QIS@Sharif

تشخیص
کانال‌های کوانتومی اکستریم
با استفاده از
شبکه‌های عصبی

لاله معمارزاده اصفهانی

محمد مهدی ماستری فراهانی

آبان ۱۴۰۲

فهرست مطالب

- طرح مسئله
- حل مسئله به صورت الگوریتمی
- حل مسئله با روش شبکه‌های عصبی
  - شبکه‌های عصبی
  - نتایج
- آینده پژوهی

طرح مسئله

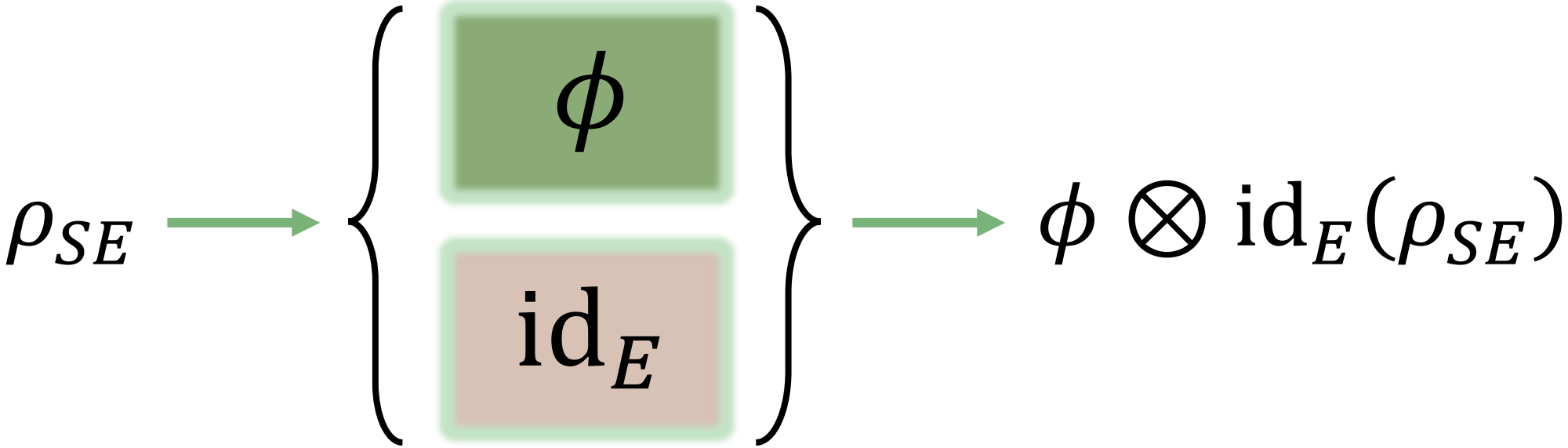
Quantum Evolutions, Quantum Simulations,
Extreme Quantum Channels

Quantum Evolutions (PTP maps)

$$\rho \longrightarrow \boxed{\phi} \longrightarrow \phi(\rho)$$

$$\begin{cases} \phi(\rho) > 0 \\ \mathrm{Tr}(\rho) = \mathrm{Tr}\big(\phi(\rho)\big) = 1 \end{cases} \qquad \forall \rho \in \mathcal{D}(\mathcal{H})$$

Quantum Evolutions (CPTP maps)



$$\begin{cases} \phi \otimes \mathrm{id}_E > 0 \\ \mathrm{Tr}(\rho) = \mathrm{Tr}\big(\phi(\rho)\big) = 1 \end{cases} \qquad \forall E$$

Representations of CPTPs (Choi Matrix)

$$C_\phi = (\phi \otimes I_d)|\Gamma\rangle\langle\Gamma| = \frac{1}{d}\sum_{i,j=0}^{d-1} \phi(|i\rangle\langle j|) \otimes |i\rangle\langle j|$$

$$\phi \text{ is } CPTP \quad \Leftrightarrow \quad \boxed{C_\phi \geq 0}$$

Choi, M. (1975). Completely positive linear maps on complex matrices. Linear Algebra and Its Applications, 10(3), 285–290.

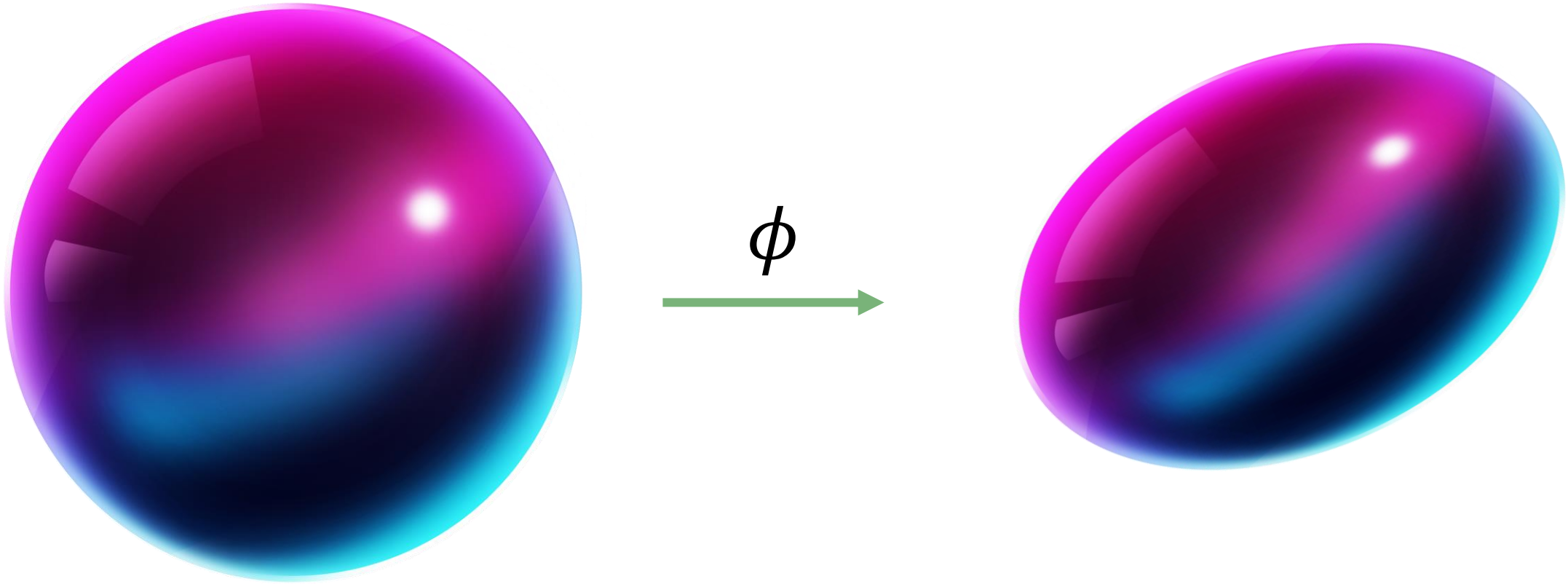
Representations of CPTPs (Kraus Operators)

$$\phi \text{ is } CP \quad \Leftrightarrow \quad \phi(\rho) = \sum_{i=1}^{K} A_i \rho A_i^{\dagger}$$

$$\phi \text{ is } TP \quad \Leftrightarrow \quad \sum_{i=1}^{K} A_i^{\dagger} A_i = I$$

Kraus, K. (1983). States, effects, and operations: Fundamental Notions of Quantum Theory. Springer.

Representations of CPTPs (Affine)



Nielsen, M. A., & Chuang, I. L. (2010). Quantum computation and quantum information: 10th Anniversary Edition. Cambridge University Press.

Representations of CPTPs (Affine)

$$\rho = \sum_{i=0}^{d^2-1} r_i \, \sigma_i \qquad \boldsymbol{r} = \begin{pmatrix} 1 \\ r_1 \\ \vdots \\ r_{d^2-1} \end{pmatrix}$$

$$\boldsymbol{r} \in S^{d^2-2} \cup \{\text{inside of } S^{d^2-2}\}$$

Bengtsson, I., & Życzkowski, K. (2017). Geometry of quantum states: An Introduction to Quantum Entanglement. Cambridge University Press.

Representations of CPTPs (Affine)

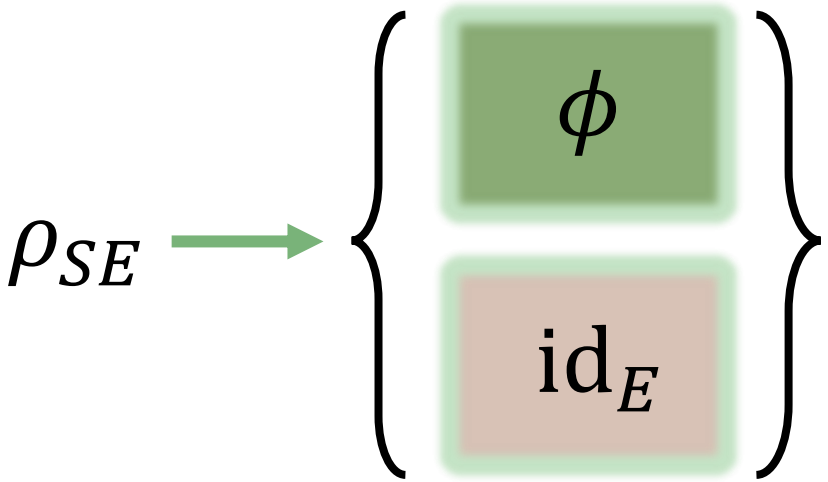
$$\rho' = \phi(\rho) \quad \xrightarrow{Rep} \quad \boldsymbol{r}' = T_\phi \boldsymbol{r}$$

$$T_\phi = \begin{pmatrix} 1 & 0 \\ \boldsymbol{t}_\phi & M_\phi \end{pmatrix}$$

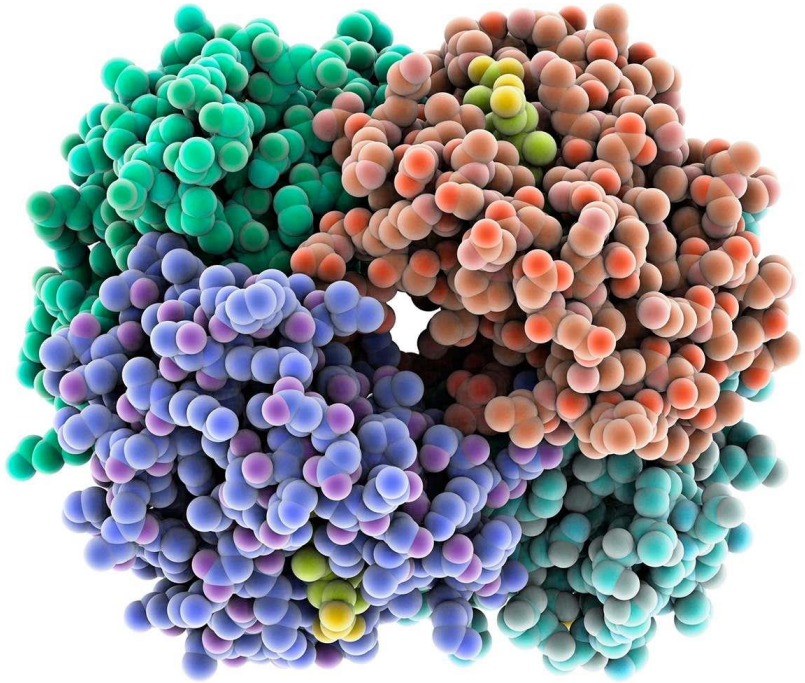
Nielsen, M. A., & Chuang, I. L. (2010). Quantum computation and quantum information: 10th Anniversary Edition. Cambridge University Press.

Importance of CPTP maps

Theory



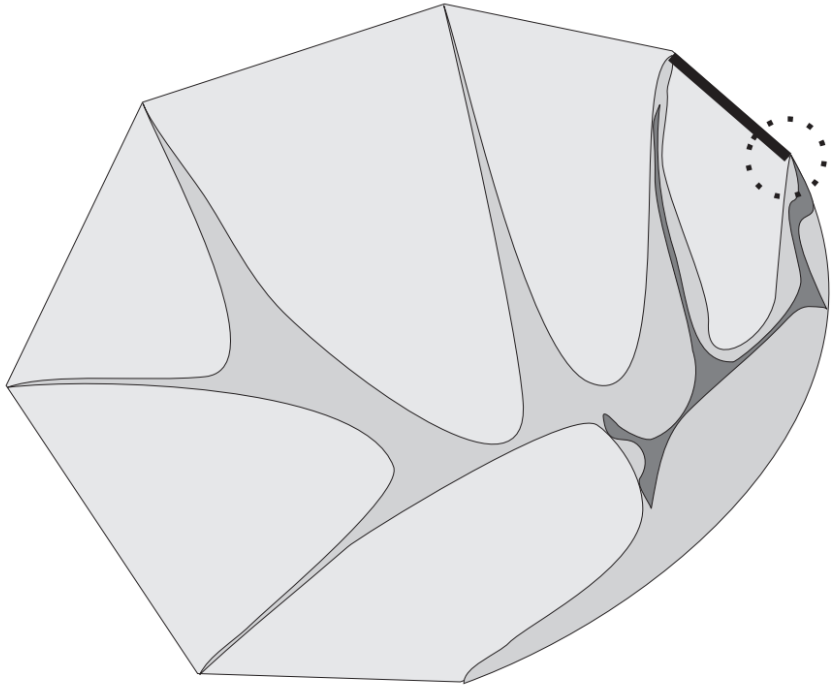
Simulation



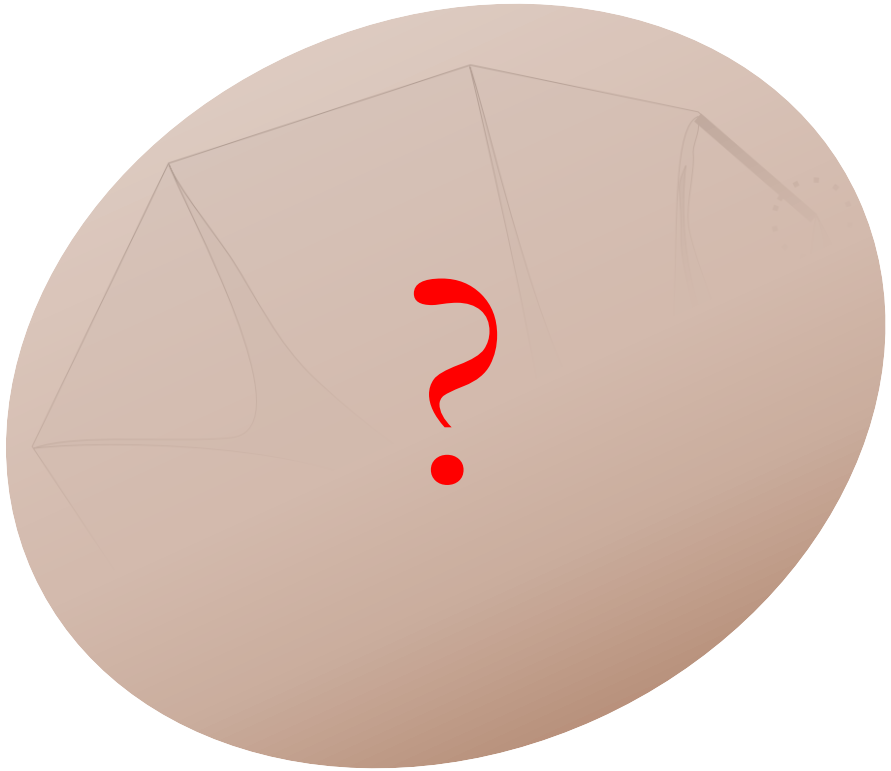
https://www.thoughtco.com/protein-function-373550

The Problem

$d = 2$

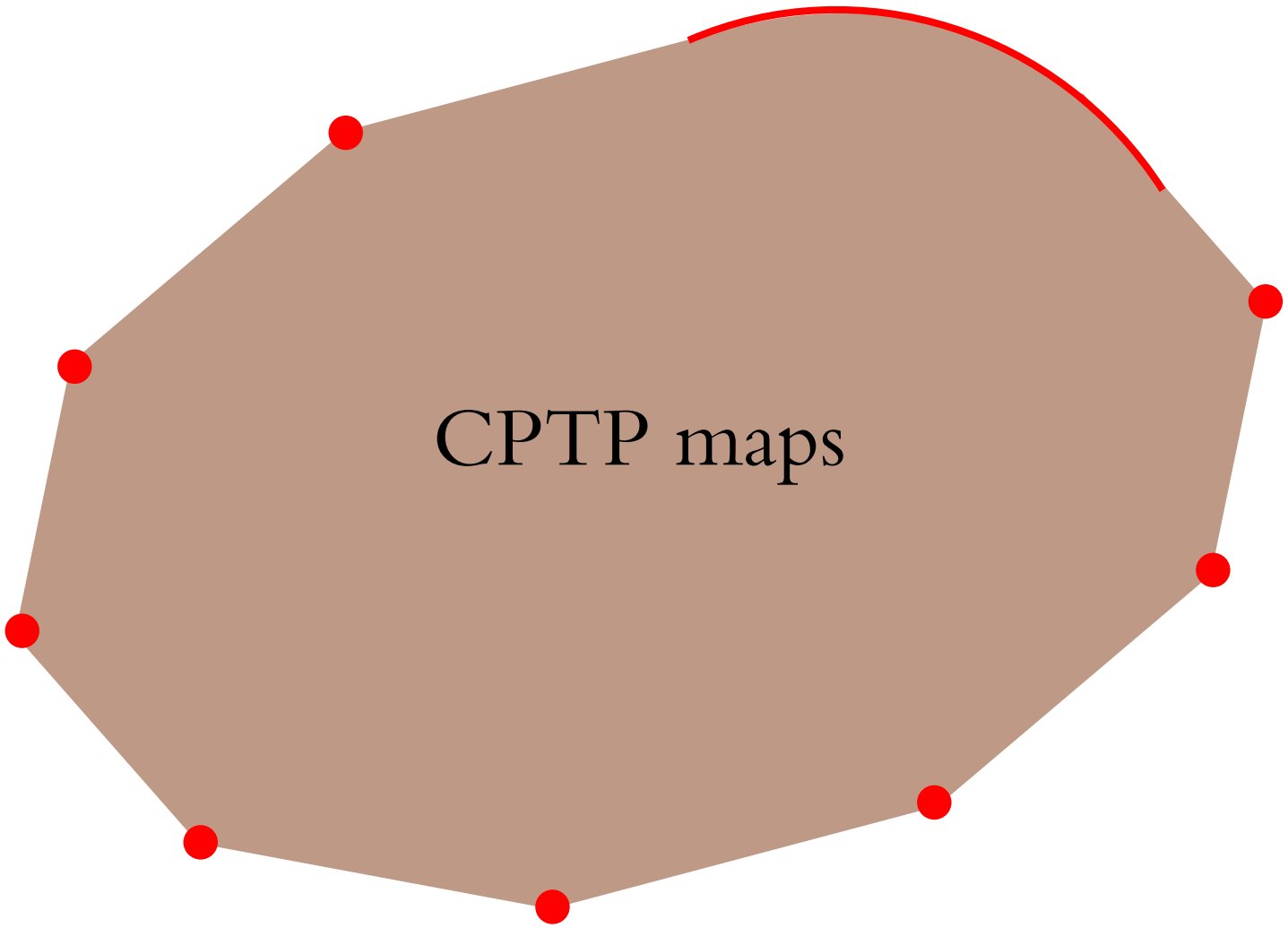


$d > 2$

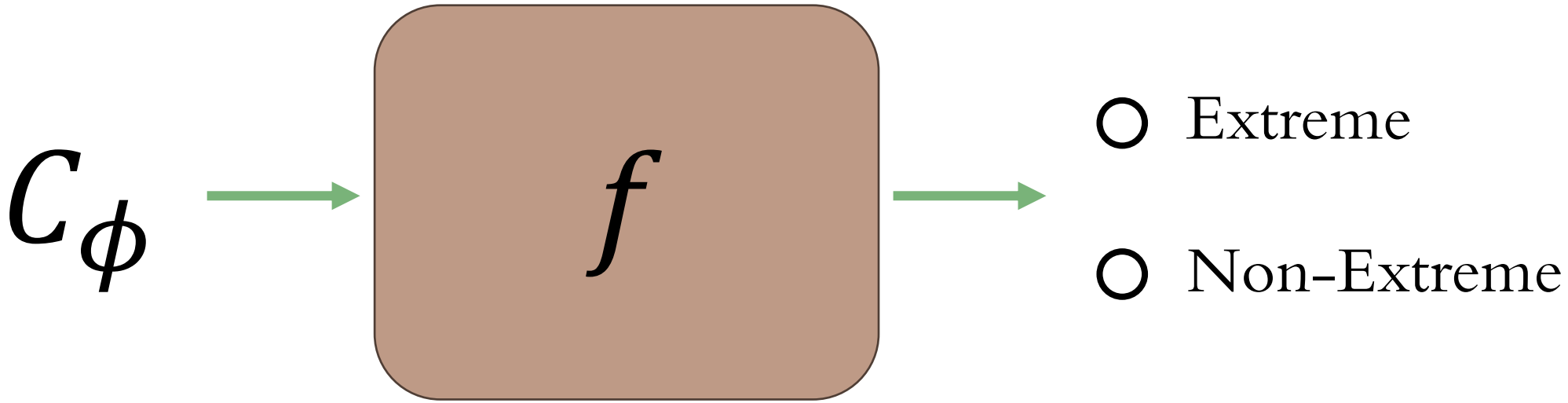


Wolf, M. M., Eisert, J., Cubitt, T. S., & Cirac, J. I. (2008). Assessing Non-Markovian quantum dynamics. Physical Review Letters, 101(15).

The Problem



The Main Problem



حل مسئله به صورت الگوریتمی

Choi Theorem, Unital and non-Unital
Extreme Qubit Channels

Choi Theorem

$$\phi(\rho) = \sum_{i=1}^{K} A_i \rho A_i{}^{\dagger} \qquad \left\{A_i{}^{\dagger} A_j\right\}_{i,j=1}^{K}$$

Linear Independence of $\left\{A_i{}^{\dagger} A_j\right\}_{i,j=1}^{K} \quad \Leftrightarrow \quad \phi \; is \; Extreme$

Choi, M. (1975). Completely positive linear maps on complex matrices. Linear Algebra and Its Applications, 10(3), 285–290.

Complexity

$$O(d^6) + O(d^{4.6}) \sim O(d^6)$$

Find $\{A_i\}_{i=1}^{K}$

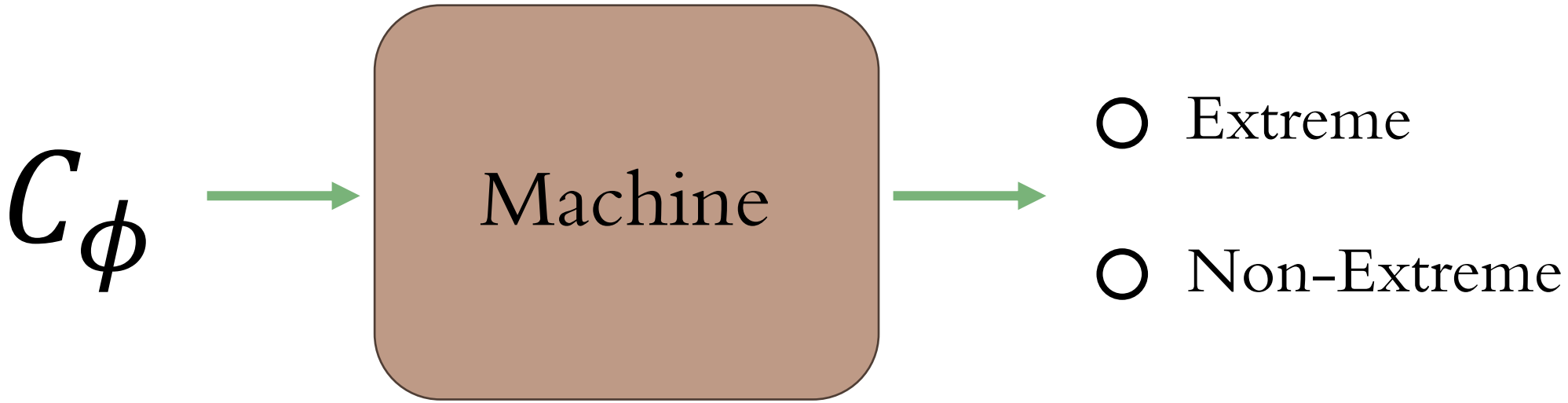
(Numerical)

Linear Independence of $\{A_i{}^\dagger A_j\}_{i,j=1}^{K}$

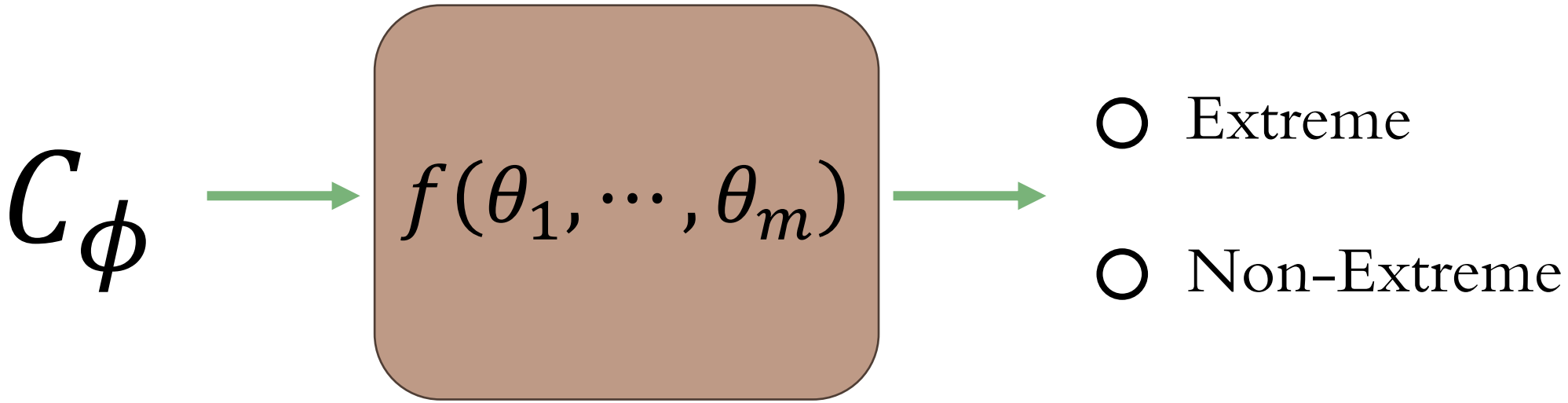
حل مسئله با روش شبکه‌های عصبی

Neural Networks

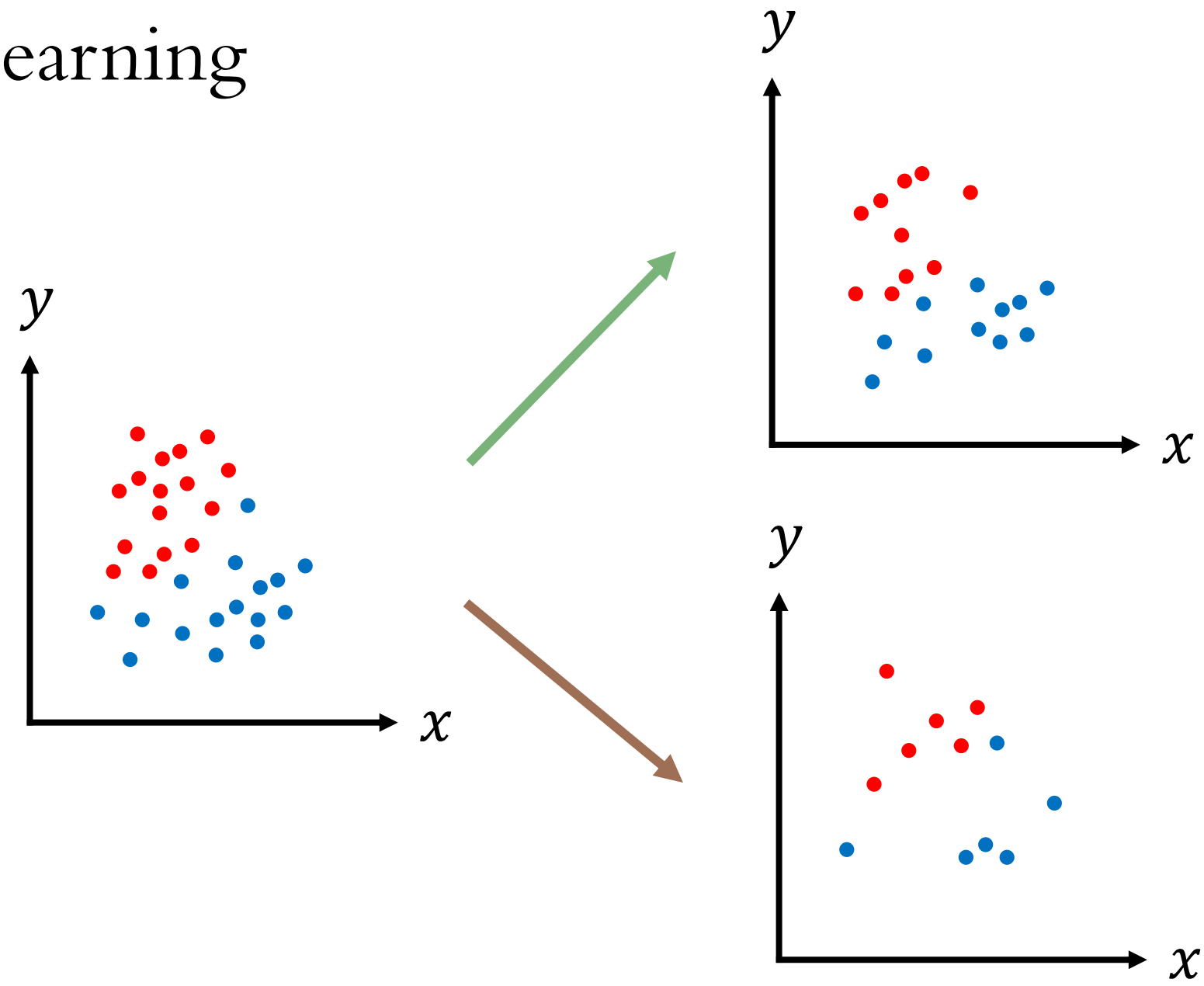
Extreme Classifier



Extreme Classifier



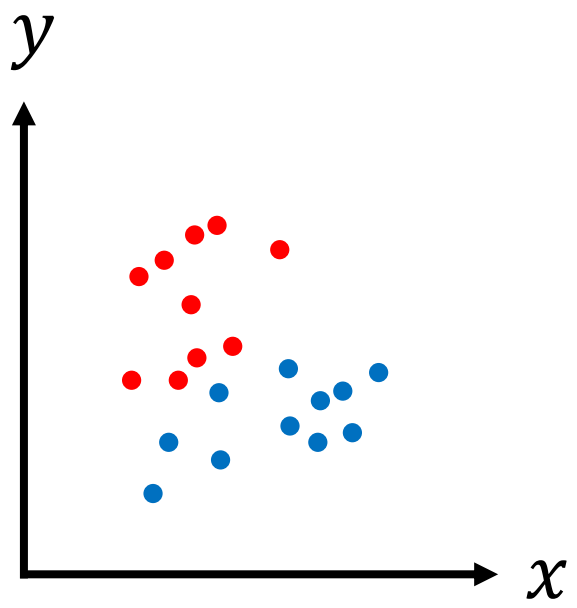
Learning



Training set

Test set

Learning



Training set

$$f_{\theta_1,\cdots,\theta_m}(x,y)$$

Training Loss

Learning

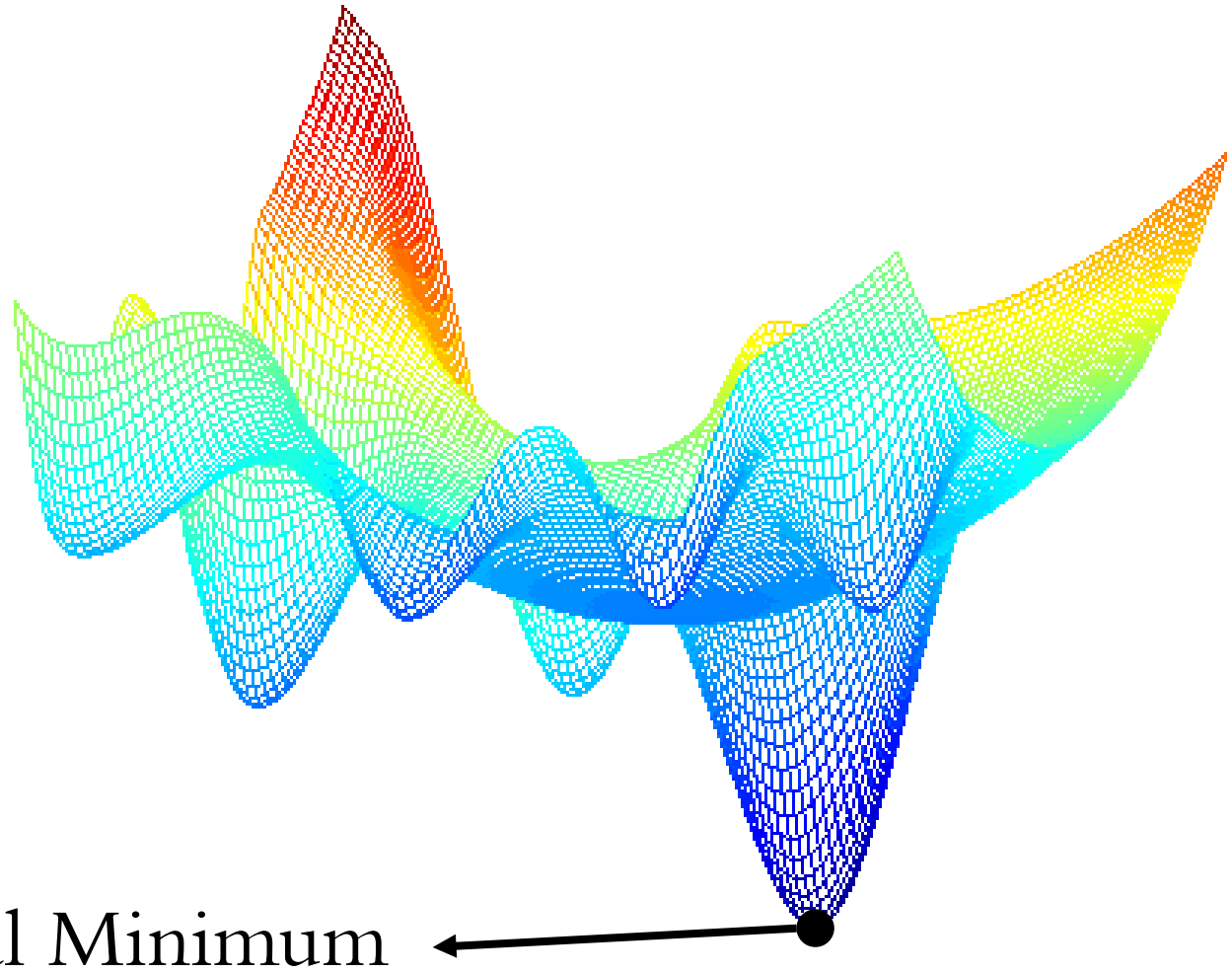
Training Loss

$$\mathcal{L} = \left\| f_{\theta_1,\cdots,\theta_m}(x, y) - L(x, y) \right\|$$

$(x, y) \in$ Train set

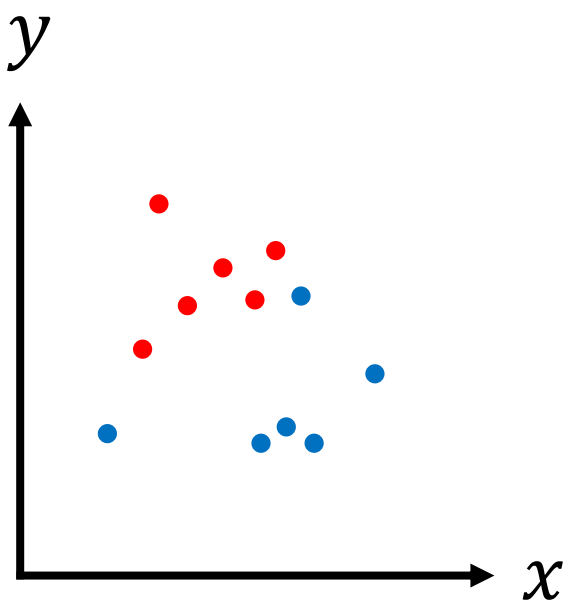
Learning (Optimization)

$$\mathcal{L} \to 0 \quad \Leftrightarrow$$



https://www.aic.fel.cvut.cz/research-areas/optimization

Evaluation



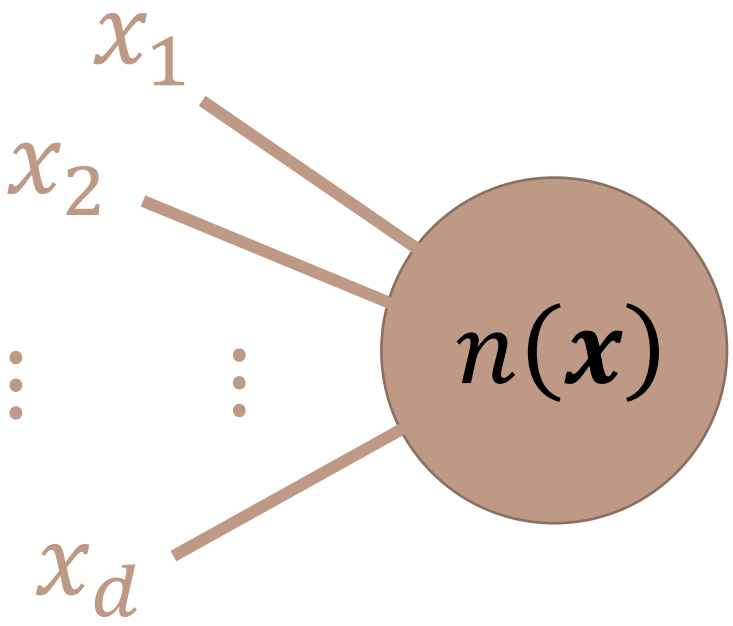
Test set

$$f_{\theta_1,\cdots,\theta_m}(x,y)$$

Test Loss

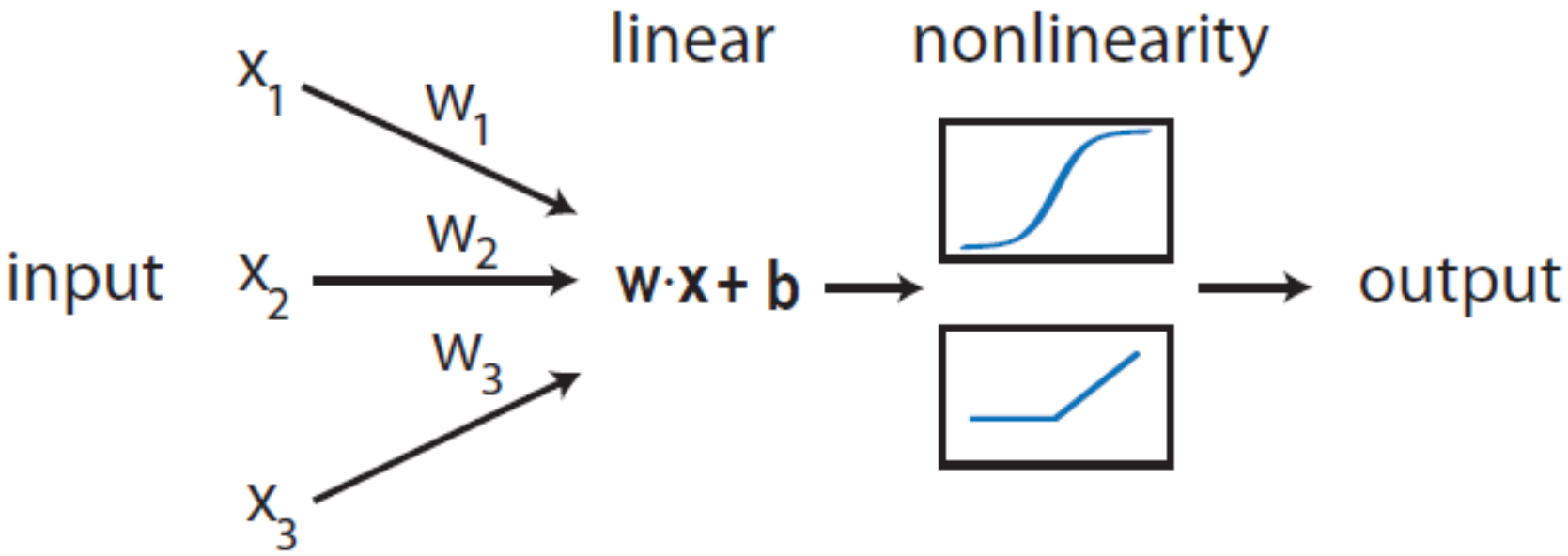
https://www.aic.fel.cvut.cz/research-areas/optimization

A Neuron



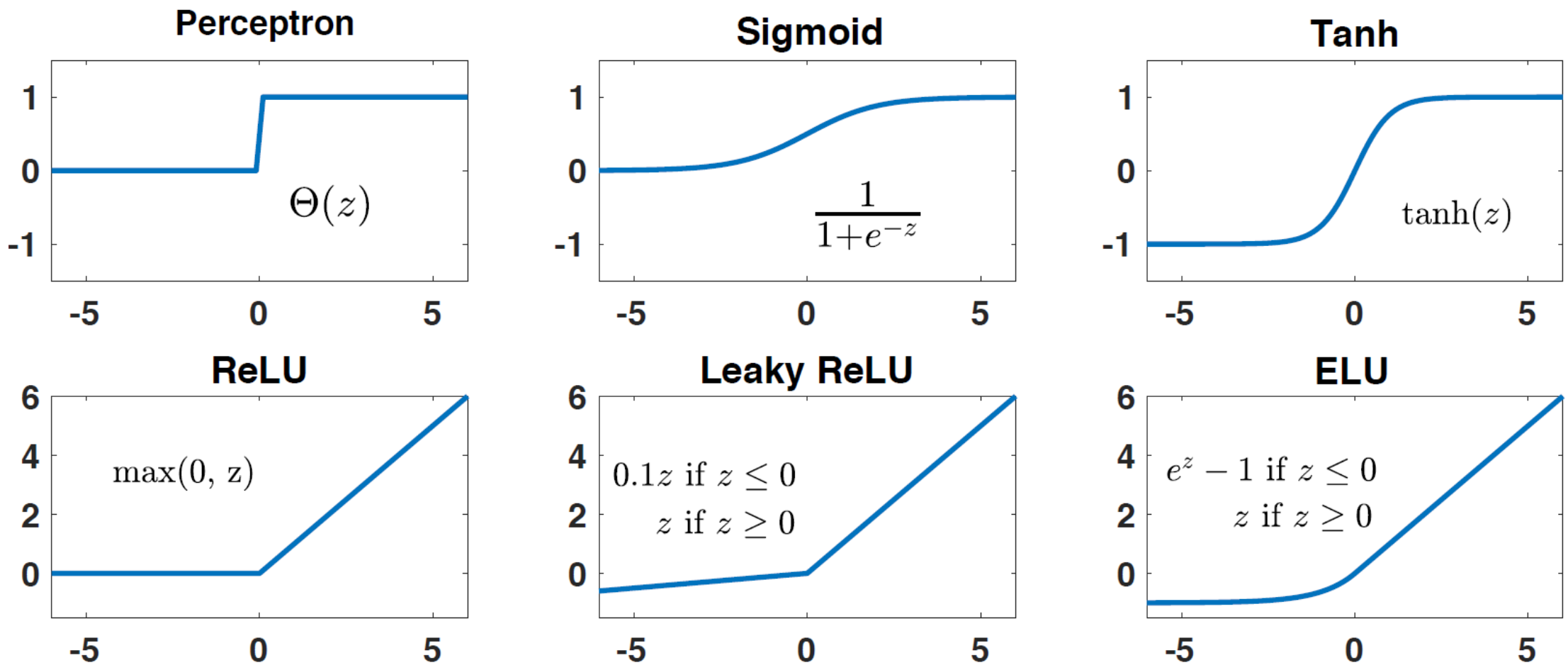
$$n(\boldsymbol{x}) \in \mathbb{R}$$

A Neuron (Mathematical Description)



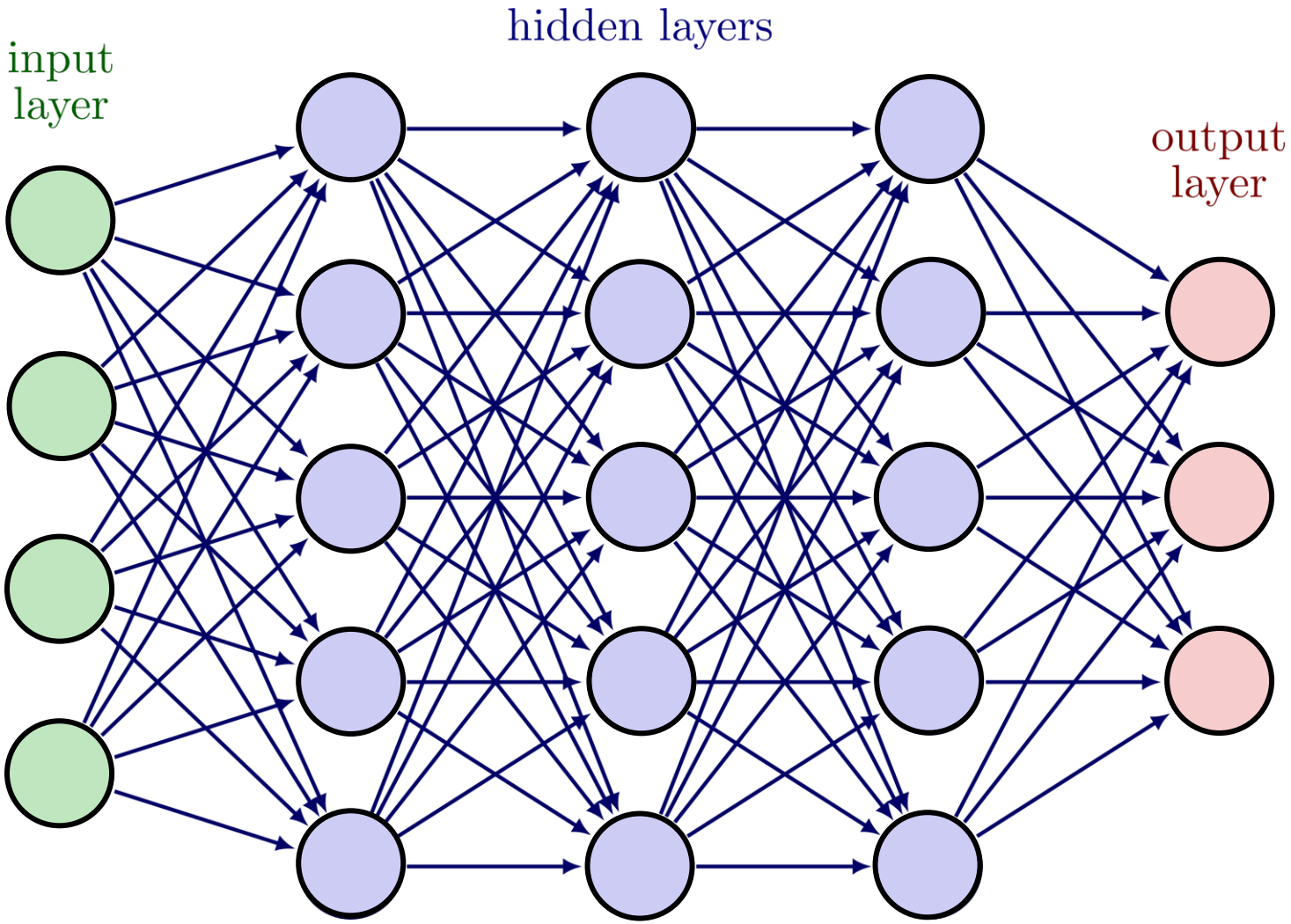
Mehta, P., Bukov, M., Wang, C., Day, A. G. R., Richardson, C. C., Fisher, C. G., & Schwab, D. J. (2019). A high-bias, low-variance introduction to Machine Learning for physicists. Physics Reports, 810, 1–124.

Possible non-linear activation function



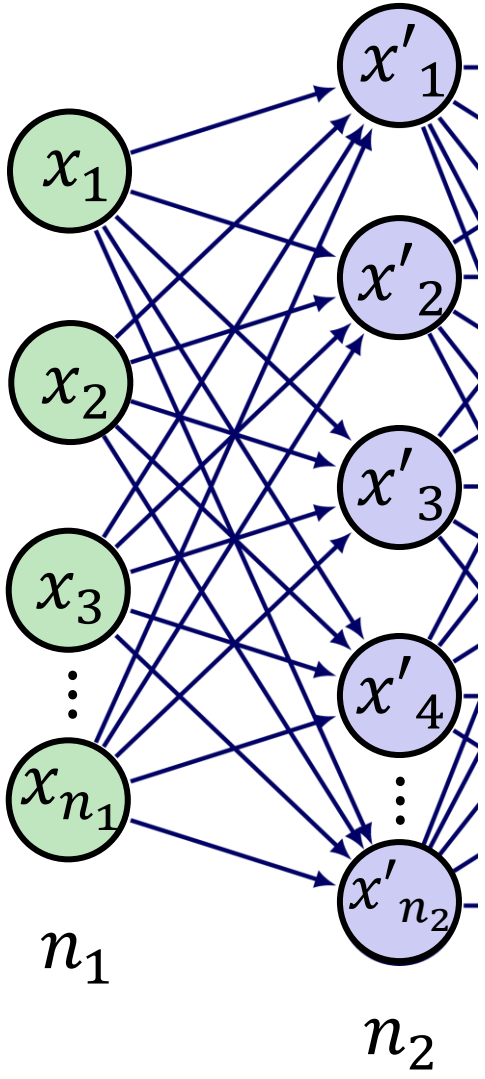
Mehta, P., Bukov, M., Wang, C., Day, A. G. R., Richardson, C. C., Fisher, C. G., & Schwab, D. J. (2019). A high-bias, low-variance introduction to Machine Learning for physicists. Physics Reports, 810, 1–124.

A Neural Network (NN)



https://tikz.net/neural_networks/

A Neural Network (NN)

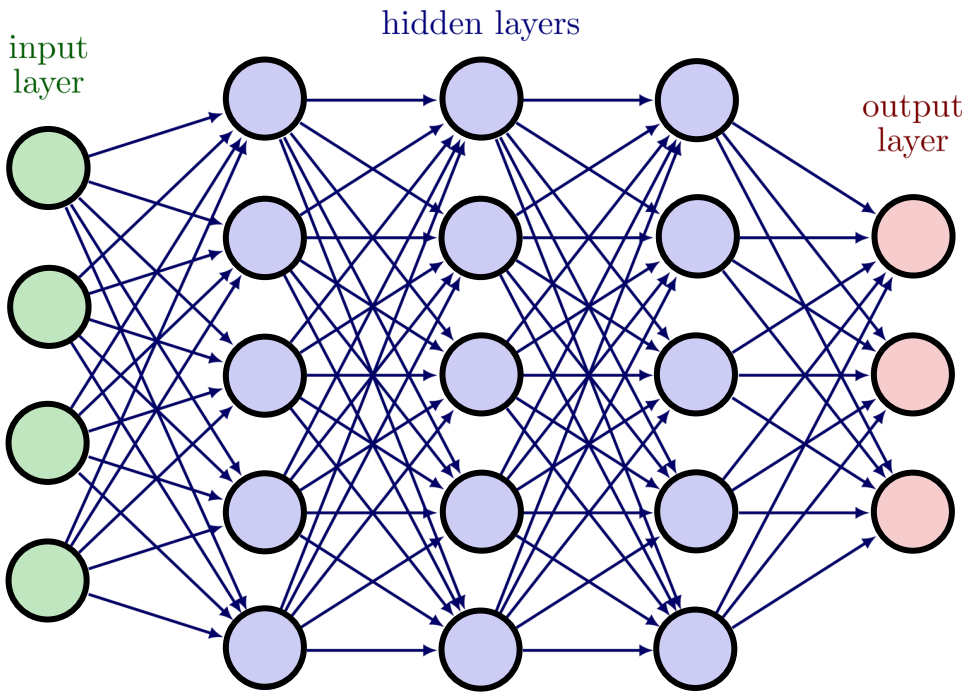


$$x'_i = a(\boldsymbol{w_i} \cdot \boldsymbol{x_i} + b_i) \quad \Leftrightarrow \quad \boldsymbol{x}' = a(w\boldsymbol{x} + \boldsymbol{b})$$

$$w = \begin{pmatrix} \boldsymbol{w}_1 \\ \vdots \\ \boldsymbol{w}_{n_2} \end{pmatrix}_{n_2 \times n_1} \qquad \boldsymbol{b} = \begin{pmatrix} b_1 \\ \vdots \\ b_{n_2} \end{pmatrix}_{n_2 \times 1}$$

https://tikz.net/neural_networks/

Learn a NN

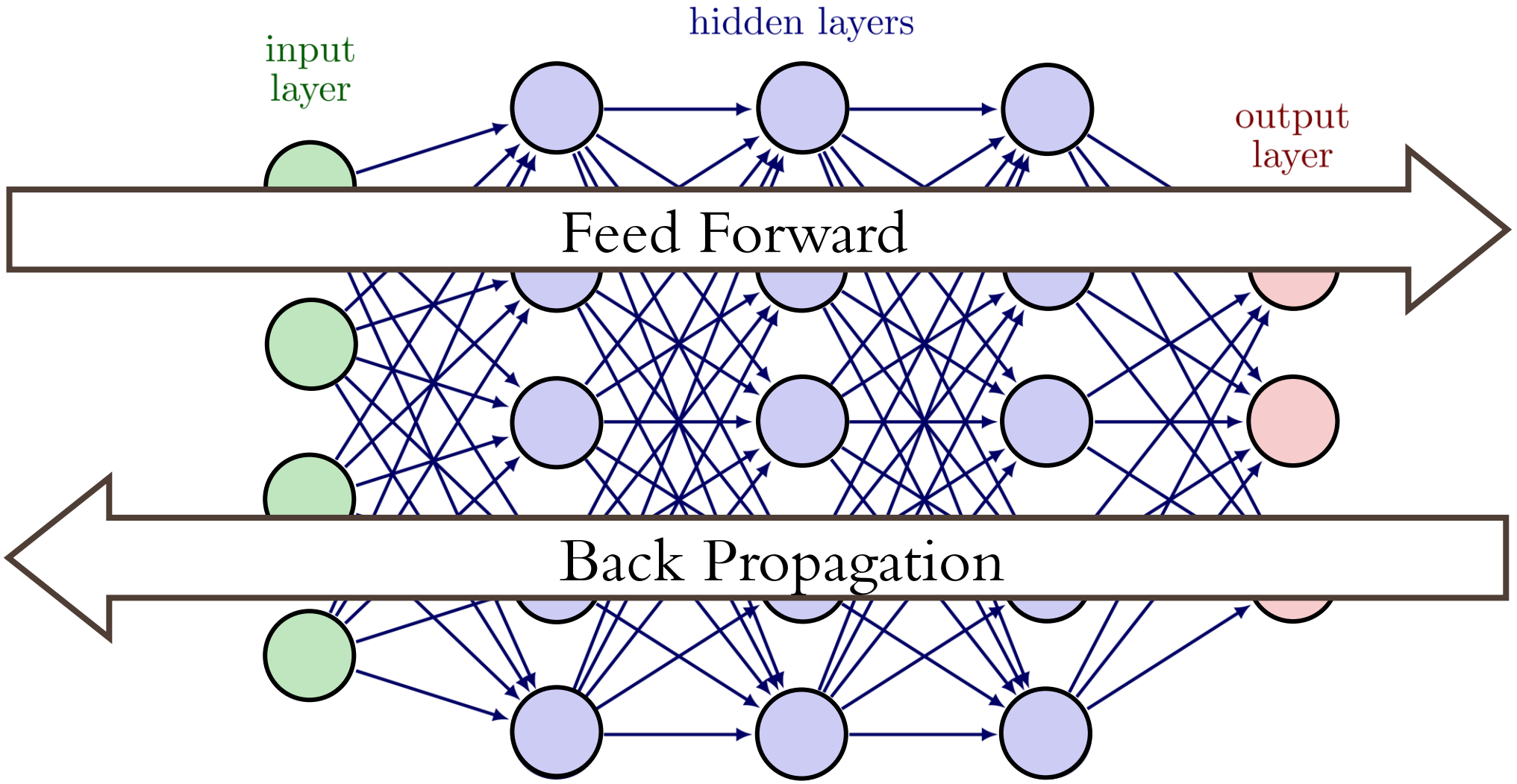


$\Leftrightarrow$

$$f_{w_i, \boldsymbol{b}_i}(\boldsymbol{x}_{\text{in}})$$

https://tikz.net/neural_networks/

Learn a NN



$\mathcal{L}$

Adam

https://tikz.net/neural_networks/

Universal Approximation Theorem

- A neural network with a single hidden layer can approximate any continuous, multi-input/multi-output function with arbitrary accuracy.

Mehta, P., Bukov, M., Wang, C., Day, A. G. R., Richardson, C. C., Fisher, C. G., & Schwab, D. J. (2019). A high-bias, low-variance introduction to Machine Learning for physicists. Physics Reports, 810, 1–124.

Complexity for Training a NN

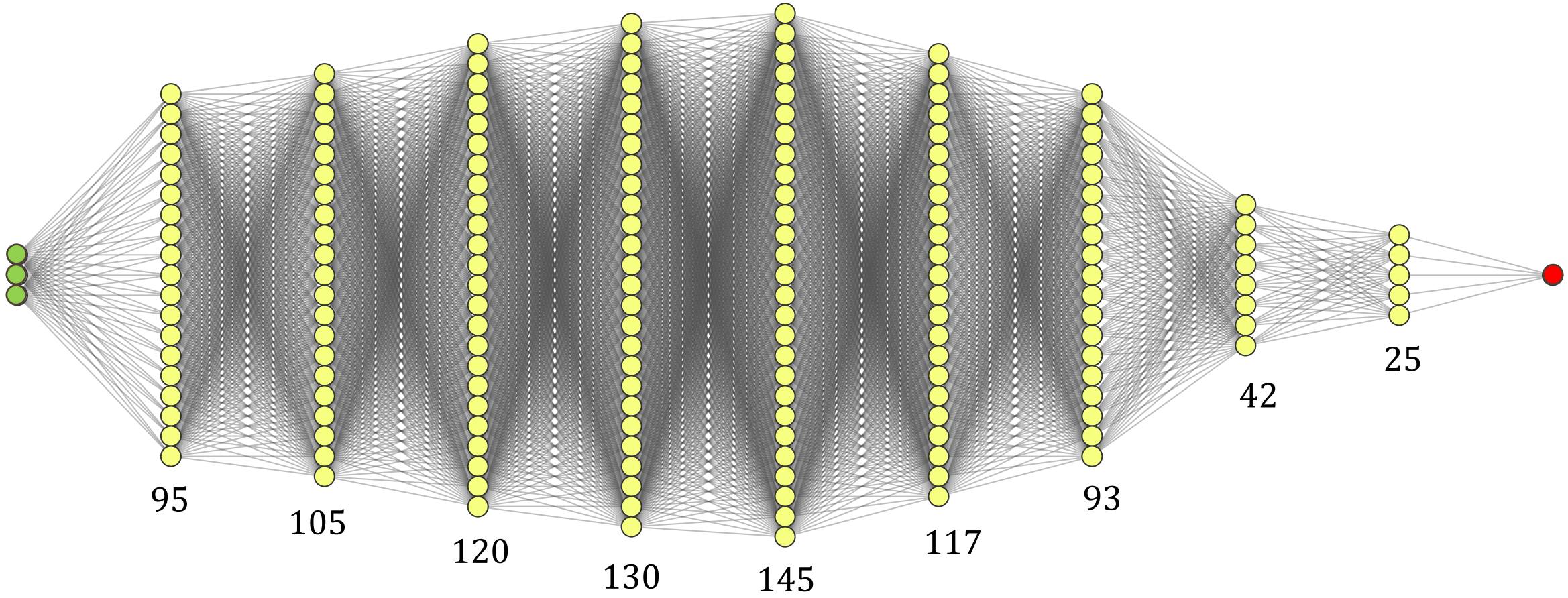
$$O\left(n_{train} \times n_{epoch} \times \sum_{i=0}^{l-1} n_i n_{i+1}\right)$$

Mehta, P., Bukov, M., Wang, C., Day, A. G. R., Richardson, C. C., Fisher, C. G., & Schwab, D. J. (2019). A high-bias, low-variance introduction to Machine Learning for physicists. Physics Reports, 810, 1–124.

حل مسئله با روش شبکه‌های عصبی (نتایج)

Qubit Extreme Detection,
Qutrit Extreme Detection

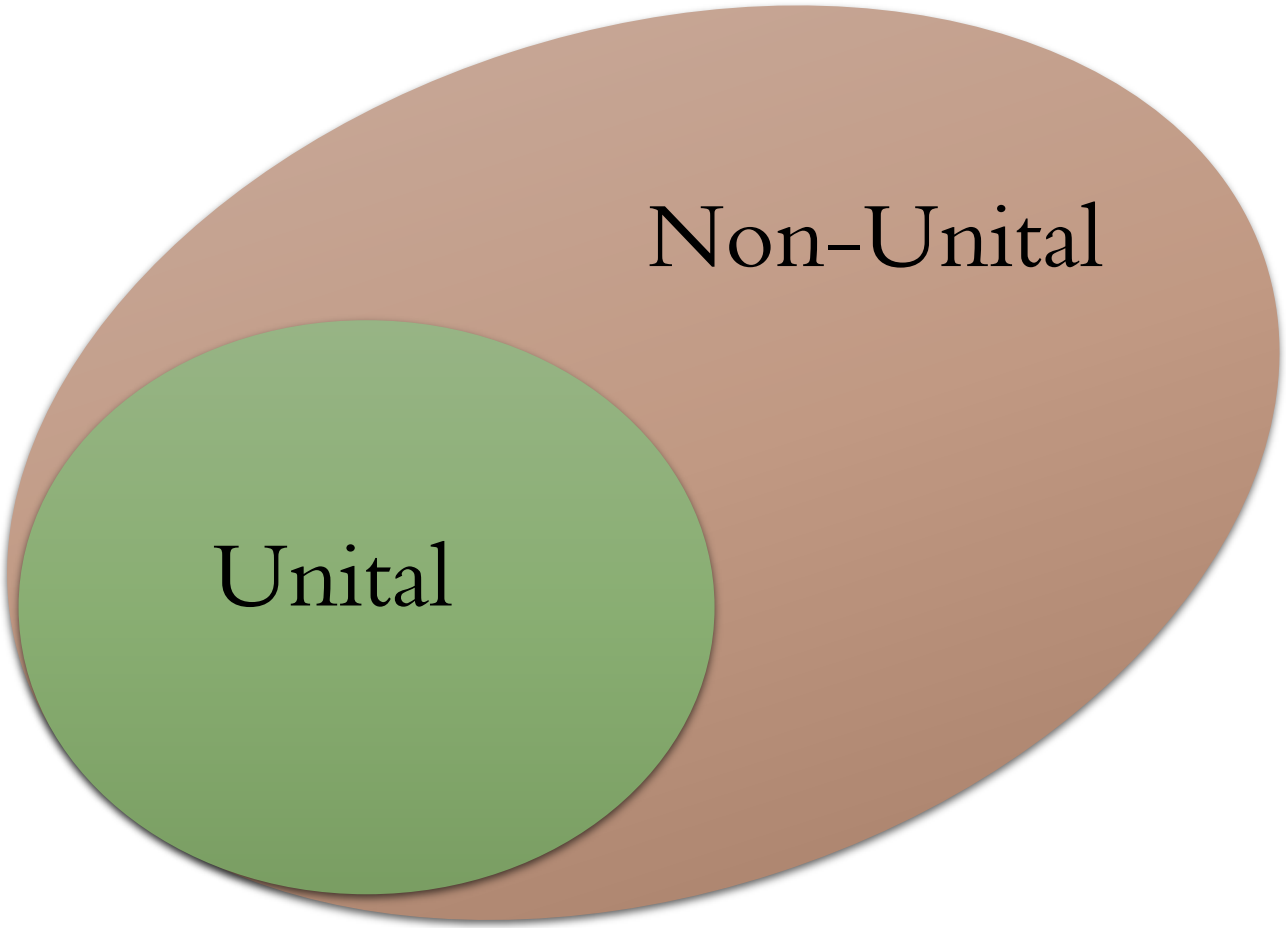
Architecture of our NN:



$$\# \, parameters = 106\,115$$

Qubit Extreme Channels

Extremal Qubit Channels (d=2)



Extremal Qubit Channels (d=2)

Unital

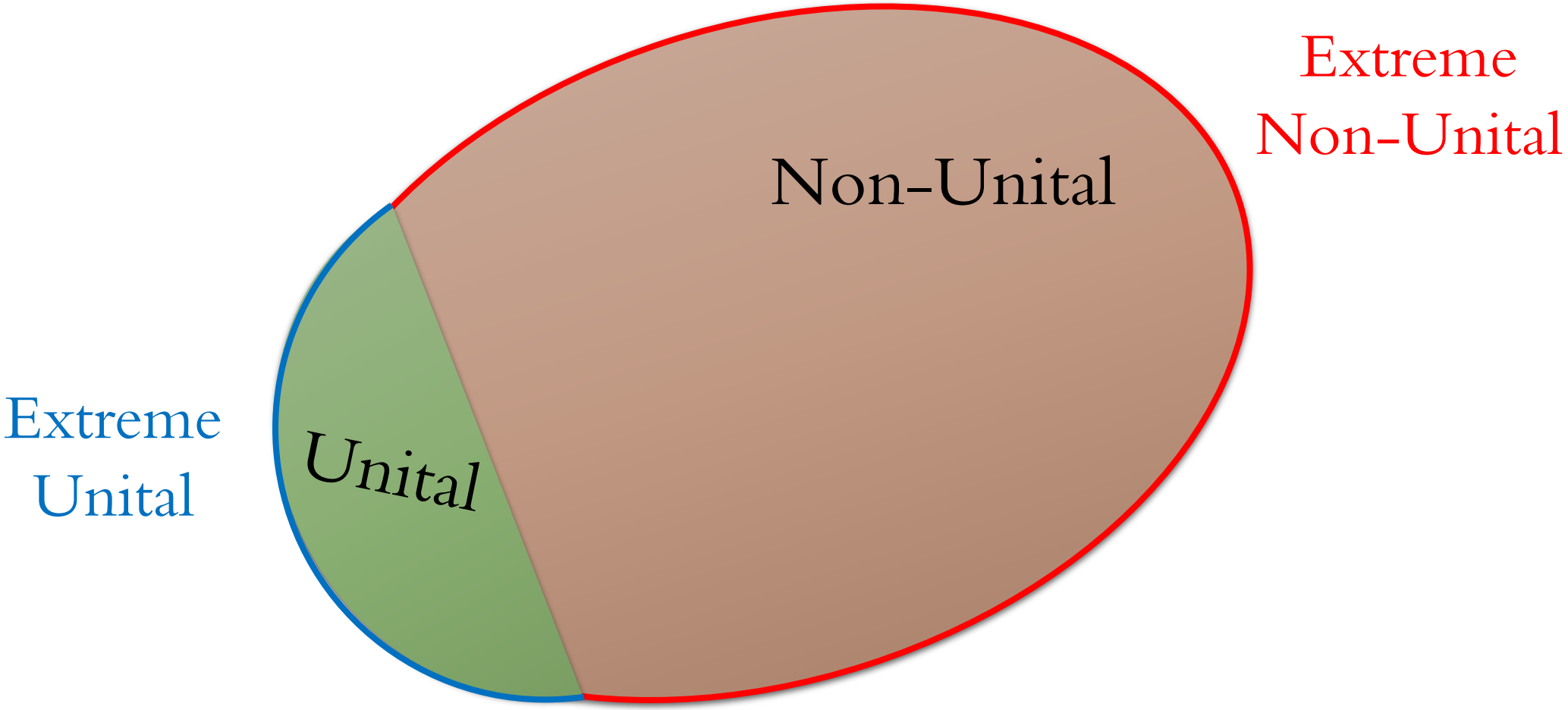
$$\mathcal{U}(\rho) = U\rho U^{\dagger}$$

$$U \in \mathrm{SU}(2)$$

Ruskai, M. B., Szarek, S. J., & Werner, E. M. (2002). An analysis of completely-positive trace-preserving maps on M2. Linear Algebra and Its Applications, 347(1–3), 159–187.

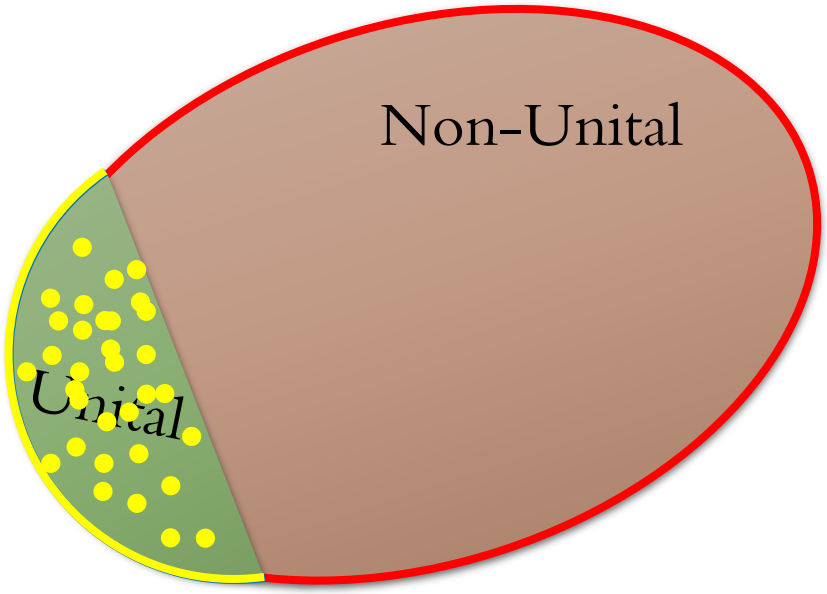
Mendl, C. B., & Wolf, M. S. (2009). Unital Quantum Channels – convex structure and revivals of Birkhoff's Theorem. Communications in Mathematical Physics, 289(3), 1057–1086.

Extremal Qubit Channels (d=2)



Detection of Extremal Qubit Channels using NN

Training set

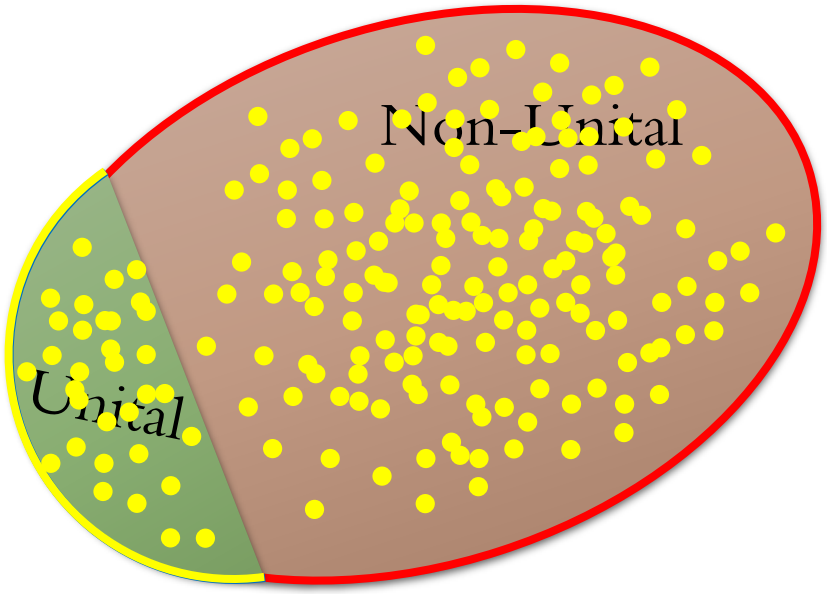


Test Accuracy = 99.6%

Test Accuracy
Of Non-unital Extreme = 12.6%

Detection of Extremal Qubit Channels using NN

Training set

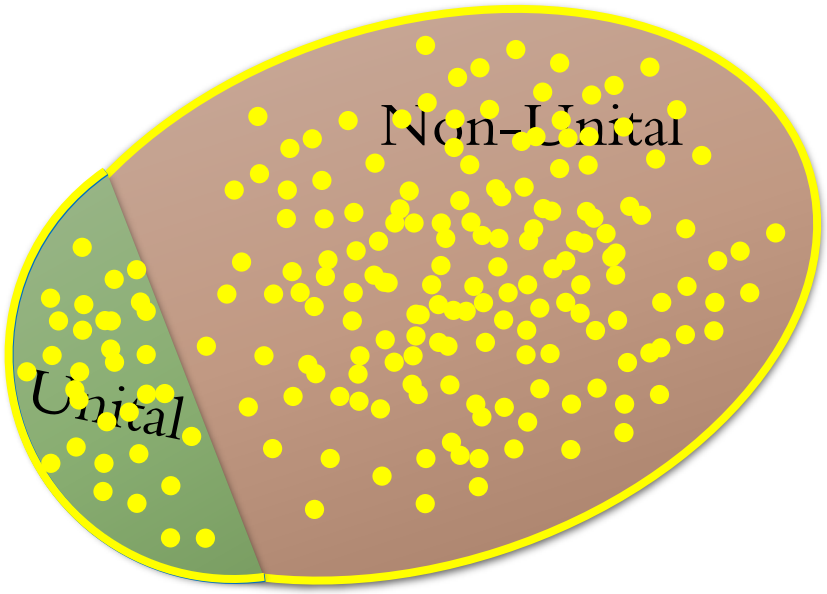


Test Accuracy = 100%

Test Accuracy
Of Non-unital Extreme = 17.7%

Detection of Extremal Qubit Channels using NN

Training set



Test Accuracy = 100%

Test Accuracy
Of Non-unital Extreme = 100%

Comparison of Complexities

Algorithmic

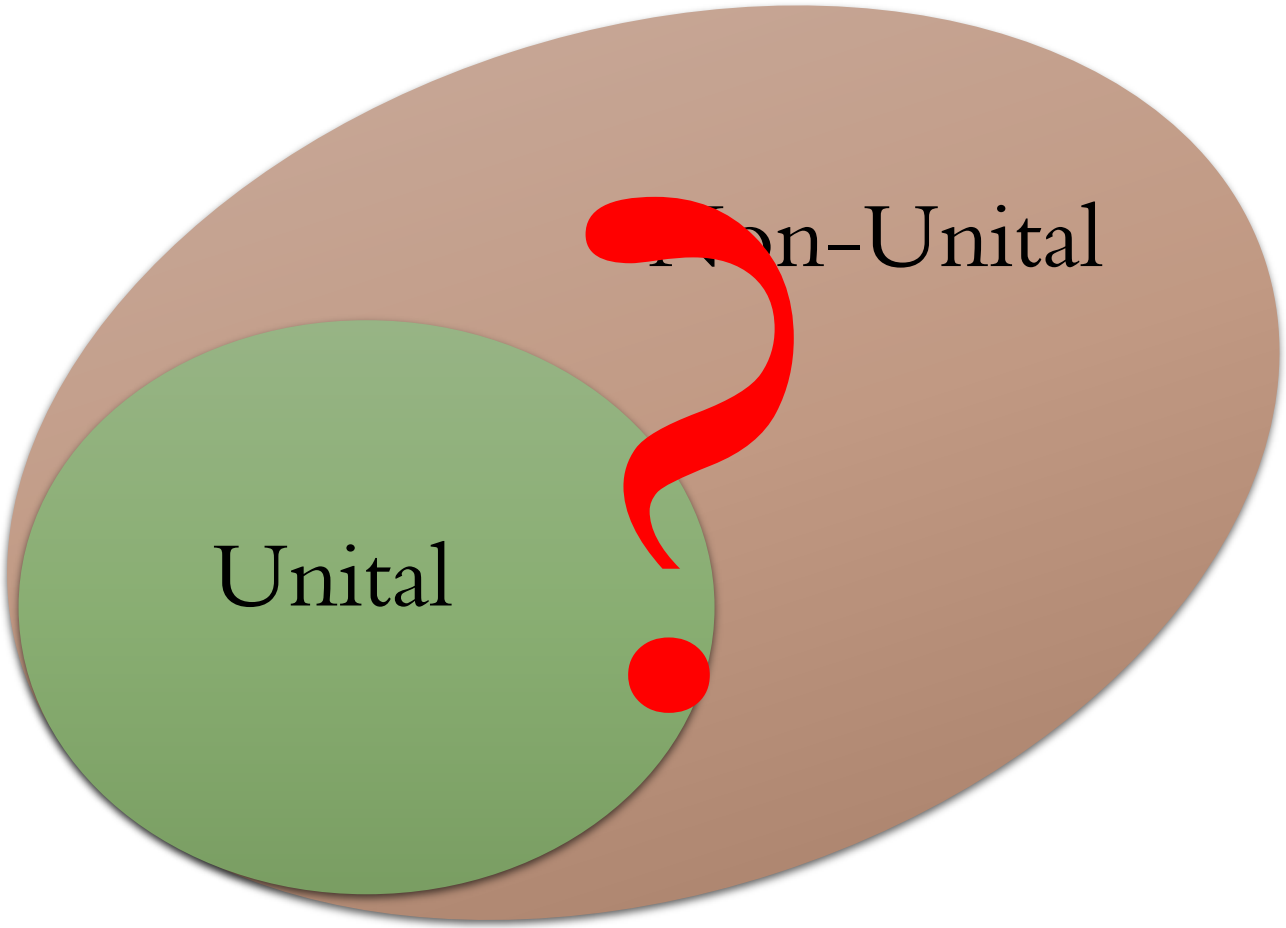
$$O(d^6) \sim O(2^6)$$

NN

$$O(d^4) \sim O(2^4)$$

Qutrit Extreme Channels

Extremal Qudit Channels (d>2)



Extremal Qutrit Channels (d=3)

$$A_1 = \begin{pmatrix} 0 & 0 & 0 \\ \sqrt{\alpha} & 0 & 0 \\ \sqrt{\beta} & 0 & 0 \end{pmatrix}, \qquad A_2 = \begin{pmatrix} \sqrt{1-\alpha-\beta} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$B_1 = \begin{pmatrix} 0 & \sqrt{\alpha} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \qquad B_2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \sqrt{1-\alpha} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$C_1 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & \sqrt{\alpha} & 0 \end{pmatrix}, \qquad C_2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \sqrt{1-\alpha} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Detection of Extremal Qutrit Channels (d=3)

Training set

$$A_1 = \begin{pmatrix} 0 & 0 & 0 \\ \sqrt{\alpha} & 0 & 0 \\ \sqrt{\beta} & 0 & 0 \end{pmatrix}, \quad A_2 = \begin{pmatrix} \sqrt{1-\alpha-\beta} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$B_1 = \begin{pmatrix} 0 & \sqrt{\alpha} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad B_2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \sqrt{1-\alpha} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$C_1 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & \sqrt{\alpha} & 0 \end{pmatrix}, \quad C_2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \sqrt{1-\alpha} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Test Accuracy = 100%

Test Accuracy Of C = 100%

Detection of Extremal Qutrit Channels (d=3)

Training set

$$A_1 = \begin{pmatrix} 0 & 0 & 0 \\ \sqrt{\alpha} & 0 & 0 \\ \sqrt{\beta} & 0 & 0 \end{pmatrix}, \quad A_2 = \begin{pmatrix} \sqrt{1-\alpha-\beta} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$B_1 = \begin{pmatrix} 0 & \sqrt{\alpha} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad B_2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \sqrt{1-\alpha} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$C_1 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & \sqrt{\alpha} & 0 \end{pmatrix}, \quad C_2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \sqrt{1-\alpha} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Test Accuracy = 100%

Test Accuracy Of B = 100%

Detection of Extremal Qutrit Channels (d=3)

Training set

$$A_1 = \begin{pmatrix} 0 & 0 & 0 \\ \sqrt{\alpha} & 0 & 0 \\ \sqrt{\beta} & 0 & 0 \end{pmatrix}, \qquad A_2 = \begin{pmatrix} \sqrt{1-\alpha-\beta} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$B_1 = \begin{pmatrix} 0 & \sqrt{\alpha} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \qquad B_2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \sqrt{1-\alpha} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$C_1 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & \sqrt{\alpha} & 0 \end{pmatrix}, \qquad C_2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \sqrt{1-\alpha} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Test Accuracy = 100%

Test Accuracy Of A = 0.8%

Detection of Extremal Qutrit Channels (d=3)

Training set

$$A_1 = \begin{pmatrix} 0 & 0 & 0 \\ \sqrt{\alpha} & 0 & 0 \\ \sqrt{\beta} & 0 & 0 \end{pmatrix}, \quad A_2 = \begin{pmatrix} \sqrt{1-\alpha-\beta} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$B_1 = \begin{pmatrix} 0 & \sqrt{\alpha} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad B_2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \sqrt{1-\alpha} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

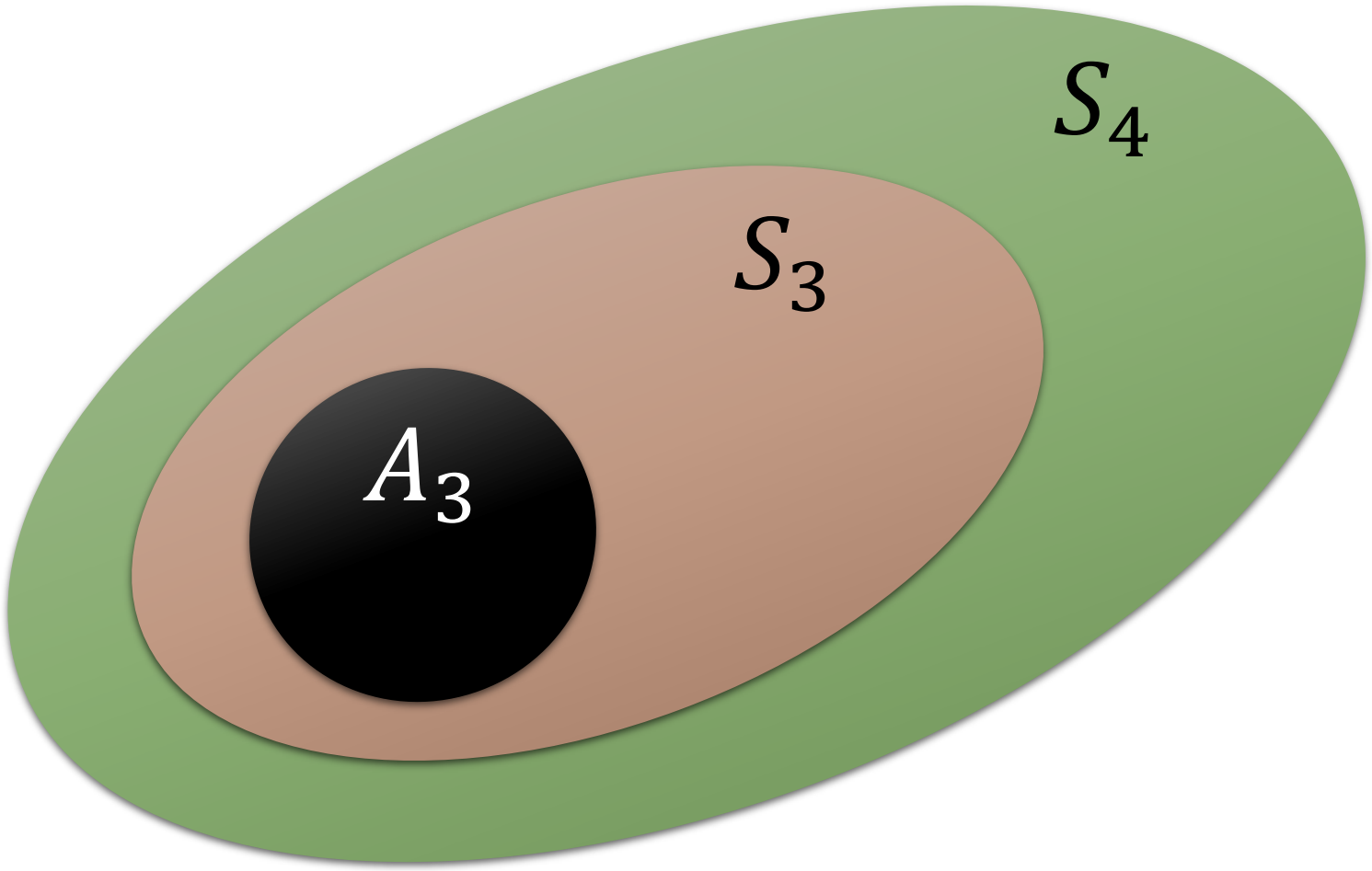
$$C_1 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & \sqrt{\alpha} & 0 \end{pmatrix}, \quad C_2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \sqrt{1-\alpha} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Test Accuracy = 100%

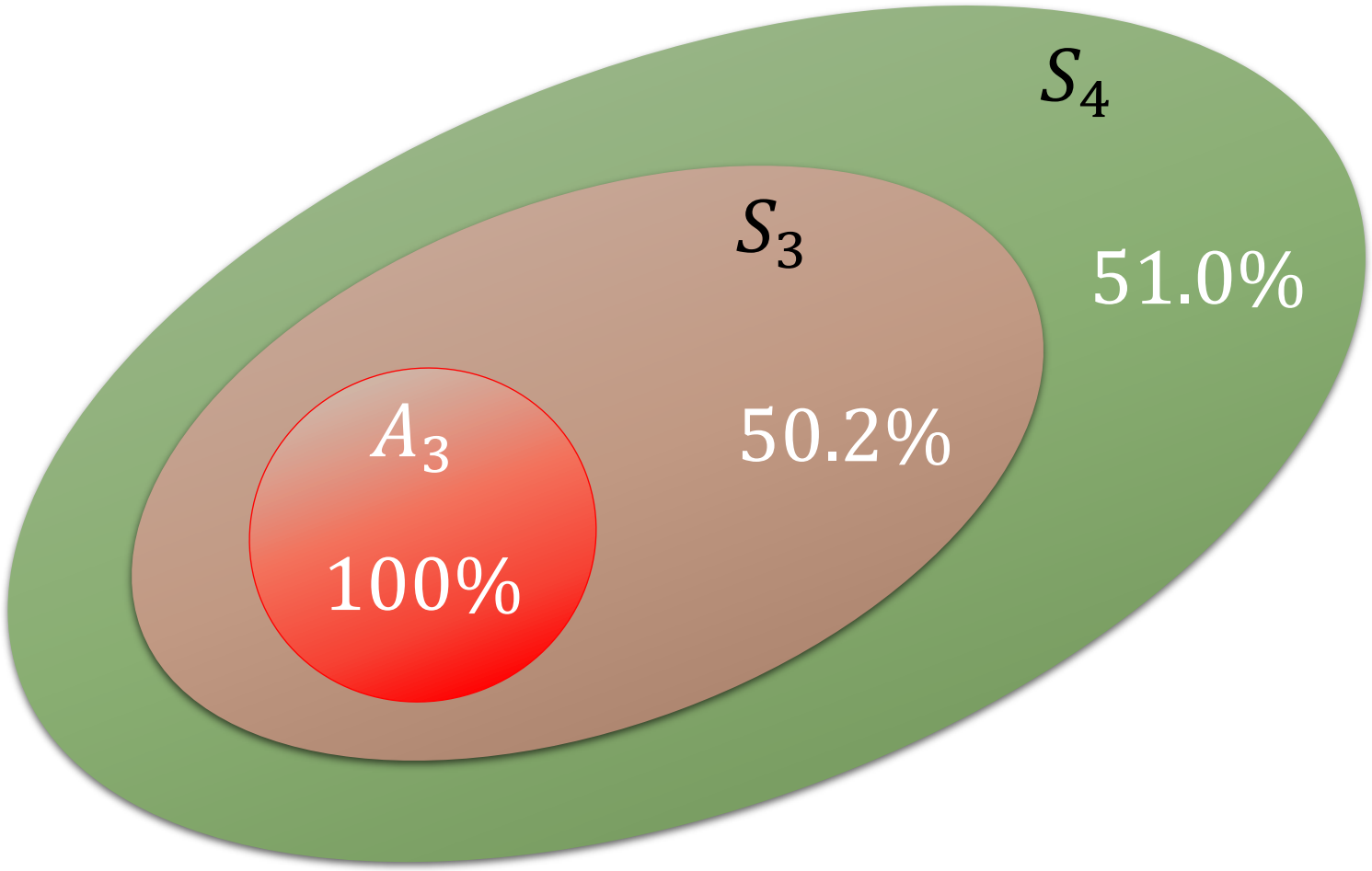
Test Accuracy Of B = 73.2%

Test Accuracy Of C = 94.5%

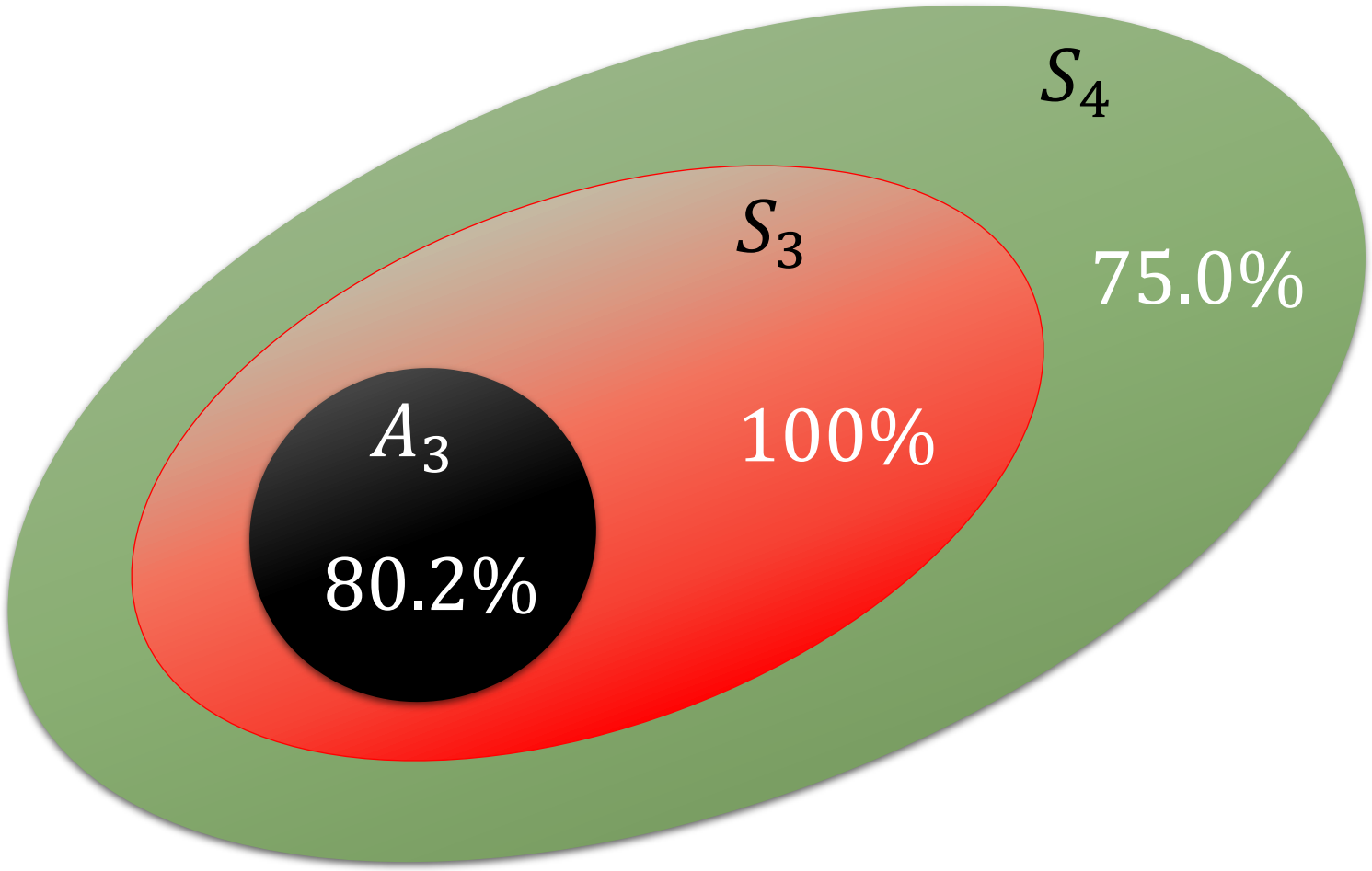
Extremal G-covariant Qutrit Channels



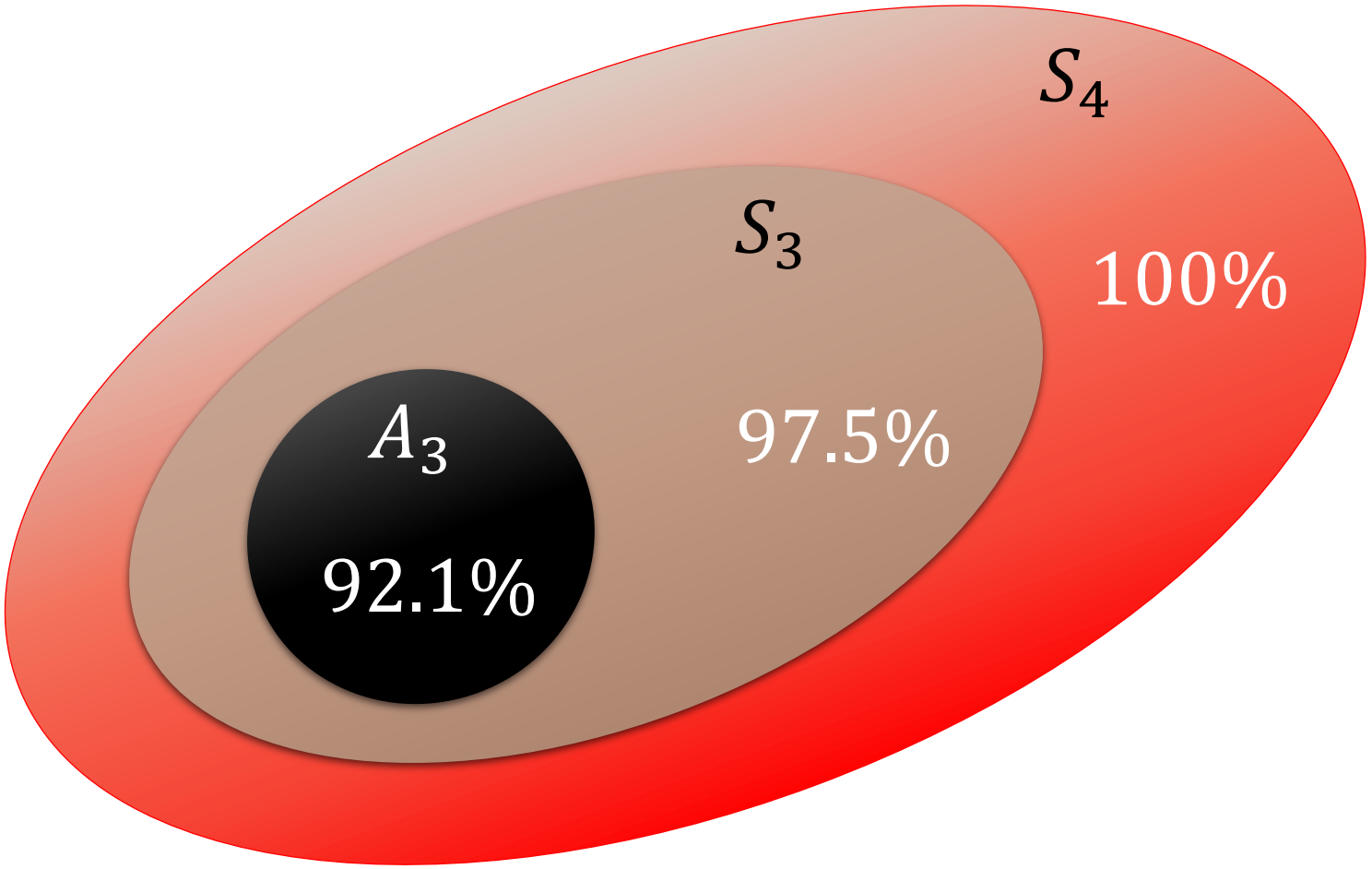
Detection of Extremal G-covariant Qutrit Channels



Detection of Extremal G-covariant Qutrit Channels



Detection of Extremal G-covariant Qutrit Channels



Comparison of Complexities

Algorithmic

$$O(d^6) \sim O(3^6)$$

NN

$$O(d^4) \sim O(3^4)$$

آینده‌پژوهی

آینده پژوهی

- دادن وابستگی بُعد به لایه‌های پنهان و بررسی عملکرد شبکه‌ی عصبی در بُعدهای بالاتر
- تولید کانال‌های اکستریم با استفاده از شبکه‌های عصبی مولد

تشکر از توجه شما!

پیوست

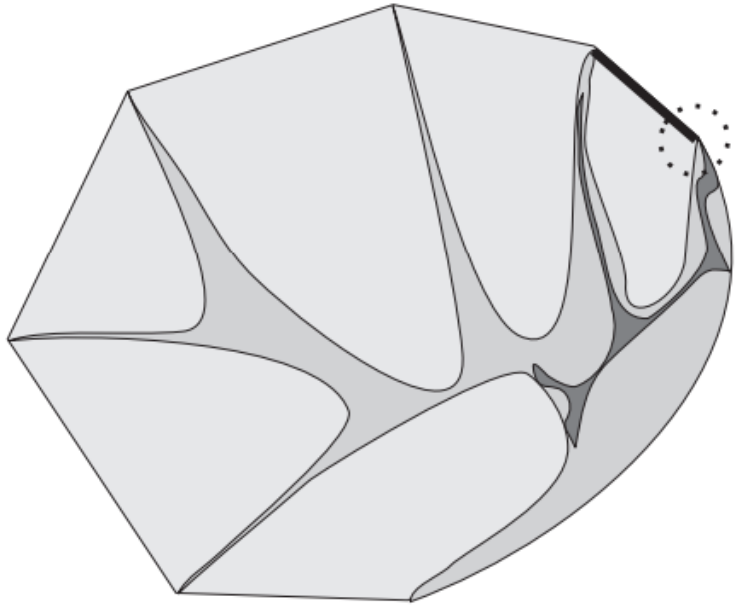


FIG. 1: Schematic depiction of the (12-dimensional) convex set of qubit channels. The (dark grey) subset of Markovian channels is non-convex and contains 2% of the channels. The larger still non-convex set of time-dependent Markovian channels (17%) contains all extremal channels. All sets, including the measure zero set of indivisible channels (black line) can be found in the neighborhood of the identity (dotted circle).

Extremal Qubit Channels (d=2)

Non-Unital

$$T_\phi = \begin{pmatrix} 1 & 0_{1\times 3} \\ \boldsymbol{t}_\phi & M_\phi \end{pmatrix} \rightarrow M_\phi = \begin{pmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{pmatrix} \quad , \boldsymbol{t}_\phi = \begin{pmatrix} 0 \\ 0 \\ t_3 \end{pmatrix}$$

$$\lambda_3 = \lambda_1 \lambda_2$$

$$t_3^2 = (1-\lambda_1^2)(1-\lambda_2^2) \qquad , t_1 = t_2 = 0$$

$$\begin{tikzcd} 1 \arrow[r] & 2 \arrow[dl] \\ & 3 \arrow[ul] \end{tikzcd}$$

Ruskai, M. B., Szarek, S. J., & Werner, E. M. (2002). An analysis of completely-positive trace-preserving maps on M2. Linear Algebra and Its Applications, 347(1–3), 159–187.

Mendl, C. B., & Wolf, M. S. (2009). Unital Quantum Channels – convex structure and revivals of Birkhoff's Theorem. Communications in Mathematical Physics, 289(3), 1057–1086.