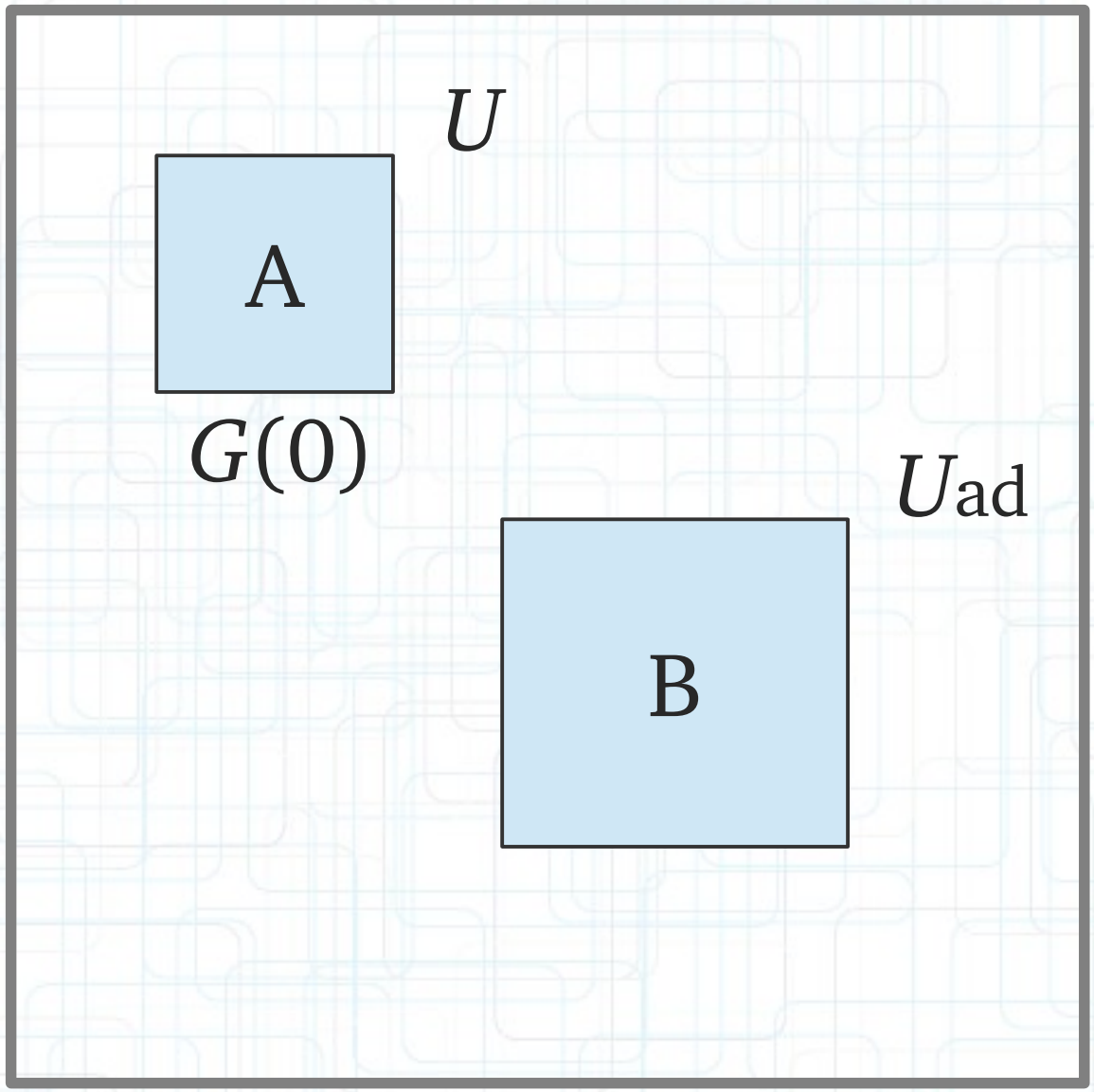
$$\text{Basis} = \{|i\rangle \,|\, i \in \mathbb{Z}\}$$

$$\mathcal{Z} \subset \text{Basis}$$

$$|H_{ij}| < Ae^{-\mu|j-i|}$$



$$|\langle j|U(t)|i\rangle| < \exp(at - \mu|j-i|)$$



Basis: $H(0)$

$$H_{\mathrm{eff}}(t) = H(t) - H_{\mathrm{ad}}(t)$$

$$H_{\mathrm{eff}}(t) = \sum_{\mathcal{Z}(t)} \left(H_{\mathrm{eff}}(t)\right)_{\mathcal{Z}(t)}$$

$$V_{\mathrm{LR}}^{\mathrm{m}} < \epsilon\, \Delta_{\mathrm{min}}$$

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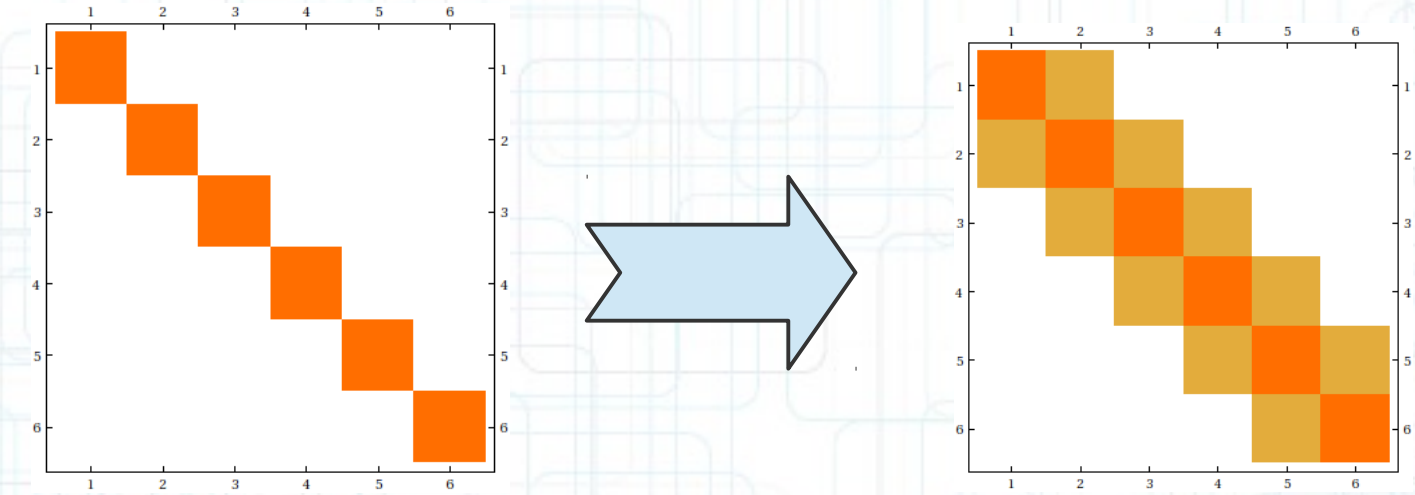
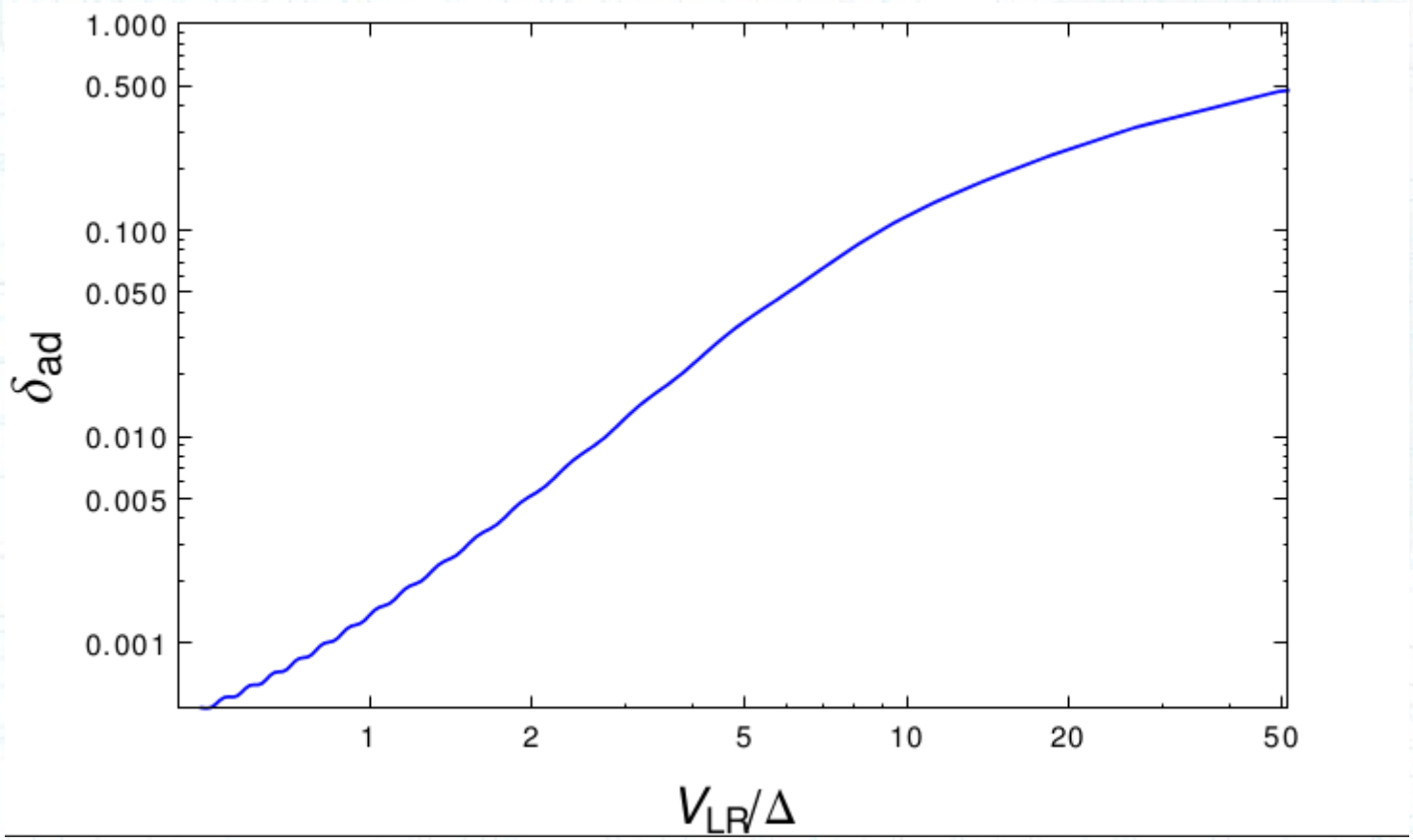
$$V_{\mathrm{LR}}^{\mathrm{m}} < \epsilon\, \Delta_{\mathrm{min}}$$

$$\Rightarrow \quad \frac{\max_t \|\dot{H}(t)\|}{\Delta_{\mathrm{min}} T} < \underbrace{\mu |\mathcal{G}| \epsilon}_{\delta}\, \Delta_{\mathrm{min}}$$


1
2
3
4
5
6


1.000
0.500
0.100
0.050
0.010
0.005
0.001
δ_{ad}
1
2
5
10
20
50
V_{LR}/Δ