Quantum memories and error correction

By Faezeh Kimiaee

Based on:

James R. Wootton, Journal of Modern Optics, 59 (2012) 1717.

Contents

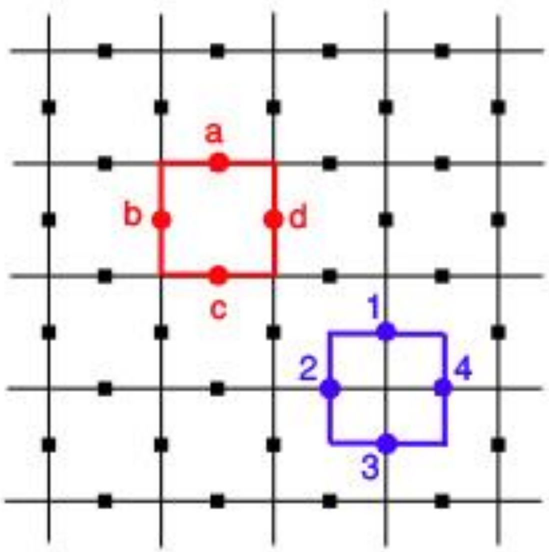
- Toric code model
- Toric code with open boundary condition
- Square lattice coding
- Holes coding

Toric code model

$$H = -J\sum_S A_S - K\sum_P B_P$$

$$A_s = \prod_{j\in star\ \ s} \sigma_j^x = \sigma_1^x\sigma_2^x\sigma_3^x\sigma_4^x$$

$$B_p = \prod_{j\in plaquette\ \ p} \sigma_j^z = \sigma_a^z\sigma_b^z\sigma_c^z\sigma_d^z$$



$$[A_S, B_P] = 0, \quad A_S^2 = B_P^2 = 1, \quad \prod_s A_S = \prod_p B_P = 1$$

- # qubits =2N
- # independent stabilizers =2N-2

→ $\text{Degeneracy} = \frac{2^{2N}}{2^{2N-2}} = 4$

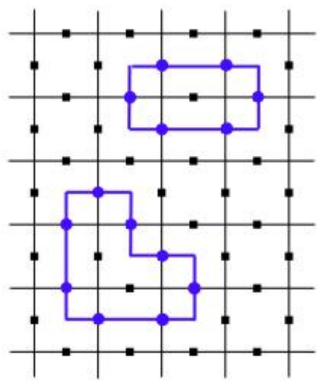
A. Y. Kitaev Ann. Phys. 303, 2 (2003)

Ground state

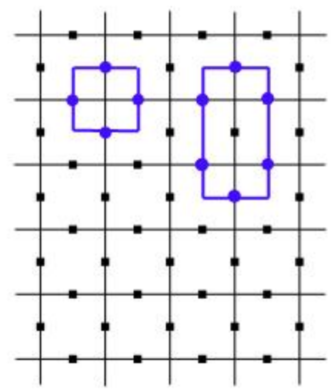
$$|g.s.\rangle = \frac{1}{\sqrt{2^{N_S}}}\left[\prod_{Star\ S}(A_S+1)\right]|o\rangle^{\otimes 2N}$$

$$A_S\,|g.s.\rangle = |g.s.\rangle, \quad B_P\,|g.s.\rangle = |g.s.\rangle$$

$$|g.s.\rangle = \frac{1}{\sqrt{2^{N_S}}}\left\{ \left| \vphantom{\frac{1}{1}} \right. \right.$$



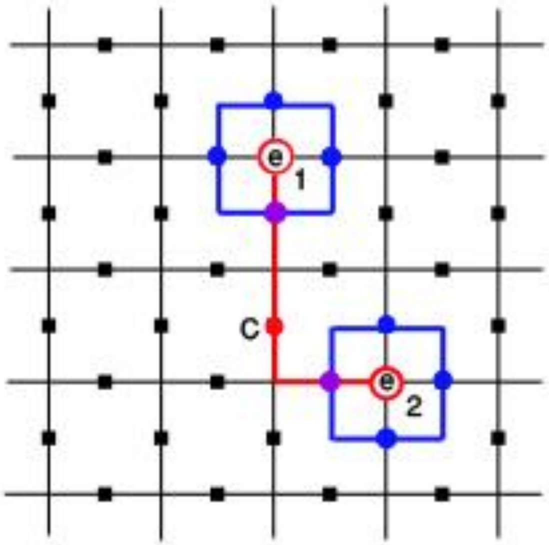
$$\left. \right\rangle + \left| \vphantom{\frac{1}{1}} \right.$$



$$\left. \left. \right\rangle + \ldots \right\}$$

$$H = -J\sum_S A_S - K\sum_P B_P \rightarrow H\,|g.s.\rangle = \left(-JN_S - KN_P\right)|g.s.\rangle$$

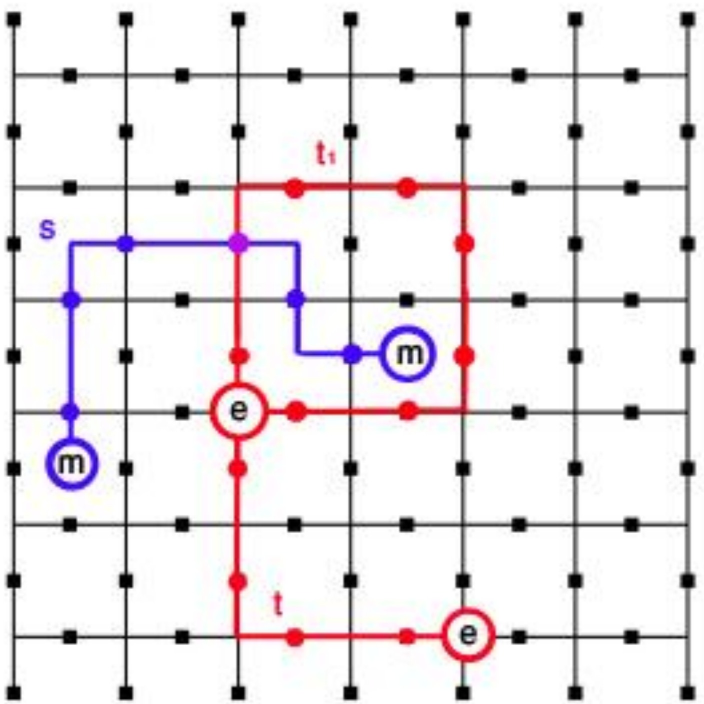
Excitations



$$A_s S_c^z = -S_c^z A_s \text{ for } S \in 1,2 \rightarrow A_s S_c^z \,|\, g.s.\rangle = -S_c^z \,|\, g.s.\rangle$$

Eigenstate of eigenvalue -1 for thoes 2

Statistics of the Excitations



$$|\psi_i\rangle = S_t^Z S_s^X \,|\, g.s.\rangle$$

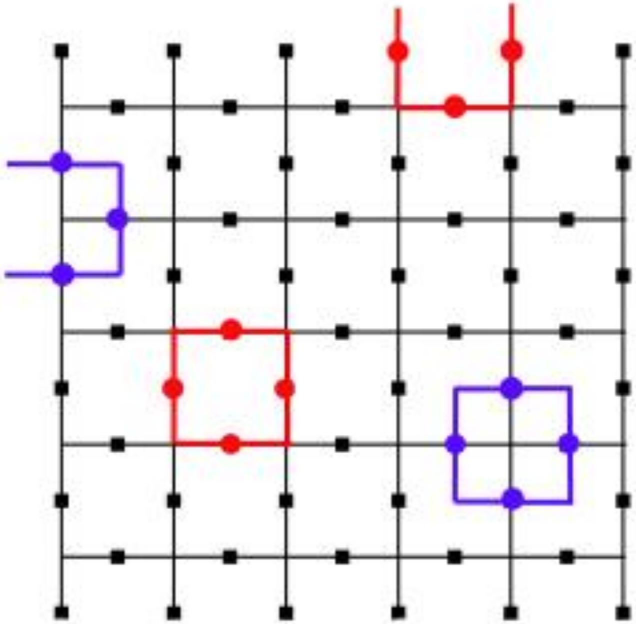
$$|\psi_f\rangle = S_{t_1}^Z \,|\psi_i\rangle$$
$$= S_{t_1}^Z S_t^Z S_s^X \,|\, g.s\rangle$$
$$= -S_t^Z S_s^X S_{t_1}^Z \,|\, g.s.\rangle$$
$$= -|\psi_i >$$

Toric code with open boundary condition

$$H = -\sum_{S} A_S - \sum_{S'\in\partial m} A_{S'} - \sum_{P'\in\partial m} B_{P'} - \sum_{P} B_P$$

$$[A_S, B_P] = 0, \qquad A_S^2 = B_P^2 = 1,$$

$$\prod_s A_s \neq 1, \quad \prod_p B_p \neq 1$$

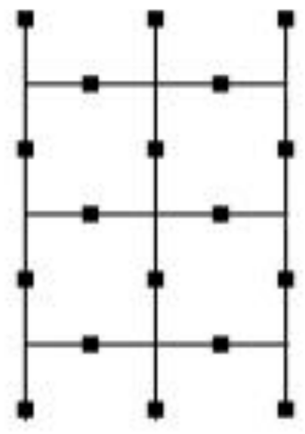


qubits =2N

independent stabilizers =2N-1

$$\rightarrow \quad Degeneracy = \frac{2^{2N}}{2^{2N-1}} = 2$$

Degeneracy!



N=18

Hilbert space: 2^{18}

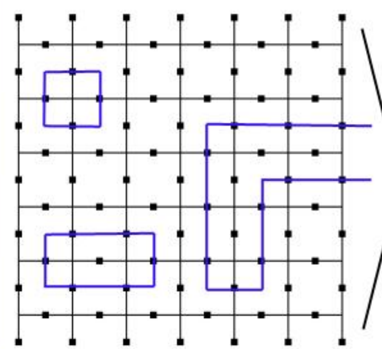
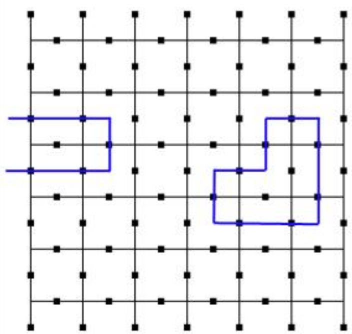
Plaquett=8

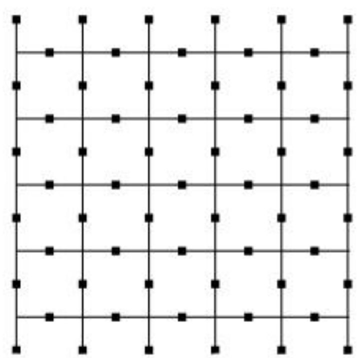
Star=9

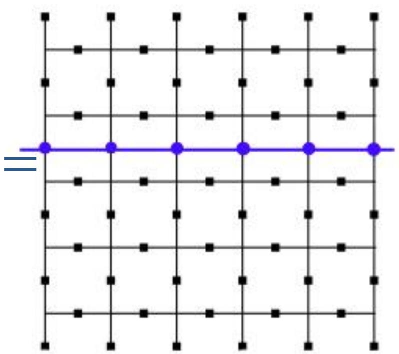
$\longrightarrow 2^{17}$

Ground states and coding

$$|\xi\rangle = \frac{1}{\sqrt{2^{N_S}}}\left[\prod_{Star\ S}(A_S+1)\right]|0\rangle^{\otimes 2N}$$

$$|\xi\rangle = \frac{1}{\sqrt{2^{N}}}\Bigg\{ \Bigg|$$  $$\Bigg\rangle + \Bigg|$$  $$\Bigg\rangle + \ldots \Bigg\}$$

$$|0\rangle \equiv |g.s.\rangle =$$ 

$$|1\rangle \equiv T_X|g.s.\rangle =$$ 

What about top and bottom boundaries ?!

$$|g.s.\rangle = \frac{1}{\sqrt{2^{\#S}}}\prod_S (A_S+1)|0\rangle^{\otimes 2N}, \qquad |g.s.'\rangle = T_x|g.s.\rangle$$

$$|\Phi\rangle = \frac{1}{\sqrt{2^{\#P}}}\prod_P (B_P+1)|+\rangle^{\otimes 2N}, \qquad |\Phi'\rangle = T_z|\Phi\rangle$$

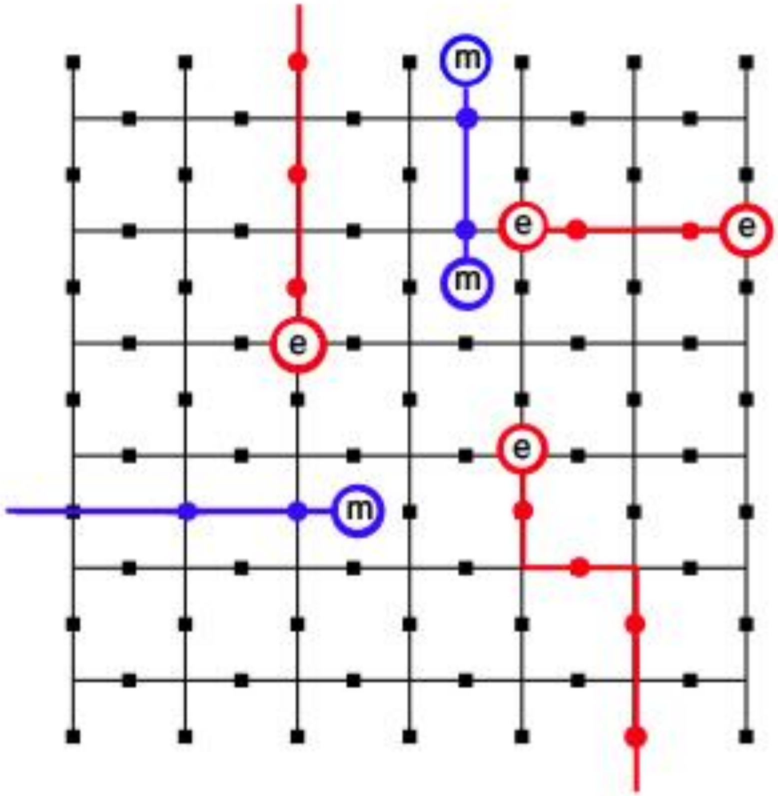
$$|\Phi\rangle = \alpha|g.s.\rangle + \beta|g.s.'\rangle, |\Phi'\rangle = \alpha'|g.s.\rangle - \beta'|g.s.'\rangle$$

$$\langle g.s.|\Phi\rangle = \frac{1}{\sqrt{2^{\#S}2^{\#P}}} {}^{\otimes 2N}\langle 0|\prod_S (A_S+1)\prod_P (B_P+1)|+\rangle^{\otimes 2N}$$

$$= \frac{1}{\sqrt{2^{\#S}2^{\#P}}} 2^{\#S}2^{\#P} {}^{\otimes 2N}\langle 0|+\rangle^{\otimes 2N} = \frac{1}{\sqrt{2}} = \alpha$$

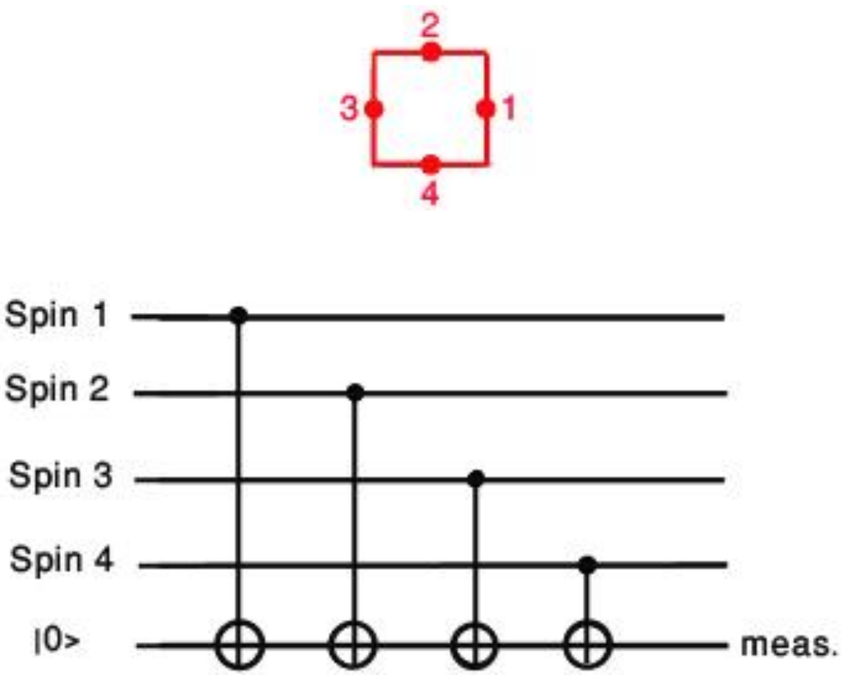
Coding: $|\Phi\rangle \to |+\rangle, \quad T_Z|\Phi\rangle \to |-\rangle$

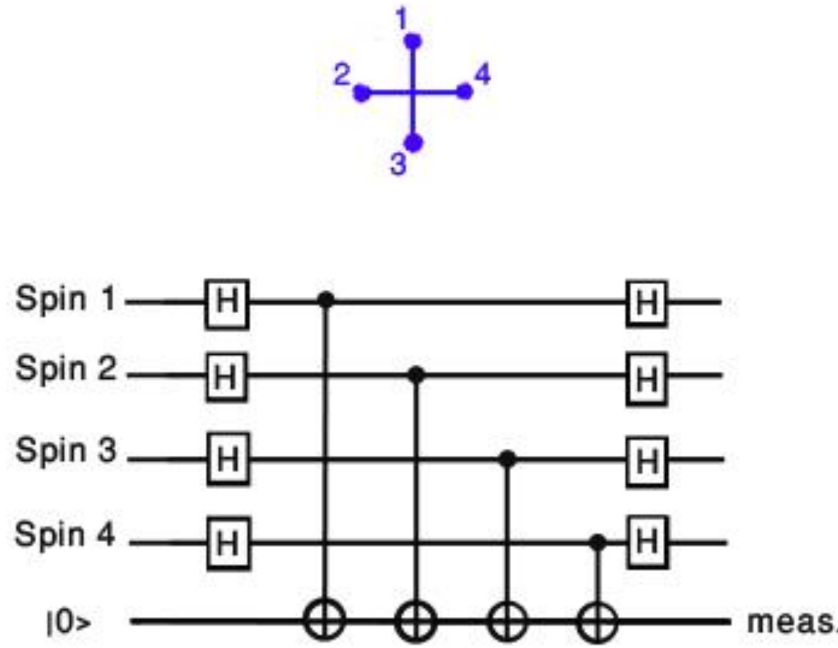
Excitations!



Some strings are able to create a single anyon!

Stabilizer measurements

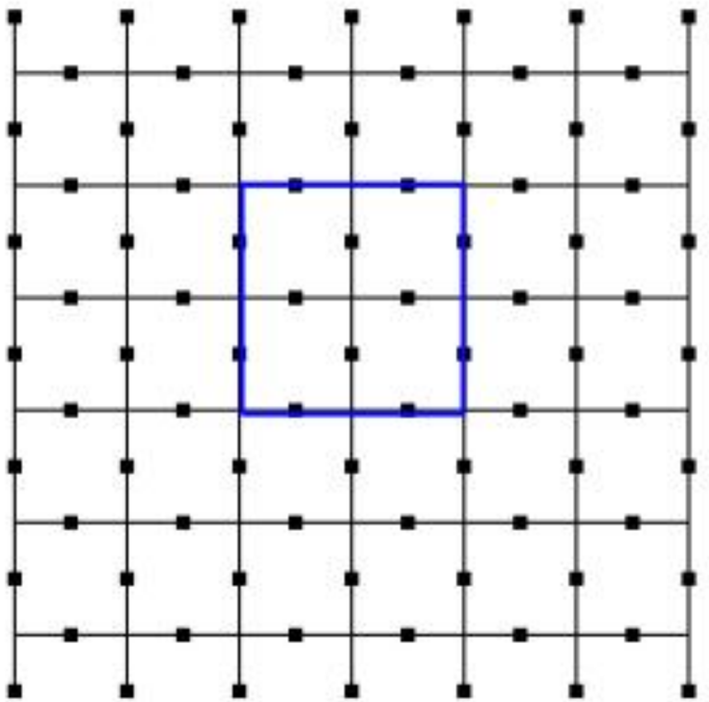




Barbara M. Terhal, arxiv:1302.3428v2

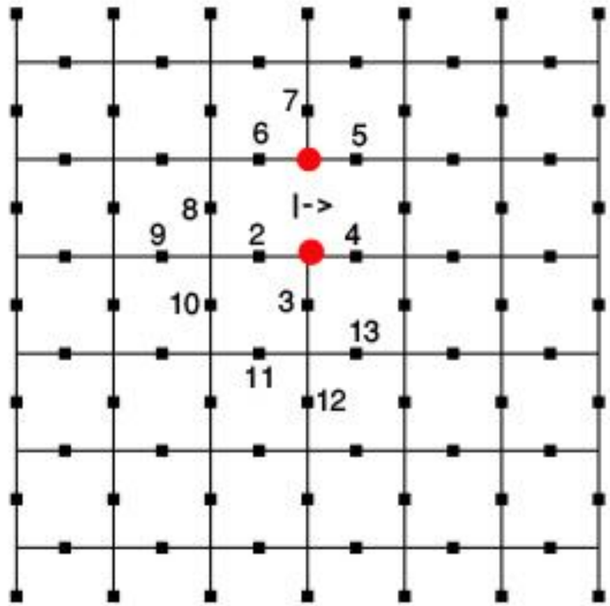
Creation of holes

- We first choose a selection of plaquette stabilizers that we no longer wish to enforce.



Creation of holes…

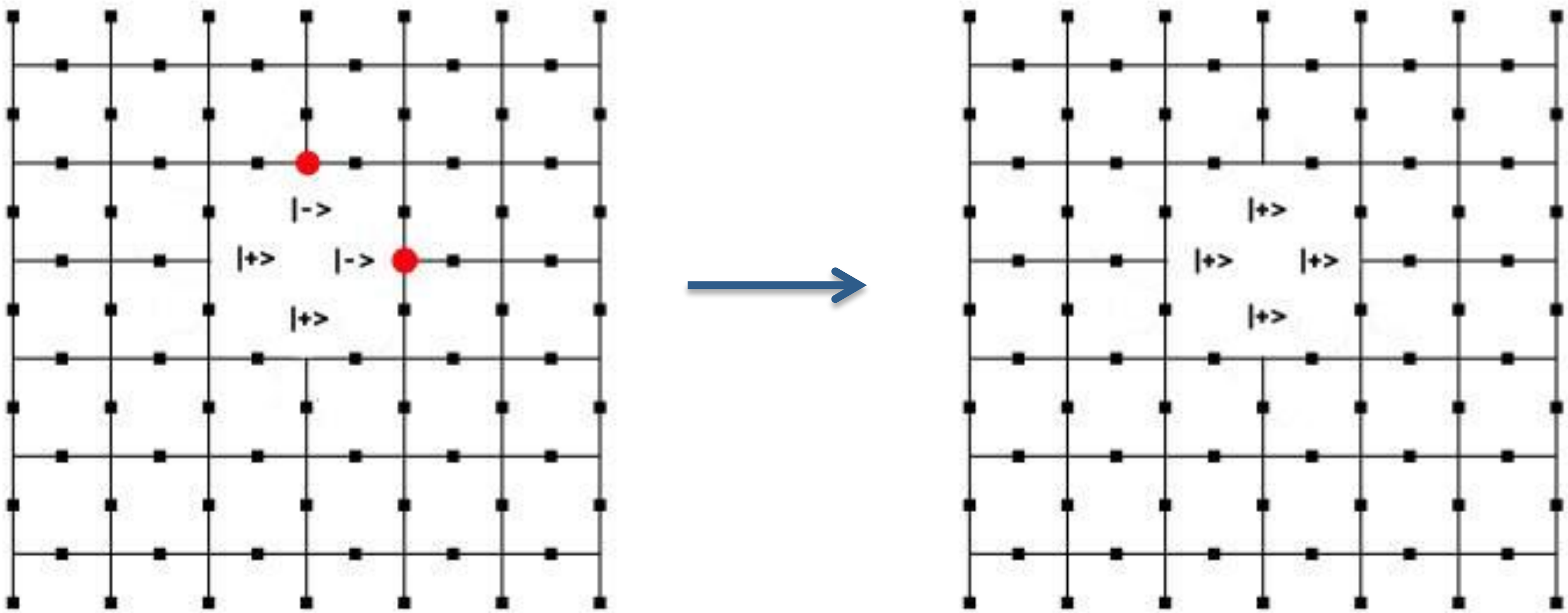
- Measuring in the x basis



$$\frac{\left(1+\sigma_1^x\right)}{2}|\xi\rangle = \ldots\left(1+\sigma_{2,3,4}^x\right)\left(1+\sigma_{5,6,7}^x\right)\ldots|+\rangle_1|0\rangle^{\otimes 2N-1}$$

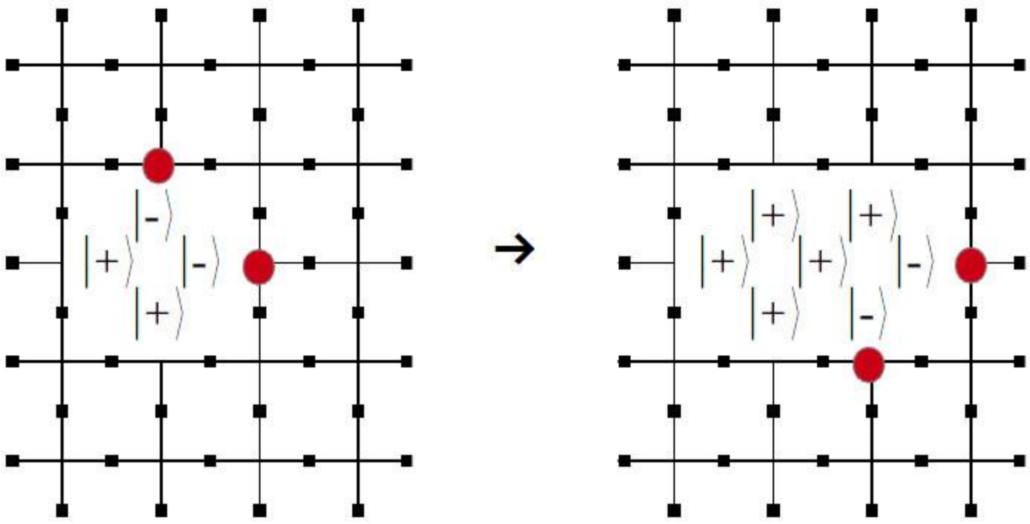
$$\frac{\left(1-\sigma_1^x\right)}{2}|\xi\rangle = \ldots\left(1-\sigma_{2,3,4}^x\right)\left(1-\sigma_{5,6,7}^x\right)\ldots|-\rangle_1|0\rangle^{\otimes 2N-1}$$

Creation of holes…

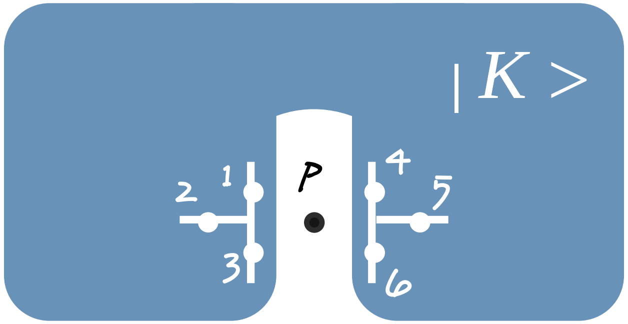


Expansion of holes

- Expansion is simply achieved through more creation.



Contract a hole

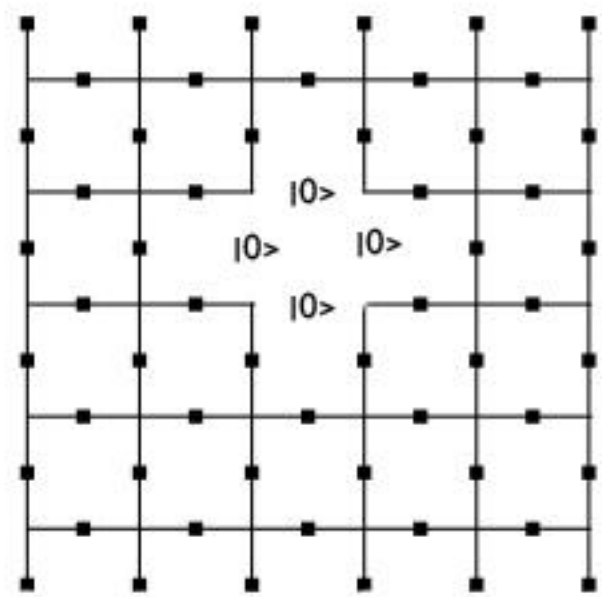


$$|\Psi> = C_{1p}C_{4p}|K>|0>_p$$

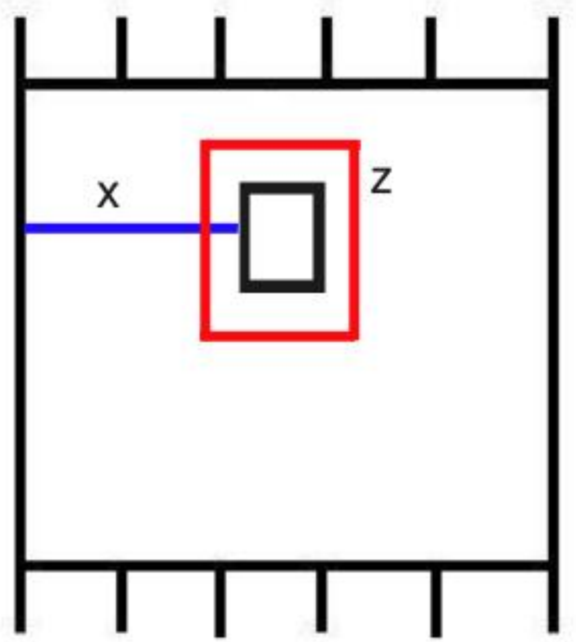
$$X_1X_2X_3X_p|\Psi> = |\Psi>$$

$$X_4X_5X_6X_p|\Psi> = |\Psi>$$

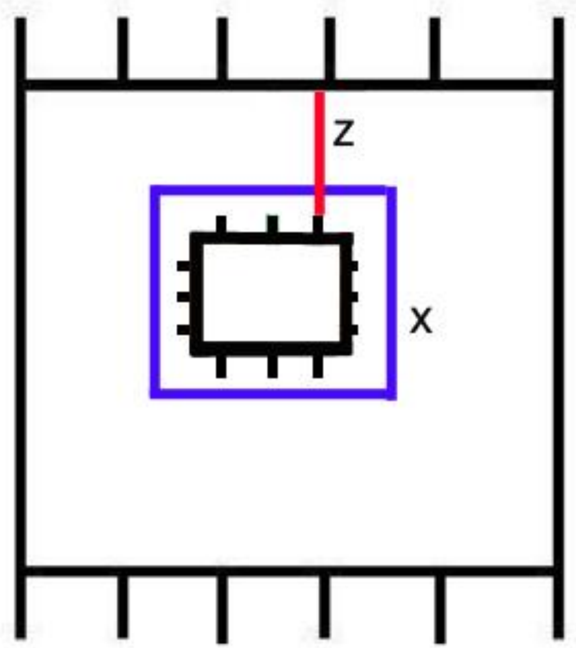
Rough holes



Hole coding

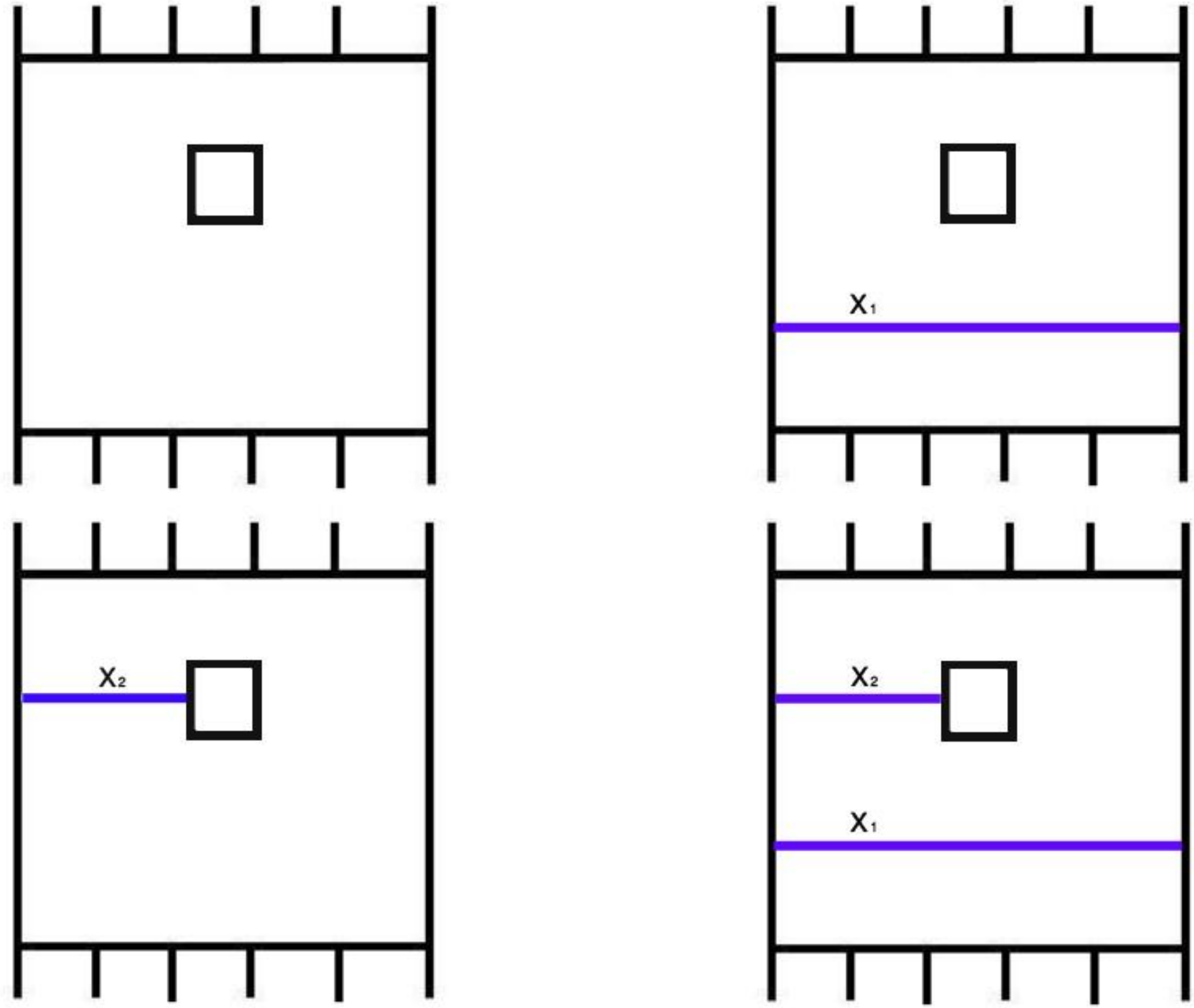


Smooth hole
(smooth qubit)

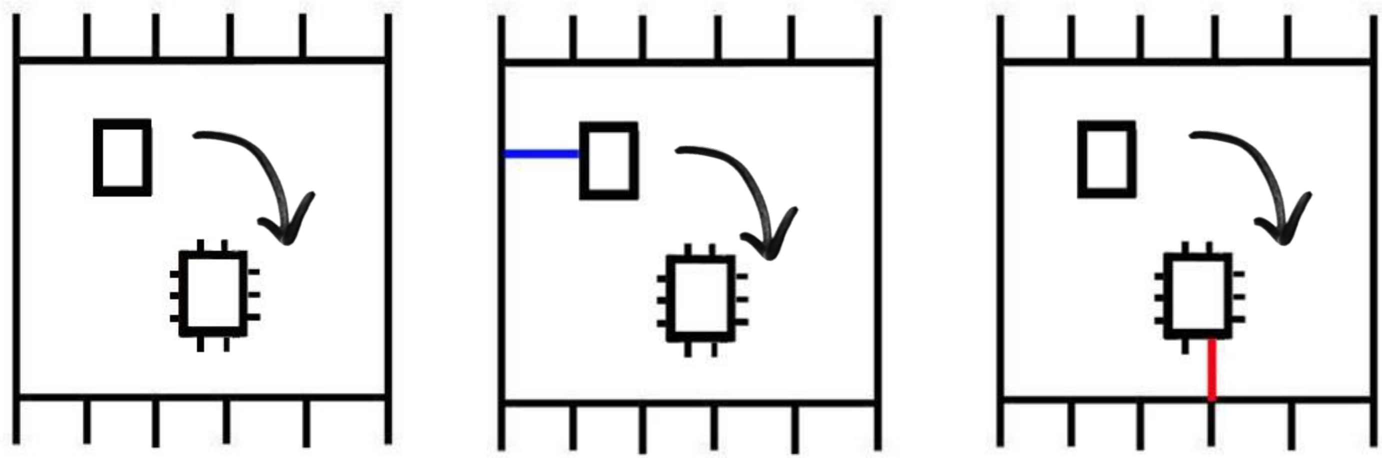


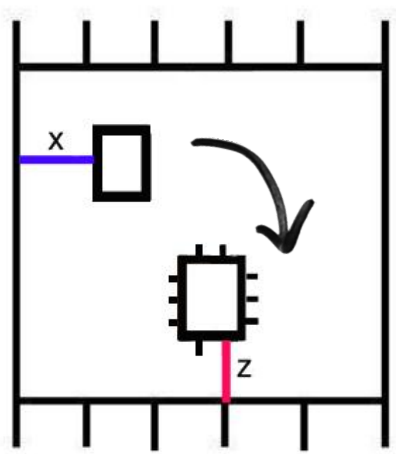
Rough hole
(rough qubit)

Hole coding

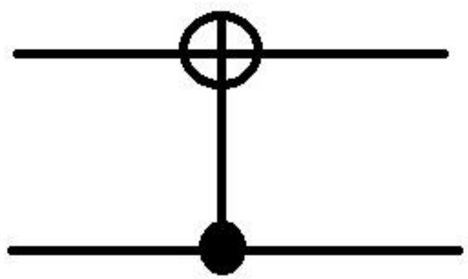


CNOT gate

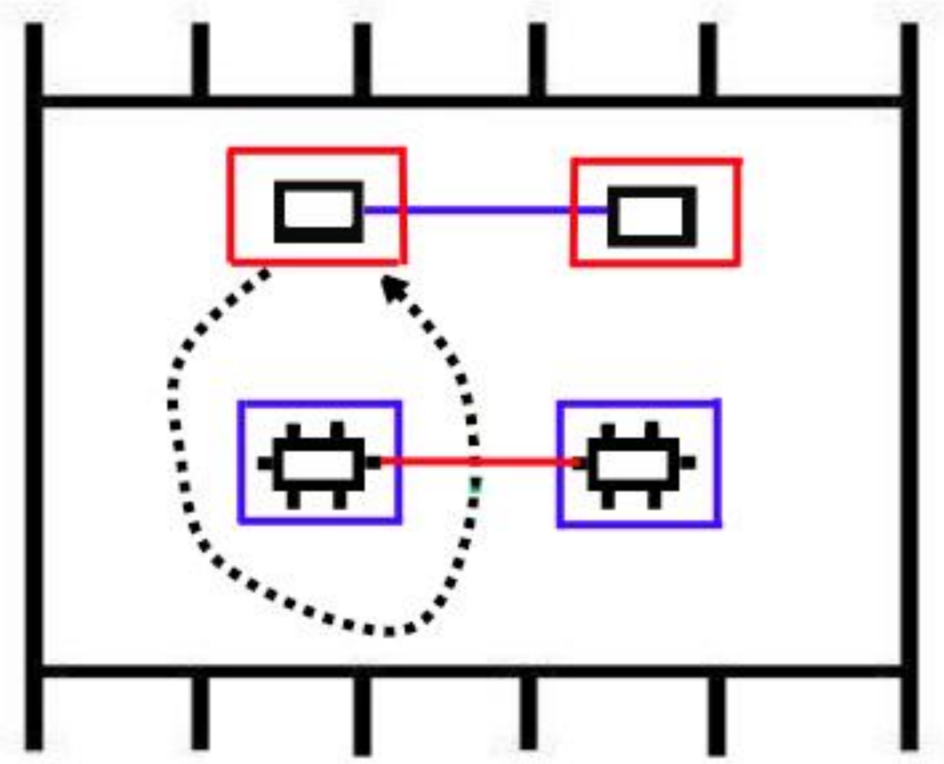




generating a phase of -1

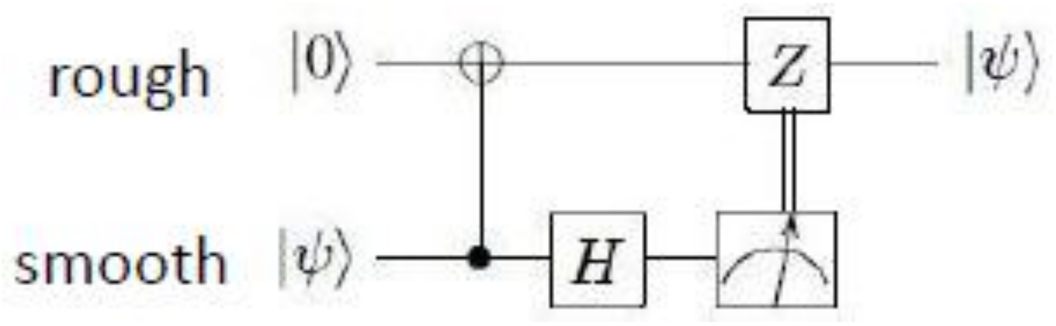


CNOT

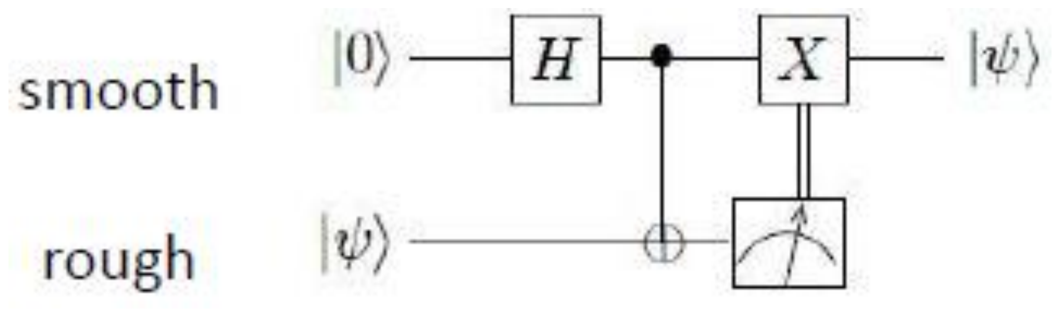


Converting a rough qubit to a smooth qubit

Converting a smooth qubit into a rough qubit



Converting a rough qubit to a smooth qubit



Universal gates!

S. Bravyi and A. Kitaev, Phys. Rev. A 71, 022316 (2005).

Thank you